Particle cosmology lectures Kenji Kadota (CTPU, IBS)

Plan of Lectures

1. Introduction to Standard Cosmology

Brief History of the Universe
FRW cosmology
Thermodynamics in the Expanding Universe

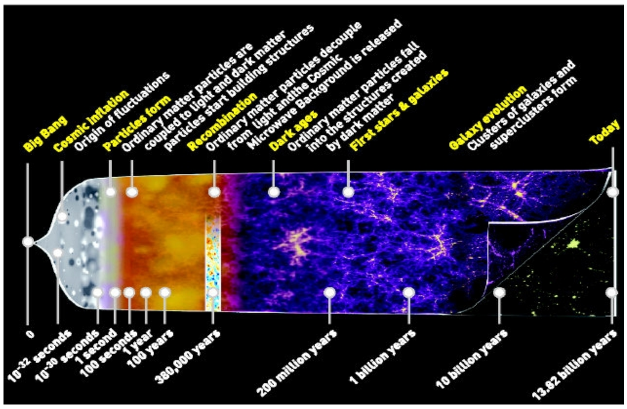
2. The Early Universe Phenomenology

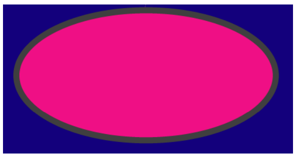
Inflation
Big Bang Nucleosynthesis
Baryogenesis/Leptogenesis

3. CMB and Large Scale Structure of the Universe

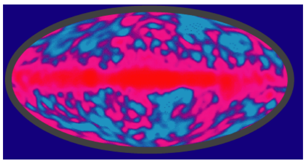
Cosmic Microwave Backgrounds
Baryon Acoustic Oscillations

History of the Universe





$\Delta T/T=10^{-3}$

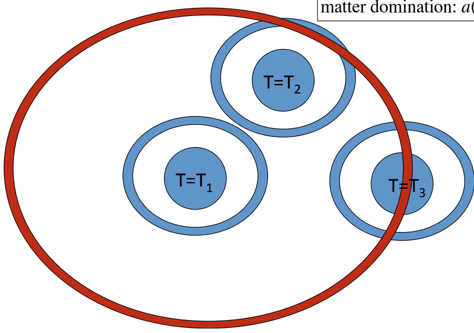


$\Delta T/T=10^{-5}$
(seeds structure
formation of the Universe)

Horizon problem

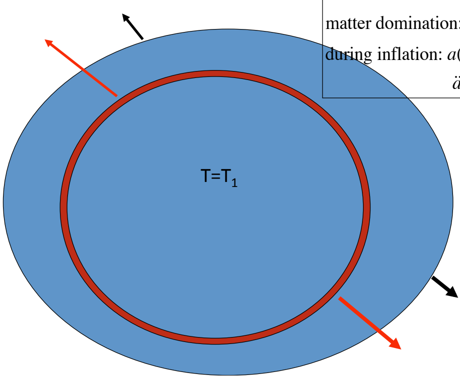
$$ds^2 = dt^2 - a^2(t)dx^2$$

Causally connected region: $L_H \propto ct$
Physical volume size: $L_p \propto a(t)$
radiation domination: $a(t) \propto t^{1/2}$
matter domination: $a(t) \propto t^{2/3}$

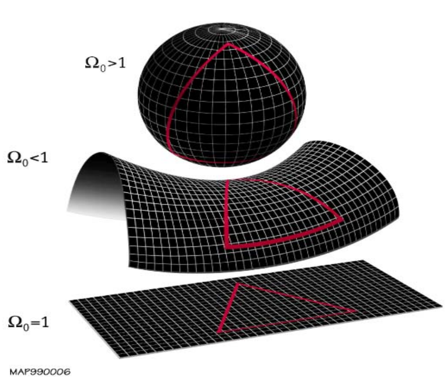


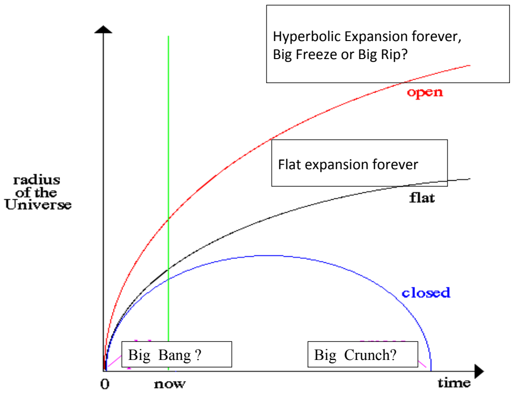
$$ds^2 = dt^2 - a^2(t)dx^2$$

Causally connected region: $L_H \propto ct$
Physical volume size: $L_p \propto a(t)$
radiation domination: $a(t) \propto t^{1/2}$
matter domination: $a(t) \propto t^{2/3}$
during inflation: $a(t) \propto t^{\gamma}, \gamma > 1$
$\ddot{a}(t) \propto \gamma(\gamma - 1)$

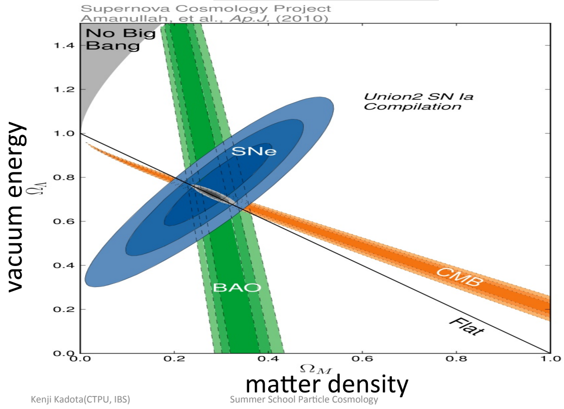


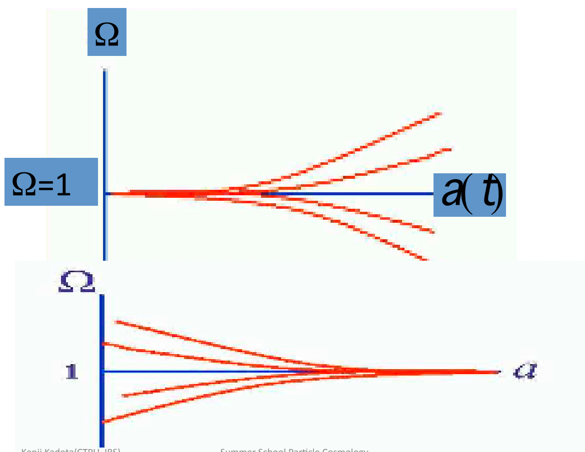
Flatness problem





Flatness problem





Flatness problem

$$a(t) \propto t^{\gamma}, \gamma < 1 \qquad \ddot{a}(t) \propto \gamma(\gamma - 1)$$

$$\dot{a}^2(t)(\Omega(t)-1) = \text{const} = \dot{a}^2(t = t_{\text{now}})(\Omega(t_{\text{now}})-1)$$

$$\Omega(t_{\text{now}}) = 1.003^{+0.013}_{-0.017}$$

Recall from Lecture 1

$$\frac{\Omega_{early}^{-1}-1}{\Omega_0^{-1}-1} \sim \frac{T_0}{T_{eq}}\left(\frac{T_{eq}}{T_{early}}\right)^2$$

Ω(t)≡energy/(critical energy density)

Flatness Problem: Why flat today?

Flat today $\Rightarrow$ Even flatter before!

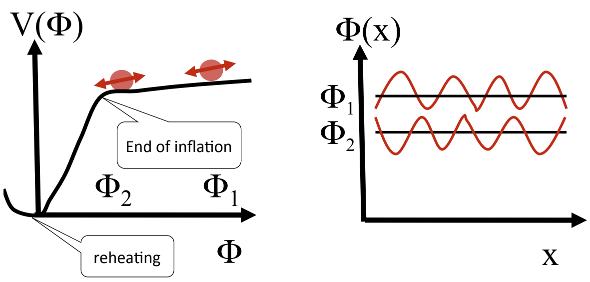
Inflation

- What is inducing the Inflation?

Candidates:

- Constant term (no symmetry prohibits it)
- Fermion (not enough vacuum energy)
- Boson (let it mimic a constant term!)

Slow-roll inflation by a scalar field ("inflaton")



Inflation

- Nice idea

Can explain

seeds of structure formation, horizon problem, flatness problem etc.

- Currently inflation is “dynamics”

- The realization in a sensible particle theory is another story.

e.g. Inflation by right-handed scalar neutrino
(Murayama et al ’94, Ellis et al ’04, Antusch et al ’05, KK & Yokoyama ’06, …)

Motivated from the neutrino physics.
Economical:
matter/antimatter asymmetry of the Universe too (leptogenesis).
SUSY dark matter.

What do we need to accelerate the Universe?

SLOW ROLL !!! Required condition: $\dot{\phi}^2 < V$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(E+3p)$$

Equation of state parameter $w \equiv \frac{p}{E} < -\frac{1}{3}$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), E = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

e.g. Cosmological constant Λ

$$w = -1, p = -\Lambda = -E$$

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\Lambda \Rightarrow a \propto \exp(\sqrt{\Lambda}t)$$

$$E + 3p = 2\left(\dot{\phi}^2 - V\right) < 0 \qquad \dot{\phi}^2 < V$$

Potential energy need dominate potential energy

Scalar field dynamics

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$$

$$g_{\mu\nu} = diag(1, -a(t)^2, -a(t)^2, -a(t)^2)$$

$$\text{determinant: } g \equiv \left|\det g_{\mu\nu}\right|$$

$$\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = (-g)^{-1/2}\partial_\mu[(-g)^{1/2}g^{\mu\nu}\partial_\nu\phi]$$

Klein-Gordon equation

$$\Box\phi + \frac{\partial V}{\partial\phi} = 0$$

$$\ddot{\phi} - \nabla^2\phi + 3H\dot{\phi} + \frac{\partial V}{\partial\phi} = 0$$

(e.g. during inflation, the non-zero k modes redshifted away)

$\nabla^2\phi$ spatial gradient term neglected in considering the spatially homogeneous classical solution

Friedmann equation $H^2 = \frac{1}{3}\rho = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$

ex: Show that a scalar field oscillating in a quadratic potential behaves as pressureless dust $\rho \propto 1/a^3$

ex: A scalar field oscillating in a quadratic potential behaves as pressureless dust $\rho \propto 1/a^3$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

For H<<m, $\phi(t) \propto \exp[\pm imt - (3/2)Ht]$

Using the virial theorem, $\langle V \rangle = \langle \dot{\phi}^2/2 \rangle = \rho_\phi/2$

$$\langle P \rangle = \langle \dot{\phi}^2/2 - V \rangle = 0$$

$$\langle \rho \rangle = \langle \dot{\phi}^2/2 \rangle + \langle V \rangle \sim \langle \dot{\phi}^2 \rangle \sim e^{-3Ht}$$

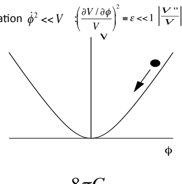
$$H = \frac{da/dt}{a} \to a \sim e^{Ht}$$

Number of e-folds $N \equiv \ln\frac{a(t_{end})}{a(t)} = \int_t^{t_{end}} H\,dt$
(the space is stretched by a factor e^N)

Spatially coherent oscillation for the quadratic potential behaves as no-relativistic matter.

Exercise: How about for a quartic potential? Ans. it behaves like the radiation $\rho \propto 1/a^4$

Exercise: Derive the slow roll conditions using the condition for the acceleration $\dot{\phi}^2 << V$: $\left(\frac{\partial V/\partial\phi}{V}\right)^2 \equiv \varepsilon << 1$ $\left|\frac{V''}{V}\right| = |\eta| << 1$



$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial\phi} = 0$$

Hubble dragging

Slow-roll approximation:

$$3H\dot{\phi} + \frac{\partial V}{\partial\phi} = 0 \to \dot{\phi}^2 \sim \frac{\left(\frac{\partial V}{\partial\phi}\right)^2}{H^2} \sim \frac{\left(\frac{\partial V}{\partial\phi}\right)^2}{V} \qquad H^2 \sim \frac{8\pi G}{3}V$$

Required condition for acceleration: $\dot{\phi}^2 << V \to \left(\frac{\partial V/\partial\phi}{V}\right)^2 \equiv \varepsilon << 1$

$$3H\dot{\phi} + \frac{\partial V}{\partial\phi} = 0 \Rightarrow 3\dot{H}\dot{\phi} + 3H\ddot{\phi} + \frac{\partial^2 V}{\partial\phi^2}\dot{\phi} = 0 \Rightarrow \dot{H} \sim -V'' \qquad \dot{H} = -4\pi G(\rho + P) = -4\pi G\dot{\phi}^2$$

Required condition for acceleration: $\dot{\phi}^2 << V \to \dot{H} << V \qquad V'' << V \to \left|\frac{V''}{V}\right| \equiv |\eta| << 1$

$N = \ln\frac{a(t_{end})}{a(t)} = \int_t^{t_{end}} H\,dt$ During the inflation, the number of e-folds ~exp[50] may be required

ex: Calculate the slow-roll parameters evaluated at N=50 from the end of inflation
for $V = m^2\phi^2/2$

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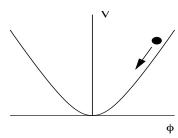
for $V = m^2\phi^2/2$

$$\varepsilon = \eta = 2/\phi^2$$

$$\varepsilon \text{ or } \eta \sim 1 \text{ for } \phi_{end} \sim \sqrt{2}M_p$$



$$N \equiv \ln\frac{a(t_{end})}{a(t)} = \int_t^{t_{end}} H\,dt \sim \int_{\phi_{end}}^{\phi}\frac{V}{V'}d\phi \qquad N = 50 \to \phi_* = 14M_{Pl} \qquad \varepsilon \sim \eta \sim 2/14^2 \sim 0.01$$

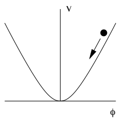
ex: Derive the condition for the eternal inflation for $V = m^2\phi^2/2$

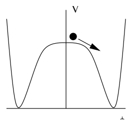
During a Hubble time, $\Delta t = 1/H$ inflaton gets a quantum kick of $H/2\pi$
while classically rolling down by $\dot{\phi}\Delta t \sim \dot{\phi}/H$

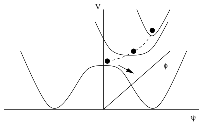
$H >> \dot{\phi}/H \to \frac{V^3}{V'^2} >> 1$ Eternal inflation for $\phi > 1/\sqrt{m}$

From the CMB: $m \sim 10^{13} GeV$

inflation models







$$V = \frac{1}{2}m^2\phi^2 \qquad V = V_0 - \frac{1}{2}m^2\phi^2 + .. \qquad V = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda(\psi^2 - M^2)^2 + \frac{1}{2}\lambda'\phi^2\psi^2$$

Large field inflation | Small field inflation | Hybrid inflation

how much inflation needed?

- ex. How many e-folds are required for the inflation energy scale E?

the horizon size ~1/H:

$$\frac{1}{H_{\rm inf}} \Rightarrow \frac{1}{H_{\rm inf}} \times (\text{stretched by inflation}) \times (\text{post-inflation redshift}) > \frac{1}{H_0}$$

The patch during inflation of order $\frac{1}{H_{\rm inf}}$ needs to cover our current Horizon size $\frac{1}{H_0}$ now

$$\frac{1}{H_{\rm inf}} \sim \frac{1}{\rho_{\rm inf}^{1/2}}, \text{post-inflation redshift} = \frac{E_{\rm inf}}{kT_{CMB}} = \left(\frac{\rho_{r,\rm inf}}{\rho_{r,0}}\right)^{1/4}$$

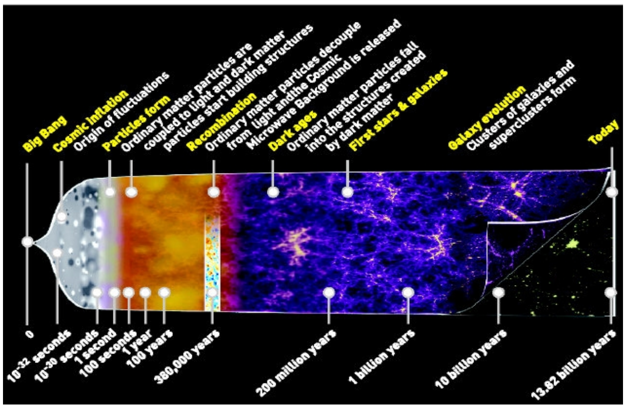
$$\text{The stretch by inflation} > \left(\frac{\rho_{r,\rm inf}}{\rho_{r,0}}\right)^{1/4} = \frac{a_0}{a_{\rm inf}}$$

In other words, the number of inflationary e-folds > post-inflationary e-folds

e.g. GUT scale inflation

$$\Delta N = \ln\left(\frac{a_0}{a_{\rm inf}}\right) = \ln\left(\frac{\rho_{r,\rm inf}}{\rho_{r,0}}\right)^{1/4} \sim \ln\left(\frac{(10^{15}GeV)^4}{(10^{-4}eV)^4}\right)^{1/4} \sim \ln 10^{28} \sim 64$$

History of the Universe



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Big Bang Nucleosynthesis(BBN)

Formation of nuclei (first three minutes of the Universe).
The nucleus formation occurs around 1 MeV which is a typical nuclear binding energy scale.

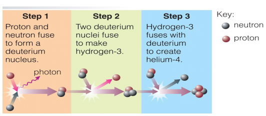
(e.g. The deutrium: D mass=1875.62 MeV and the mass of proton plus neutron 1877.84 MeV. the binding energy is hence 2.22 MeV)

Even though many baryonic matter(p, n, π, Λ) are created after the QCD phase transition (~150 MeV), only protons and neutrons survive till T~1MeV.

p and n then combine to form a long-lasting stable ^{4}He (binding energy~28 MeV) (3 minutes old Universe)

$$e.g.\ ^4He: p + n \to D + \gamma, D + D \to n + {}^3He, {}^3He + D \to p + {}^4He$$

^{4}He has a big binding energy (~28 MeV) and pretty much all neutrons form into ^{4}He.



In the first approximation, no elements heavier than ^{4}He are produced because of large ^{4}He binding energy and the low reaction rate due to small baryon density.

Neutron/Proton ratio

$$\nu + n \Leftrightarrow p + e^-$$
$$e^+ + n \Leftrightarrow p + \bar{\nu}$$

(important because essentially all neutrons end up with Helium 4 most tightly bound nucleus)

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} \exp\left(-\frac{Q_n}{kT}\right) \qquad n_n / n_p \sim 1 \text{ if } m_n = m_p!$$

$$m_n = 939.6\text{MeV} \quad m_p = 938.3\text{MeV}$$

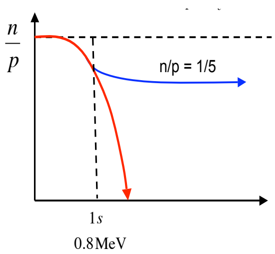
$$Q_n = (m_n - m_p)c^2 = 1.29\text{MeV}$$

Freeze-out:
Weak interaction rate ~Hubble expansion rate

$$G_F^2 T^5 \sim H = T^2$$

$$t \approx 1\text{s} \quad kT \approx 0.8\text{MeV}$$

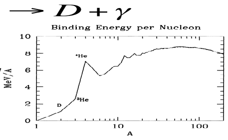
$$\frac{n}{p} = \exp\left(-\frac{1.29}{0.8}\right) \approx \frac{1}{5}$$



Nucleosynthesis starts with deuterium production to from heavier elements

Deuterium production changes n/p further $n + p \longrightarrow D + \gamma$

Deuterium however has a small binding energy 2.2 MeV



High energy tails of blackbody photons can break up D easily

Need to wait until the photons get cooled down

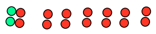
$$T \sim 0.1 MeV (t \sim 300 s)$$

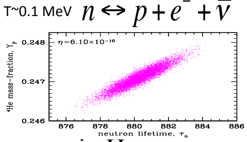
The equilibrium densities of other nuclei elements are not reached till this time because they are all produced from D (around this time when D and He are produced, the reaction rates are by now too slow to produce any heavier particles). DEUTERIUM BOTTLENECK

While waiting, the neutrino decay becomes important for T~0.1 MeV $n \leftrightarrow p + e^- + \bar{\nu}$

$$n_n = n_0 e^{-t/\tau}, \tau \sim 880 s$$

$$\frac{n}{p} \sim \frac{1}{5} e^{-300/880} \sim \frac{1}{7}$$





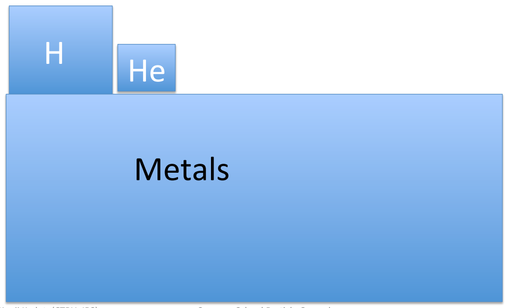
Primordial Abundances $X_p \equiv \dfrac{\text{mass in H}}{\text{total mass}} \sim 0.75, Y_p \equiv \dfrac{\text{mass in He}}{\text{total mass}} \sim 0.25$

Cosmology predicts that the early Universe is filled with hydrogen and helium

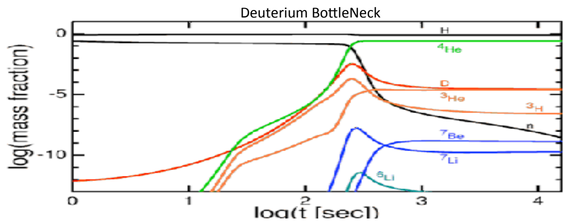
The early Universe is metal free ("metal" refers to heavier elements not H or He)

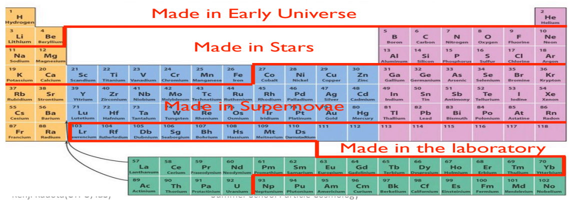
All metals were made later in the stars (dense and hot to overcome the binding energy)

Simplified periodic table



Deuterium BottleNeck





Using BBN to constrain the particle physics models

- Additional relativistic degrees of freedom: $H^2 \propto \rho \propto g T^4$

relativistic degrees of freedom modifies the expansion rate and hence the freeze-out temperature. e.g. additional neutrino species can make the neutron freeze-out earlier with a larger number density -> More ^{4}He

- The baryonic abundance $\eta_B \ (n_b / n_\gamma (n_\gamma \sim 400 / cm^3))$

Less baryons decreases the reaction to burn D into heavier elements. -> More D

$(e.g. \text{less collision rate } D + p \rightarrow {}^3He + \gamma)$

Smaller n_b / n_γ also means to wait longer for a longer cooling time -> more neutrons decay -> Less He

-The lepton asymmetry

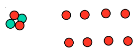
The lepton asymmetry change can change neutron to proton ratio -> ^{4}He changes

$$\nu + n \Longleftrightarrow p + e^-$$
$$e^+ + n \Longleftrightarrow p + \bar{\nu}$$
$$n \Longleftrightarrow p + e + \bar{\nu}$$

- Exercise: How can the BBN be sensitive to expansion rate of the Universe?

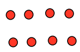
If cooling is huge, no time for the neutrons to decay

$$\frac{n_n}{n_p} \sim 5, X_p \sim 2/3, Y_p \sim 1/3$$



If cooling is negligible, all neutrons decay

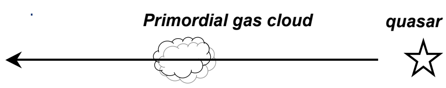
$$X_p \sim 1, Y_p \sim 0$$



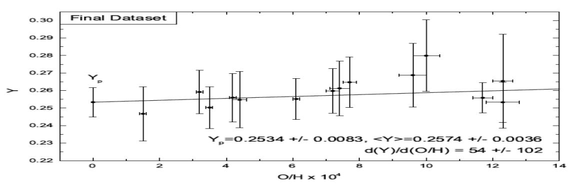
Big expansion rate-> Big cooling rate $H^2 \propto \rho \propto g T^4$

Relativistic degrees of freedom constrained by the BBN

Can we check BBN predictions by observations?



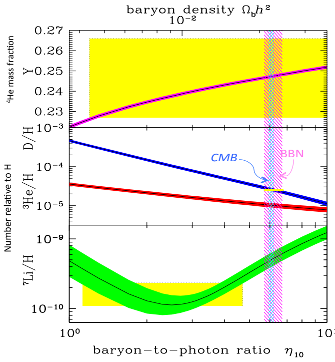
Examine the primordial (unprocessed) gas cloud not yet polluted by stars (typically identified by low metalicities)



Extrapolation to Oxygen/Hydrogen ratio=0 gives an estimate for the primordial He abundance Y

Measuring the deuterium abundance can tell us the baryon density

Sensitive to the baryon abundance



BBN gives an important constraint on

$$\Omega_b h^2 = 0.02$$

(Thanks to a big slope, a big error in D abundance gives a relatively small error in baryon density)

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Baryon asymmetry in the Universe

How can the baryon asymmetry arise from an initially symmetric condition?

Sakharov's conditions (1967)

1) B violation

2) C and CP violation
the baryon number is odd under C and CP
(If CP were an exact symmetry, a process (producing ΔB>0) and its CP conjugate process (producing ΔB<0) would have the same rate.)

CKM matrix has CP violating phase
e.g. CP is violated in Kaon $(K^0 \,\&\, \bar{K}^0)$ decay
$K^0 \to \pi^- e^+ \nu_e, \bar{K}^0 \to \pi^+ e^- \bar{\nu}_e$ (more positrons (~10^{-3}) than electrons)

3) Departure from the thermal equilibrium
(otherwise, a process and its inverse process would have the same rate)
The expansion of the Universe can realize it

Is it possible in the Standard Model?

1) B violation

The SM can violate B (strictly speaking B+L violation by axial anomaly)

sphaleron effects wash-out B+L number In the SM, the global $U(1)_{B+L}$ is anomalous

Lagrangian of chiral fermion in SU(2) gauge interactions

$$L = \bar{\psi}_L \gamma^\mu (\partial_\mu - i\frac{g}{2}\sigma^A W_\mu^A)\psi_L \quad \text{invariant under} \quad \psi \to e^{i\beta}\psi$$

Classically, from Noether theorem, there is a conserved current

$$\partial_\mu J^{5\mu} = 0 \qquad J_\mu^5 = \bar{\psi}\gamma_\mu \psi \quad Q_5 = \int d^3x J^{5,0} = \int d^3x \bar{\psi}\gamma^0 \psi$$

Quantum mechanically, there is an anomaly $\partial_\mu J^{5\mu} = -\frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$

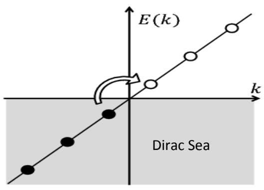
$$\int d^4x \partial_\mu J^{5\mu} = \int dt \frac{\partial}{\partial t}\int d^3x J^{5,0} = Q_5(t=\infty) - Q_5(t=-\infty)$$

$\bar{\psi}\gamma^0\psi = \psi^\dagger \psi$
number operator

Fermions are created, even though there is no perturbative interactions in Lagrangian to create it..

To see where they came from, let us consider the Dirac sea picture where SU(2) field strength tensor is non-zero. The chiral fermion is sitting in an external gauge field. Initially, all the negative states are filled and all the positive energy states are empty

In the early Universe, before Higgs gets vev, all gauge fields and fermions were massless



Baryon number violation due to Fermion fluctuations in the thermally fluctuating W backgrounds

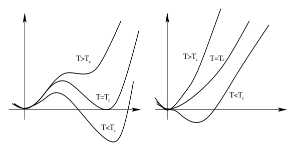
For a lepton: $\Delta L = 1$

For a quark $\Delta B = 1$
(baryon umber 1/3 x 3 colors)

In chemical equilibrium, $B_{end} \sim 0.35(B-L), L_{end} \sim -0.65(B-L)$

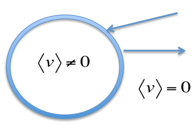
Both B and L are broken. B-L is conserved

Electroweak baryogenesis:



- B is violated by the sphaleron effects.
- CP is violated in the SM.
- First order of electroweak phase transition realizes the departure from the equilibrium

In SM: With higgs mass=125 GeV, not the first order (need mH<50 GeV)
(the strength of transition depends on the height of potential <v>/Tc)
CP violating effects too small for the desirable order of baryon asymmetry



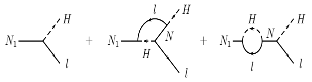
chiral flux difference because of the different reflection probability for L/R (CP violation)

SUSY? Need the light stop (~120 GeV) for the first order (less likely)
chargino sector gives an additional CP violating phases (still not enough)

Leptogenesis

- Lepton number violated by Majorana mass term
- CP violation from the complex neutrino Yukawa matrix

$$Y_N NLH + \frac{1}{2} M_R NN$$



Heavy Majorana neutrino decays. $N \to LH \qquad N \to \bar{L}\bar{H}$

Sphaleron effects: $B_{end} \sim 0.35(B-L), L_{end} \sim -0.65(B-L)$

ex: The departure from the thermal equilibrium:

The condition of out of equilibrium decay (strictly speaking, should solve Boltzmann equation):

$$\Gamma = \frac{Y^+Y}{16\pi} M_N < H(T \sim M_N) = \sqrt{\frac{\pi^2 g_*}{90}} M_N^2 \qquad M_N > \sqrt{\frac{90}{\pi^2 g_*}} \frac{Y^+Y}{16\pi} M_P$$

$$m_\nu = \frac{Y^+Y \langle v \rangle^2}{M_N} < \sqrt{\frac{\pi^2 g_*}{90}} \frac{v^2}{M_P} \sim 10^{-3} eV$$