Plan of Lectures

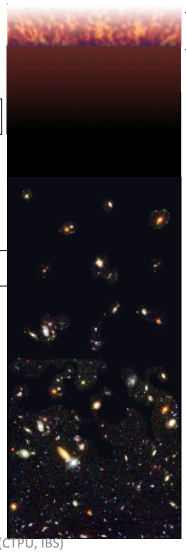
1. Introduction to Standard Cosmology	Brief History of the Universe FRW cosmology Thermodynamics in the Expanding Universe
2. The Early Universe Phenomenology	Inflation Big Bang Nucleosynthesis Baryogenesis/Leptogenesis
3. CMB and Large Scale Structure of the Universe	Cosmic Microwave Backgrounds Baryon Acoustic Oscillations

Big Picture

$$a_{rad} \sim t^{1/2}, a_{mat} \sim t^{2/3}$$

Energy/Temp	Time	Event/Epoch
10^{19}GeV	10^{-43}sec	Planck Time
??		Inflation, baryogenesis,...
TeV	10^{-12}sec	Electroweak phase transition
200MeV	10^{-5}sec	QCD phase transition
1MeV	1sec	Neutrino decoupling.
0.5MeV		Electron-positron annihilation.
0.1MeV	100sec	Big Bang Nucleosynthesis
1eV	10^4yrs	Radiation-matter equality. Z~3200.
0.1eV	10^5yrs	Recombination and Decoupling. CMB last scattering z~1100.
	1Gyr (z=10)	First galaxies. Quasar formation
	z=3	Galaxy formation
	z=1-2	Formation of clusters and superclusters. Acceleration of the Universe.
	13.8 Gyrs	Now

Brief History of Universe



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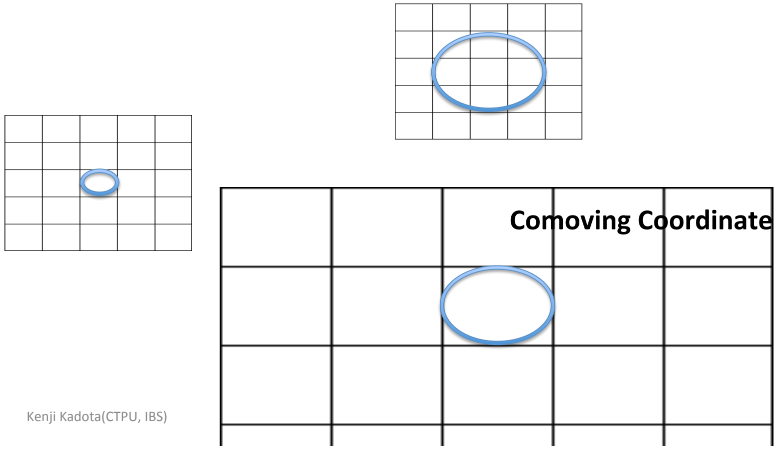
Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$$

In the flat Universe,

$$ds^2 = dt^2 - a^2(t)dx^2$$



Einstein field equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$$

In homogeneous and isotropic Universe, the uniform ideal fluid

$$T^\mu{}_\nu = \mathrm{diag}(\rho c^2, -P, -P, -P) \text{ and } T = \rho c^2 - 3P.$$

time-time component: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{P}{c^2}\right) + \frac{\Lambda c^2}{3}$

space-space component: $\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{Kc^2}{a^2} = 4\pi G\left(\rho - \frac{P}{c^2}\right) + \Lambda c^2$

Friedmann equation $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}$

Combining these two Einstein equations leads to $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{1}{3}\Lambda c^2$

-Exercise: What do we need to accelerate the Universe for $\Lambda=0$?

-Exercise: What do we need to accelerate the Universe?

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(E + 3p)$$

Equation of state parameter $w \equiv \frac{p}{E} < -\frac{1}{3}$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), E = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$E + 3p = 2\left(\dot{\phi}^2 - V\right) < 0 \quad \dot{\phi}^2 < V$$

" Slow roll "

Potential energy need dominate potential energy

e.g. Flat constant potential $V = \Lambda$

$$w = -1, p = -\Lambda = -E$$

$$\frac{\ddot{a}}{a} \propto \Lambda \Rightarrow a \propto \exp(\sqrt{\Lambda}t)$$

Friedmann equation (describing the expansion of the Universe)

$$H^2 = \frac{8\pi G}{3}\rho - k\frac{c^2}{a^2}, H \equiv \frac{\dot{a}}{a} \qquad H^2 = H_0^2\left(\Omega_v + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} - \frac{(\Omega_{total} - 1)}{a^2}\right)$$

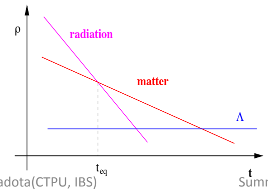
Flat Universe with k=0 : Critical Density $\rho_c = \frac{3H^2}{8\pi G}$

Density Parameter $\Omega_i \equiv \frac{\rho_i}{\rho_c} = \frac{8\pi G\rho_i}{3H^2}$ Flat Universe (k=0) $\Omega_{total} = 1$

$$\frac{k}{H^2a^2} = \frac{\rho}{3H^2/8\pi G} - 1 = \Omega_{total} - 1$$

Solution for the Friedmann equation: radiation domination epoch $\frac{a}{a_0} = \left(\frac{32\pi G\rho_{r,0}}{3}\right)^{1/4} t^{1/2}$ matter domination epoch $\frac{a}{a_0} = \left(\frac{3}{2}H_0 t\right)^{2/3}$

$$\rho_{mat} \sim 1/a^3, \rho_{rad} \sim 1/a^4, \quad a_{rad} \sim t^{1/2}, a_{mat} \sim t^{2/3}$$



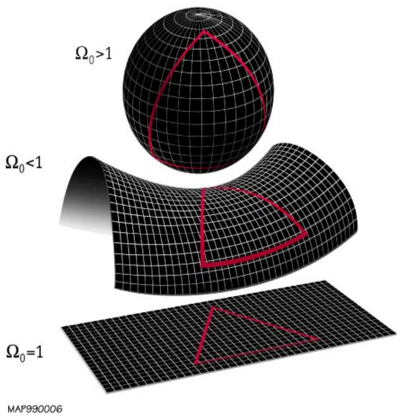
$$\Omega_0 \equiv \frac{\rho_0}{\rho_c}$$

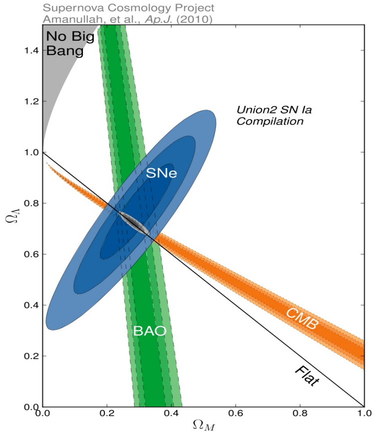
$$\rho_0 \sim 1.9\times10^{-26}\Omega_0 h^2 kg/m^3$$

$$H_0 = 100h/\sec/Mpc$$

$$1Mpc \sim 3\times10^{22}m, h \sim 0.7$$

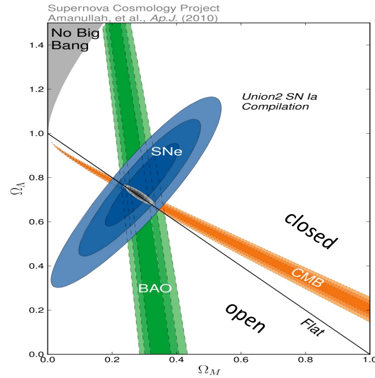
Flatness problem





-Exercise: *Flatness problem (Why Ω=1?)*
Show that (T is the temperature):

$$\frac{\Omega_{early}^{-1}-1}{\Omega_0^{-1}-1} \sim \frac{T_0}{T_{eq}}\left(\frac{T_{eq}}{T_{early}}\right)^2$$



exercise: Flatness problem

Using Friedmann equation:

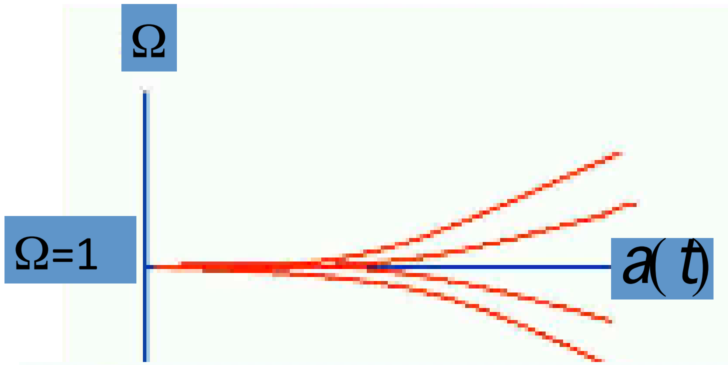
$$\Omega(a)^{-1}-1=\frac{3Kc^2}{8\pi G\rho(a)a^2} \qquad \rho_{early}\sim\rho_0\times\left(\frac{a_0}{a_{eq}}\right)^3\times\left(\frac{a_{eq}}{a_{early}}\right)^4, T\sim a^{-1}$$

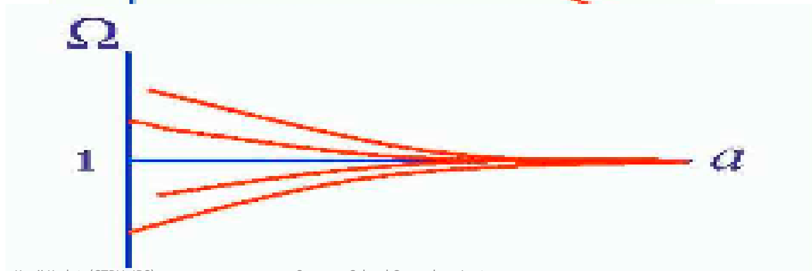
$$\frac{\Omega_{early}^{-1}-1}{\Omega_0^{-1}-1} \sim \frac{T_0}{T_{eq}}\left(\frac{T_{eq}}{T_{early}}\right)^2$$

$$(\text{e.g. For } T_{early}=T_{Planck})\sim\frac{3\times10^{-5}eV}{1eV}\left(\frac{1eV}{10^{19}GeV}\right)^2\sim10^{-60}$$

Ω_{Planck} was 60 orders of magnitude close to the unity than Ω_{today}

$$\Omega_0=1-\Omega_K, \Omega_K=0.000\pm0.005$$
$$\Omega_{Planck}^{-1}\sim1\pm0.005\times10^{-60}$$





-Exercise: <Expansion Age of the Universe>
What is the age of the Universe at a redshift z?
(Guess how old the Universe was when z=1)

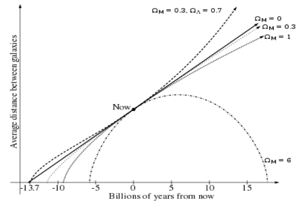
Ex: What is the age of the Universe at a redshift z?
Ans: Just integrate the Friedmann equation.

$$t(z) \equiv \int_0^{a(z)} \frac{da}{\dot{a}} = \frac{1}{H_0} \int_z^{\infty} \frac{dz}{(1+z)\sqrt{\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{vac,0} + (1-\Omega_0)(1+z)^2}}$$

$$(a = a_0/(1+z), H = \dot{a}/a,$$

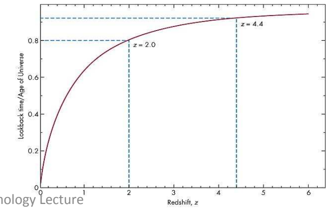
$$H^2 = H_0^2[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_{vac} - (\Omega - 1)a^{-2}])$$

$$t_H \equiv \frac{1}{H_0} = \frac{1}{67.8km/(sMpc)} \sim 14.4 \text{billion years}$$



The age of the Universe 13.8 billion years

The lookback time defined as $t_0 - t(z)$



In the radiation dominated epoch:

$$t(z) \sim \left(\frac{1+z}{10^{10}}\right)^{-2} s$$

In the matter dominated epoch:

$$t(z) = \frac{1}{H_0}\frac{2}{3}(1+z)^{-3/2} \sim \frac{2}{3}(1+z)^{-3/2} \times 10^{10} h^{-1} yr$$

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

Proper distance between two fundamental observers (one at the origin r=0 and the other at r)

$$l = a(t)\int_0^r \frac{dr'}{\sqrt{1-Kr^2}} = a(t)\chi(r)$$

$$\chi(r) = \begin{cases} \sin^{-1} r (for K = +1) \\ r (K = 0) \\ \sinh^{-1} r (K = -1) \end{cases}$$

χ : comoving ditance

In terms of the conformal time $\tau = \int_0^t \frac{cdt'}{a(t')}$

$$ds^2 = a^2[d\tau^2 - d\chi^2 - f^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)]$$

$$f = \begin{cases} \sin\chi \\ \chi \\ \sinh\chi \end{cases}$$

e.g. The photon trajectory along the radial direction ($d\theta = d\phi = 0$):
$d\tau = d\chi$(photons travel along null geodesics)

$a(t)$ is dimensionless. Some books (e.g. Kolb-Turner) uses $R(t)$ which is dimensionful. $a(t) = R(t)/R(0)$ ($R(0) = 1$, so $a(t) = R(t)$)

Hubble Parameter

The rate at which the proper distance between the fundamental observes changes:

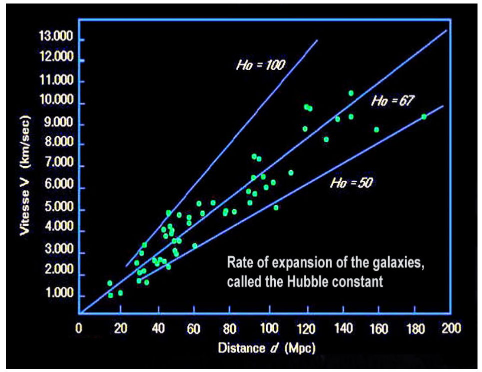
$$\frac{dl}{dt} \equiv H(t)l$$

l : proper time

$l(t) = a(t)\chi$: χ is comoving distance

ex. Show that: $H(t) = \frac{\dot{a}}{a}$

$$\frac{dl}{dt} = \dot{a}\chi = \frac{\dot{a}}{a}a\chi = \frac{\dot{a}}{a}l$$



The value of Hubble parameter at present time:

$$H_0 = 100h \; km/s/Mpc \sim 70/km/s/Mpc$$

Redshift

Suppose a fundamental observer emits the wave crest to the origin at t0 and t1.
The wavelength is then Doppler shifted;

$$\frac{\lambda_0}{\lambda_1}=\frac{a(t_0)}{a(t_1)}\equiv 1+z(t_1) \qquad \text{Present values}: z=0, a=1$$

For the relativistic particle, $pc=E=h\nu\propto a^{-1}$ due to the redshift
($p\propto a^{-1}$ holds for massive particles too)

-Exercise: What is the redshift for the epoch of matter-radiation equality?

$$\Omega_{m0}\sim 0.3(\sim\Omega_{CDM0}(0.24)+\Omega_{b0}(0.04)), \Omega_{r0}\sim 10^{-5}$$
$$1+z_{eq}=a/a_{eq}=\Omega_{m0}/\Omega_{r0}\sim 3\times 10^3 (\text{corresopnding to } T\sim 10^4 K)$$
$$(Planck 2015: z_{eq}\sim 3400)$$

The proper velocity of a particle with respect to a fundamental observer at the origin is defined as:

$$v=\frac{dl}{dt}=\frac{d(a\chi)}{dt}=\dot{a}\chi+a\dot{\chi}\equiv v_{expansion}+v_{peculiar}$$
$$v_{expansion}=\frac{\dot{a}}{a}a\chi=Hl$$

Redshift space distortion (Kaiser (1987))

Coherent infall bulk motion of galaxies towards the halo center (overdense region).

Real Space Redshift space

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Thermodynamics in the Expanding Universe

Natural units $\hbar=c=k_B=1$

$$n=\frac{g}{(2\pi)^3}\int f(\vec{p})d^3p \qquad f(\vec{p})=\frac{1}{e^{(E-\mu)/T}\pm 1}$$
$$\rho=\frac{g}{(2\pi)^3}\int E(\vec{p})f(\vec{p})d^3p$$
$$P=\frac{1}{3}n\langle pv\rangle=\frac{1}{3}n\left\langle\frac{p^2}{E}\right\rangle=\frac{g}{(2\pi)^3}\int\frac{|\vec{p}|^2}{3E}f(\vec{p})d^3p$$

For a relativistic particle (T>>m),

$$\rho=(\pi^2/30)gT^4(Bose),(7/8)\times(\pi^2/30)gT^4(Fermi)$$
$$n=(\zeta(3)(\sim 1.2)/\pi^2)gT^3(Bose),(3/4)\times(\zeta(3)/\pi^2)gT^3(Fermi)$$
$$p=\rho/3$$

For the non-relativistic limit (m>>T)

$$n=g\left(\frac{mT}{2\pi}\right)^{3/2}\exp[-m/T]$$
$$\rho=mn$$
$$p=nT$$

-Before the decoupling:
The particles can maintain the equilibrium distribution through the interactions in the plasma

$$f(\vec{p}) = \frac{1}{e^{E/T} \pm 1}$$

- Exercise: How about after the decoupling?
They can still maintain the equilibrium distribution after the decoupling if highly relativistic (T>>m) or highly non-relativistic (m>>T).

a) The physical momentum of a particle decays with the expansion of the Universe p~1/a
b) The number density ~1/a^3

$$n \sim \int d^3p f(p) = \frac{1}{a^3}\int d^3q f(q=ap) = \begin{cases} (relativistic)\frac{1}{a^3}\int d^3q \frac{1}{Exp[\frac{q}{aT}]\pm 1} \\ (non-rel)\frac{1}{a^3}\int d^3q \frac{1}{Exp[\frac{q^2}{a^2T}]\pm 1} \end{cases}$$

$$n \sim 1/a^3 \Rightarrow \begin{matrix} (rel) T \sim 1/a \\ (non-rel) T \sim 1/a^2 \end{matrix}$$

In general, however (e.g. T~m), phase space distribution does not obey an equilibrium distribution in absence of the interactions (solve the Boltzmann equation!)

The effective massless degrees of freedom

$$\rho_R = \frac{\pi^2}{30} g_* T^4 \qquad p_R = \rho_R / 3 = \frac{\pi^2}{90} g_* T^4$$

$$g_* = \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{i=fermions} g_i \left(\frac{T_i}{T}\right)^4$$

(The temperature T is the actual temperature of the background plasma, assumed to be in equilibrium. Usually T_i=T. An exception includes the neutrino temperature (to be discussed later))

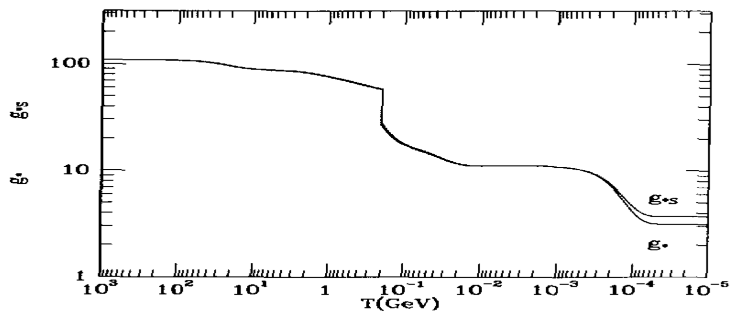
-Exercise: What is g_* at T=10GeV?

-Exercise: What is the effective massless degrees of freedom at T=10GeV?

mt~173 GeV does not contribute.

quarks except top : g*= 5 x 3 colors x 2 spin states x 2(quarks and antiquakrs)
gluon: 8 color states (no 9th state ie no color singlet) x two spins states
3 charged leptons: 3 charged leptons x two spin states x 2 (antileptons)
3 neutrinos: 3 neutrinos x 1 (only left-handed) x 2 (antineutrinos)
photons 2(two spin states, no longitudinal component)

2+16+7/8*(30+30+12+6)=86.25



History of g(T)

$T \sim 200$ GeV	all present	106.75	
$T \sim 100$ GeV	EW transition	(no effect)	
$T < 170$ GeV	top-annihilation	96.25	
$T < 80$ GeV	$W^\pm, Z^0, H^0$	86.25	
$T < 4$ GeV	bottom	75.75	
$T < 1$ GeV	charm, τ^-	61.75	
$T \sim 150$ MeV	QCD transition	17.25	(u,d,g$\to \pi^{\pm,0}$, $37 \to 3$)
$T < 100$ MeV	$\pi^\pm, \pi^0, \mu^-$	10.75	$e^\pm, \nu, \bar{\nu}, \gamma$ left
$T < 500$ keV	e^- annihilation	(7.25)	$2 + 5.25(4/11)^{4/3} = 3.36$

type		mass	spin	g
quarks	$t, \bar{t}$	173 GeV	$\frac{1}{2}$	$2 \cdot 2 \cdot 3 = 12$
	$b, \bar{b}$	4 GeV		
	$c, \bar{c}$	1 GeV		
	$s, \bar{s}$	100 MeV		
	$d, \bar{s}$	5 MeV		
	$u, \bar{u}$	2 MeV		
gluons	g_i	0	1	$8 \cdot 2 = 16$
leptons	$\tau^\pm$	1777 MeV	$\frac{1}{2}$	$2 \cdot 2 = 4$
	$\mu^\pm$	106 MeV		
	$e^\pm$	511 keV		
	$\nu_\tau, \bar{\nu}_\tau$	< 0.6 eV	$\frac{1}{2}$	$2 \cdot 1 = 2$
	$\nu_\mu, \bar{\nu}_\mu$	< 0.6 eV		
	$\nu_e, \bar{\nu}_e$	< 0.6 eV		
gauge bosons	W^+	80 GeV	1	3
	W^-	80 GeV		
	Z^0	91 GeV		
	γ	0		2
Higgs boson	H^0	125 GeV	0	1

-Exercise: Baryon asymmetry of the Universe : What is the baryon to photon ratio (Ans. 10^{-9})

-Exercise:
Derive the factor of 3/4 and 7/8 due to the spin statistics for the relativistic species

-Exercise: Sound speed $c_s^2 = \frac{dP}{d\rho}$ 1/3 for the radiation and ~T/m in the non-relativistic limit

ex: Derive the factor of 3/4 and 7/8 due to the spin statistics for the relativistic species

$$E = pc, y \equiv pc / kT$$

$$n \Rightarrow \frac{4\pi g}{(2\pi\hbar)^3}\left(\frac{kT}{c}\right)^3 \int \frac{y^2 dy}{e^y \pm 1} \qquad \varepsilon \Rightarrow \frac{4\pi g}{(2\pi\hbar)^3}\frac{(kT)^4}{c^3}\int \frac{y^3 dy}{e^y \pm 1}$$

$$\frac{1}{e^x+1} = \frac{1}{e^x-1} - \frac{2}{e^{2x}-1} \qquad \frac{n_F(T)}{g_F} = \frac{n_B(T) - 2n_B(T/2)}{g_B}$$

$$\frac{n_F(T)/g_F}{n_F(T)/g_F} = 1 - 2\left(\frac{T/2}{T}\right)^3 = 3/4$$

$$\frac{E_F(T)/g_F}{E_F(T)/g_F} = 1 - 2\left(\frac{T/2}{T}\right)^4 = 7/8$$

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Baryon asymmetry of the Universe : Baryon to photon ratio 10^{-9}

Photons: $g = 2$

$$\varepsilon_\gamma = \frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3} = \frac{0.261\ \mathrm{eV}}{\mathrm{cm}^3}\left(\frac{T}{2.725\mathrm{K}}\right)^4$$

$$n_\gamma = \frac{411}{\mathrm{cm}^3}\left(\frac{T}{2.725\mathrm{K}}\right)^3$$

Baryons:

$$\varepsilon_b = \Omega_b \frac{3H_0^2 c^2}{8\pi G} = 0.04\frac{5200\ \mathrm{eV}}{\mathrm{cm}^3}\left(\frac{h}{0.7}\right)^2 = \frac{210\ \mathrm{eV}}{\mathrm{cm}^3}\left(\frac{h}{0.7}\right)^2$$

$$n_b = \frac{E_b}{m_b} = \frac{E_b}{m_p(\sim 940 MeV)} = 0.22 / m^3$$

Photons/Baryons:

$$\eta \equiv \frac{n_\gamma}{n_b} \sim \frac{411}{0.22 \times 10^{-6}} \sim 10^9 \left(\frac{0.04}{\Omega_b}\right)\left(\frac{0.7}{h}\right)^2$$

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Sound speed for a non-relativistic monatomic gas:

$$c_s^2 = \frac{\frac{dP}{dT}}{\frac{d\rho}{dT}} = \frac{\frac{d}{dT}\left(\int \frac{p^2}{3m} e^{-\frac{p^2}{2mT}} p^2 dp\right)}{\frac{d}{dT}\left(\int m e^{-\frac{p^2}{2mT}} p^2 dp\right)}$$

$$= \frac{\int \frac{p^2}{2mT^2}\frac{p^2}{3m} e^{-\frac{p^2}{2mT}} p^2 dp}{\int \frac{p^2}{2mT^2} m e^{-\frac{p^2}{2mT}} p^2 dp} = \frac{\int \frac{1}{m} e^{-\frac{p^2}{2mT}} p^6 dp}{\int m e^{-\frac{p^2}{2mT}} p^4 dp} = \frac{5}{3}\frac{T}{m}$$

$$\int dp p^n e^{-\frac{p^2}{2mT}} = (2mT)^{\frac{n+1}{2}} \int dx e^{-x^2} x^n = 2^{(n-1)/2}(mT)^{\frac{n+1}{2}}\Gamma\left(\frac{1+n}{2}\right)$$

For a non-negative integer: $\Gamma\left(\frac{1+n}{2}\right) = \frac{(2n)!}{4^n n!}\sqrt{\pi}$

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Entropy Conservation

The entropy per comoving volume is conversed: $S = sa^3$

$$s = \frac{2\pi^2}{45} g_* T^3 \qquad g_* = \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=fermions} g_i \left(\frac{T_i}{T}\right)^3$$

g_* : Entropy degrees of freedom

(The temperature T is the actual temperature of the background plasma, assumed to be in equilibrium. Usually T_i=T)

- When g_* does not change, S=sa^3=const -> T~1/a

- When g_* changes (e.g. particles annihilate and disappear), its entropy is transferred to other relativistic particles in the thermal plasma and the thermal plasma heats up (strictly speaking, T decreases less slowly)

- Exercise: Show that $\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$

Entropy conservation : neutrino temperature $\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$

Neutrino temperature lower than the photon temperature

the energy from electron-positron annihilation goes into the thermal plasma

Neutrinos interact via the weak interactions (no EM charge)
decouples just above the electron-positron annihilation

Entropy S is conserved $S = sa^3 \propto gT^3a^3$

$$\frac{s(a_1)[\gamma + e^+ + e^- + 3\nu + 3\bar{\nu}]}{s(a_2)[\gamma + 3\nu + 3\bar{\nu}]} = \frac{T^3(a_1)[2 + (7/8)(2+2+3+3)]}{2T_\gamma^3(a_2) + (7/8)6T_\nu^3(a_2)}$$

$s(a_1)a_1^3 = s(a_2)a_2^3$ Neutrino temp scales as 1/a $a_1T(a_1) = a_2T_\nu(a_2)$ $\Rightarrow \frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$

Ex: What is the current energy density for a massive neutrino? $\Omega_\nu h^2 = \frac{m_\nu}{94eV}$

Ex: What is the current energy density for massive neutrinos?

$$n_\nu = \frac{\frac{3}{4}T_\nu^3}{T_\gamma^3} n_\gamma \sim \frac{3}{4} \times \frac{4}{11} \times \frac{411}{cm^3} \sim \frac{112}{cm^3}$$

$$\rho_\nu = m_\nu n_\nu$$

$$\Omega_\nu = \frac{\rho_\nu}{\rho_{cri}} = \frac{\rho_\nu}{1.054h^2 \times 10^4 [eV/cm^3]} \sim \frac{m_\nu}{94eVh^2}$$

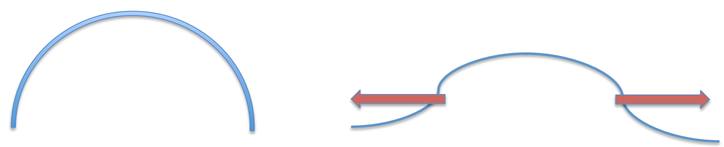
Free streaming scale

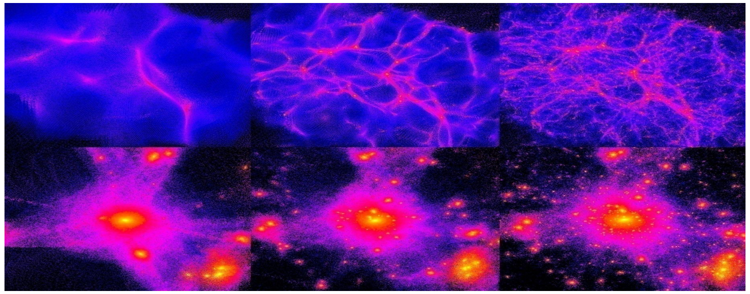
Free streaming length (c.f. Kolb and Turner)

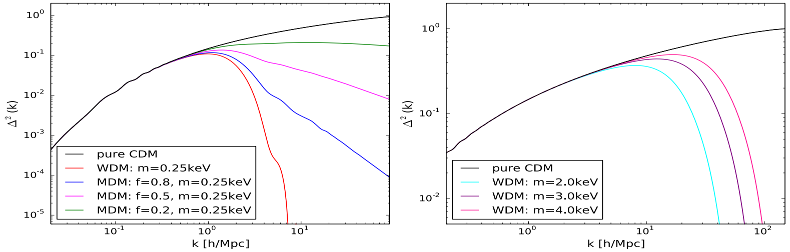
$$\lambda \equiv a(t_0) \int_{t_{dec}}^{t_0} dt v(t)/a(t) = a(t_0)a(t_{dec})v(t_{dec}) \int_{t_{dec}}^{t_0} dt 1/a^2(t)$$

$$a \sim t^{1/2}, a \sim t^{2/3}$$

$$v(t_{dec}) \sim \sqrt{T(t_{dec})/m}$$







Schneider 1412.2133

free streaming scale

1) Calculate the free streaming scale for a particle which becomes non-relativistic at tNR<teq (radiation-matter equality). Proper distance: $L_{FS}(t) = a(t)\int_0^t \frac{v(t')}{a(t')}dt'$

2) What is the free streaming length scale if the dark matter is the weakly interacting massive particle with mass 30 eV which decouples around T~MeV. Check that the mass contained in this length scale corresponds to large clusters.

(zeq~3400. The particle becomes non-relativistic at T ~m.)

(1)

$$\int_0^t \frac{v(t')}{a(t')}dt' = \int_0^{t_{NR}} \frac{v(t')}{a(t')}dt' + \int_{t_{NR}}^{t_{eq}} \frac{v(t')}{a(t')}dt' + \int_{t_{eq}}^{t} \frac{v(t')}{a(t')}dt'$$

$$t < t_{NR} : v \sim 1, a \sim t^{1/2}$$
$$t_{NR} < t < t_{eq} : v \sim 1/a, a \sim t^{1/2}$$
$$t_{eq} < t : v \sim 1/a, a \sim t^{2/3}$$

$$t < t_{NR} : L_{FS} = (2t_{NR}/a_{NR}^2)a^2 = 2t$$
$$t_{NR} < t < t_{eq} : L_{FS} = (2t_{NR}a/a_{NR})[1+\ln(a/a_{NR})]$$
$$t_{eq} < t : L_{FS} = (2t_{NR}a/a_{NR})[1+\ln(a/a_{NR})] + (3t_{NR}a/a_{NR})[1 - a_{eq}^{1/2}/a^{1/2}]$$
$$t >> t_{eq} :\sim (2t_{NR}a/a_{NR})[1+\ln(a/a_{NR})] + (3t_{NR}a/a_{NR})]$$

The matter fluctuations are suppressed at scales with L<Lfs

(2)

$$M_{fs} = \frac{4\pi}{3}\left(\frac{L_{FS}^{prop}}{2}\right)^3 \bar{\rho}_m(t_{dec}) = \frac{4\pi}{3}\left(\frac{L_{FS}^{com}}{2}\right)^3 \bar{\rho}_m(t_0)$$

$$L_{FS}^{prop}(t_0) \sim 30\,Mpc\left(\frac{m}{30eV}\right)^{-1}$$

$$M_{fs} = \frac{4\pi}{3}\left(\frac{L_{FS}^{com}}{2}\right)^3 \bar{\rho}_m(t_0) = \frac{4\pi}{3}\left(\frac{L_{FS}^{prop}}{2}\right)^3 \bar{\rho}_m(t_{dec})$$

$$\bar{\rho}_0 = \Omega_{m,0}\,\rho_{crit,0}, \text{ with } \rho_{crit,0} = 2.78 \times 10^{11} h^{-1}\,M_\odot/(h^{-1}\,Mpc)^3$$

$$M_{fs} \sim 10^{15} M_{Solar}\left(\frac{m}{30eV}\right)^{-2}$$

This mass corresponds to large clusters. All perturbations with masses smaller than this scale would be damped out, and the first objects to be formed in the early Universe would be superclusters.

History of the Universe