Topics in Electroweak Symmetry Breaking

2. The top quark in models of electroweak symmetry breaking

M. E. Peskin
Pyeongchang
July 2016

In this lecture, I will discuss models of new physics beyond the Standard Model.

Maybe it is good, first, to explain why I believe that there is new physics to be discovered at the TeV mass scale.

The Standard Model is almost perfect within its own domain. It certainly omits some aspects of nature, and so it cannot claim to be a theory of everything. Even when we add QCD, this theory omits gravity, dark matter, and an explanation for the matter-antimatter asymmetry of the universe. Neutrino masses can be included, but most theories invoke new ingredients outside the model.

But, the Standard Model also raises fundamental questions that it does not have the power to solve.

Among these are:

Why just quarks and leptons ? What is the origin of the quantum number assignments (I, Y) of these particles ?

What explains the spectrum of quark and lepton masses ? In the Standard Model, we have

$$m_f = y_f v/\sqrt{2}$$

But, the y_f are renormalized parameters. Within the Standard Model, they cannot be predicted. The presence of CKM and PMNS mixing angles adds another dimension to this problem.

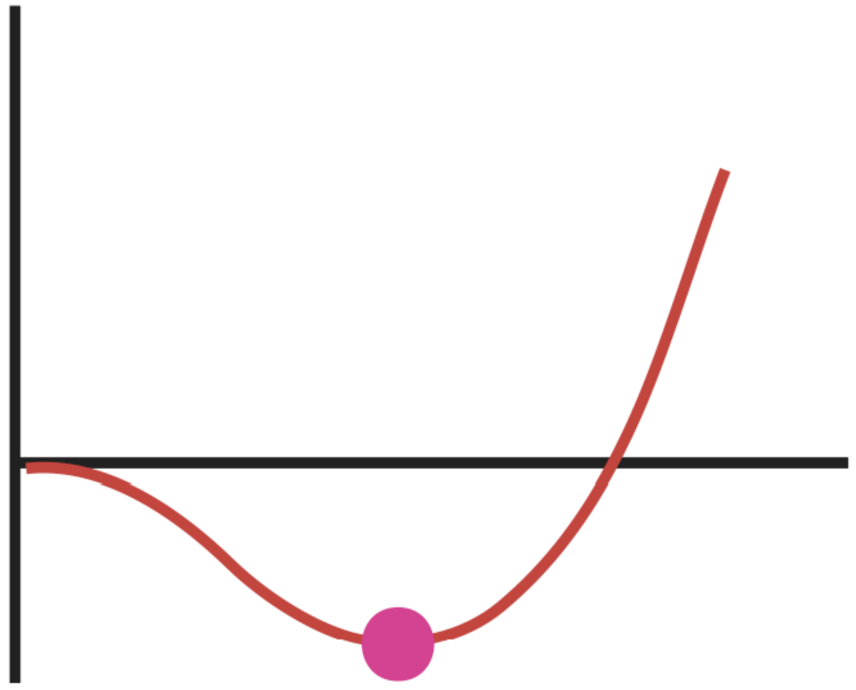
Finally, the structure of the Standard Model requires that SU(2)xU(1) be spontaneously broken. Why does this happen ? The Standard Model cannot answer this question.

Here is the explanation for SU(2)xU(1) breaking given in the Standard Model:

Write the most general renormalizable potential for φ :

$$V(\varphi) = \mu^2 |\varphi|^2 + \lambda |\varphi|^4$$

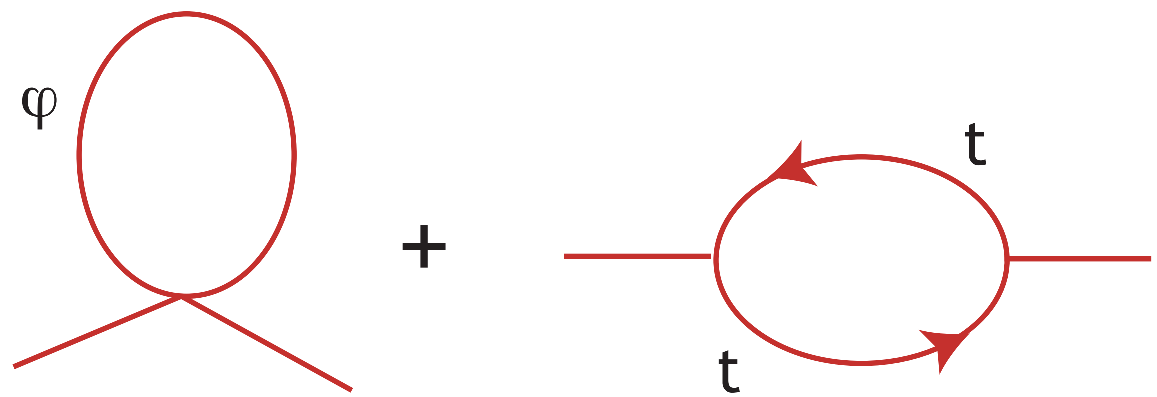
Assume $\mu^2 < 0$. Then the potential has the correct shape for symmetry breaking.



Why is $\mu^2 < 0$? That question cannot be addressed within the model.

We get into deeper trouble if we try to pursue this question by higher-order computation.

If we compute the first quantum corrections to the picture on the previous slide, we find

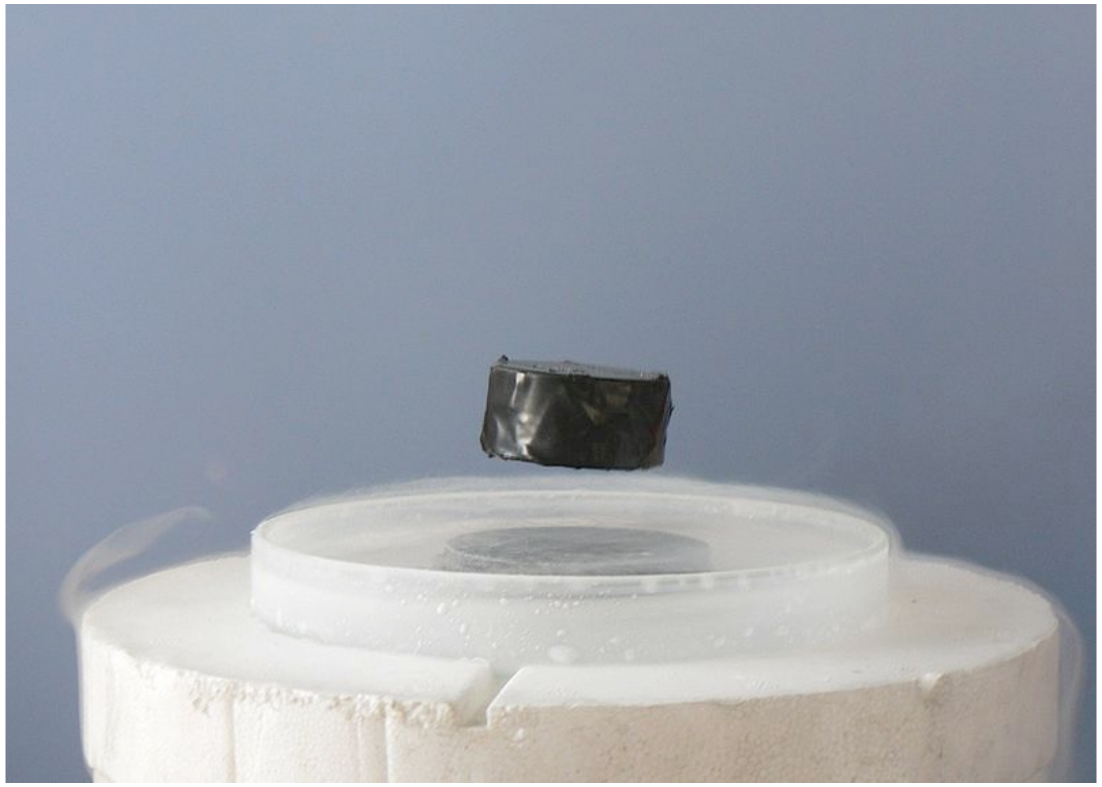


$$\mu^2 = \mu^2_{\rm bare} + \frac{\lambda}{8\pi^2}\Lambda^2 - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \cdots$$

So $|\mu^2| = (100\ {\rm GeV})^2 \ll \Lambda^2$ seems ad hoc. It might be easier to understand if there were additional diagrams that cancel these at high energy. But, for this, we need new particles at the 1 TeV mass scale.

This problem is not new to high-energy physics. It is encountered in all systems where a symmetry is spontaneously broken, especially in condensated matter physics.

Superconductivity is a property of almost any metal at cryogenic temperatures. It was discovered by Kamerlingh Onnes in 1911 (in Hg) and was quickly seen to be associated with a sharp phase transition. However, the explanation was not understood for another 45 years.



In 1950, Landau and Ginzburg proposed a phenomenological theory of superconductivity, based on a scalar field – representing the electron condensate – coupled to a U(1) gauge field – electromagnetism. The condensate acquired a ground state expectation value at low temperatures

$$G[\varphi] = \int d^3x \left[|D_\mu \varphi|^2 + A(T)\mu^2 |\varphi|^2 + B|\varphi|^4 \right]$$

This theory successfully explained the form of the phase transition, the existence of Type I and Type II superconductors, the Meissner effect, the value of the critical current, and many other features.

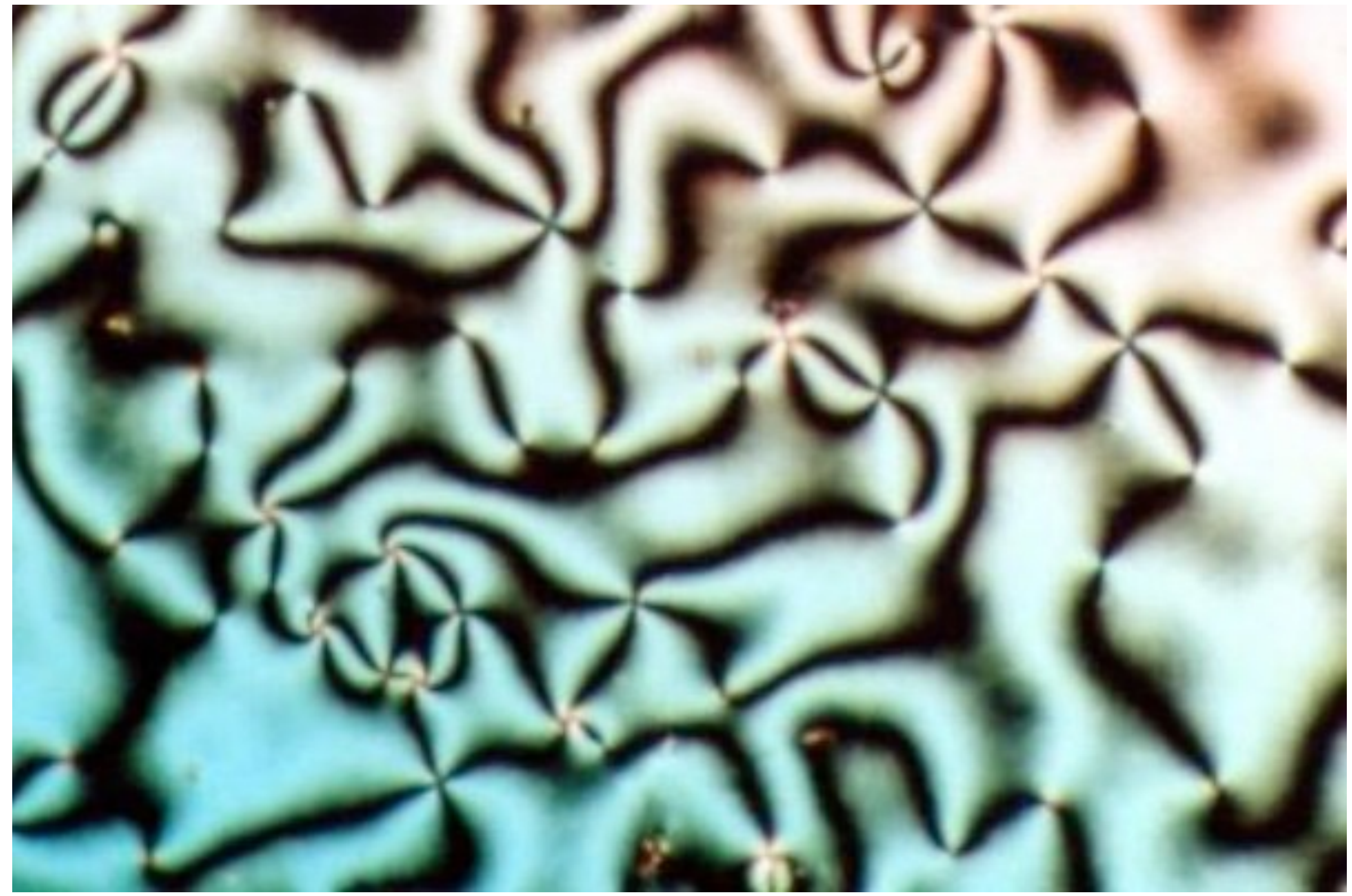
However, it could not explain why superconductivity occurs. That took until 1957, with the work of Bardeen, Cooper, and Schrieffer (BCS).

In our understanding of the phase transition to broken SU(2)xU(1), we are now at the Landau-Ginzburg stage.

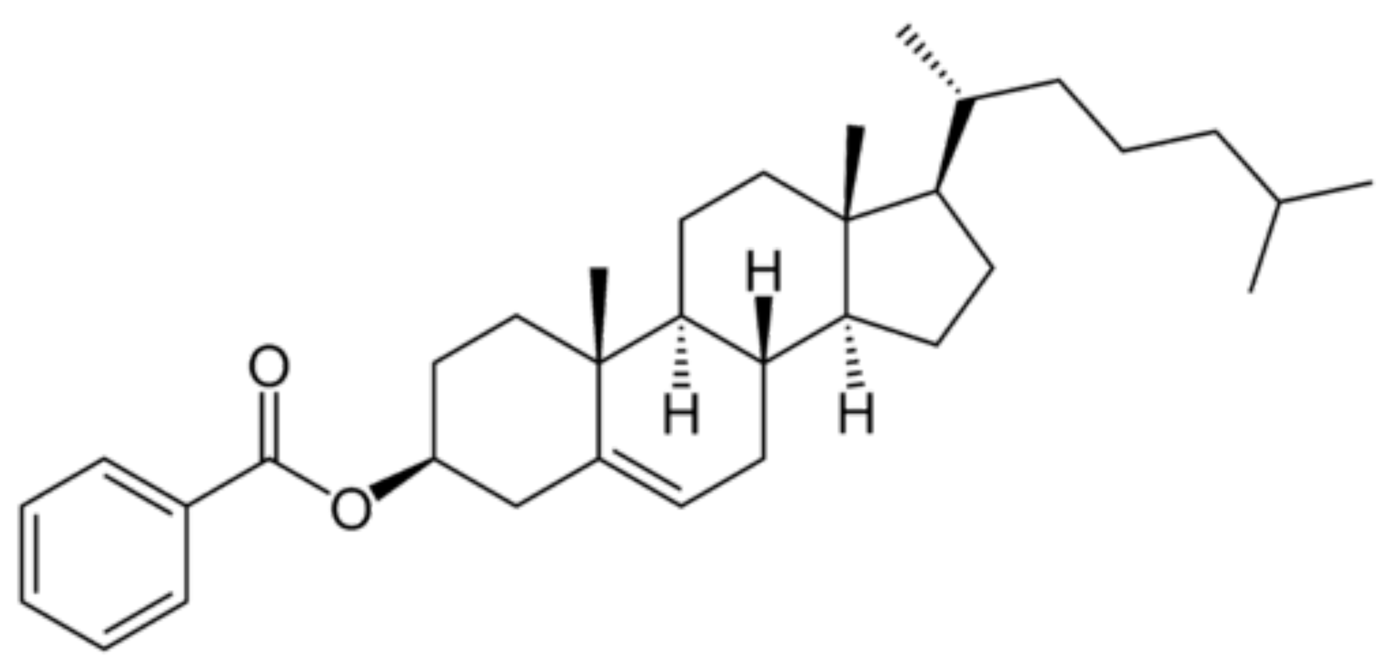
For superconductivity, physicists knew at least that the explanation had to be given in terms of the interactions of electrons and atoms.

For SU(2)xU(1), we do not know the basic ingredients out of which we must build a theory of the symmetry-breaking potential. On general principles, these must be some particles and fields. We only know that we have not discovered them yet.

nematic liquid crystal



cholesterol benzoate



As theorists, if we believe in these ideas, we ought to try to make them more concrete by suggesting specific realizations of models in which the Higgs potential can be computed.

There is an idea here that I am very fond of:

The top quark is by far the heaviest fermion of the Standard Model, and its coupling

$$\frac{y_t^2}{4\pi} = 1/13$$

is the strongest Standard Model coupling except for α_s . So, the top quark must contribute an important term to the Higgs potential.

It is possible that this term is negative and is the driver of SU(2)xU(1) breaking.

This idea is actually realized in many models of electroweak symmetry breaking. In this lecture, I will give you some examples.

There is an obstacle to setting up a quantum field theory in which we can compute the symmetry-breaking potential. If there is a Higgs scalar field, that field will have a divergent mass term, whose sign will then be ambiguous.

There are two types of solutions to this problem:

Find a symmetry that forbids radiative corrections to the scalar mass term

leads to → supersymmetry; Higgs as a Goldstone boson

Construct the scalar field out of more fundamental constituents

leads to → composite or strongly interacting Higgs

In this lecture, I will give examples in models of both types.

Begin with the example of supersymmety (SUSY)

You can find coherent introductions to supersymmetry in the book of Wess and Bagger, “Supersymmetry and Supergravity”, and in the wonderful “Primer” by Steve Martin, hep-ph/9709356. Another useful reference is my TASI lectures: arXiv:0801.1928.

Here I will give a somewhat ideosyncratic introduction, in the spirit of the considerations above.

We seek a symmetry that will prohibit the quadratically divergent radiative corrections to

$$\delta\mathcal{L} = -\mu^2|\varphi|^2$$

You know that the quadradically divergent corrections to a fermion mass term are forbidden by chiral symmetry

$$\psi_L \to e^{i\alpha}\psi_L \qquad \psi_R \to e^{-i\alpha}\psi_R$$

So, postulate a symmetry that connects the Higgs field to a fermion:

$$\delta\varphi = \overline{\xi}\ \psi$$

When discussing supersymmetry, it is most convenient to work with the most elementary fermion representations in 4 dimensions — that is, to write all fermions in terms of 2-component left-handed chiral fermions.

To do this, define

$$c = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad c^2 = -1 \qquad c^T = -c$$

Recall that a right-handed fermion in 4d can be represented as

$$\psi_R = -c\psi^*$$

so that a Dirac fermion has the form

$$\Psi = \begin{pmatrix} \psi_1 \\ -c\psi_2^* \end{pmatrix}$$

and you can check that

$$\overline{\sigma}^m = (1, -\vec{\sigma})^m$$

$$\begin{aligned} \mathcal{L} &= \overline{\Psi}(i\gamma \cdot \partial - m)\Psi \\ &= \psi_1^\dagger i\overline{\sigma} \cdot \partial\psi_1 + \psi_2^\dagger i\overline{\sigma} \cdot \partial\psi_2 - m\big[\psi_1^T c\psi_2 - \psi_1^\dagger c\psi_2^*\big] \end{aligned}$$

Now we can try to write the algebra of SUSY transformations. The basic transformation, which we now write

$$\delta\varphi = \xi^T c\psi$$

is generated by a SUSY charge Q_α:

$$\delta_\xi \varphi = \left[\xi^T c\, Q + Q^\dagger c\, \xi^*\right]\, \varphi$$

It is interesting to examine

$$\{Q_\alpha, Q^\dagger_\beta\}$$

This object is a 4-vector, and it is nonzero, because

$$\langle A|\,\{Q_\alpha, Q^\dagger_\alpha\}\,|A\rangle = |Q^\dagger_\alpha\,|A\rangle\,|^2 + |Q^\dagger_\alpha\,|A\rangle\,|^2 > 0$$

If $Q_\alpha$commutes with H, this 4-vector also commutes with H.

But, now we can invoke a powerful theorem about QFT in 4-dimensions, the Coleman-Mandula Theorem: If there is a conserved 4-vector operator other than P^m that commutes with H, the scattering is forbidden and S = 1.

$$\{Q_\alpha, Q^\dagger_\beta\} = 2\sigma^m_{\alpha\beta} P_m$$

This is a very powerful result. It says that if we want a complete theory which is supersymmetry (as we will need to be protected from quadratic divergences to all orders) then every particle in the theory must participate in the algebra and have a SUSY partner.

The SUSY algebra is equivalently written on fields

$$[\delta_\xi, \delta_\eta] = 2i[\xi^\dagger \overline{\sigma}^m \eta - \eta^\dagger \overline{\sigma}^m \xi]\partial_m$$

Here is the simplest representation of this algebra, called the chiral supermultiplet

$$\delta_\xi \phi = \sqrt{2}\xi^T c\psi \qquad \delta_\xi \phi^* = -\sqrt{2}\psi^\dagger c\xi^*$$

$$\delta_\xi \psi = \sqrt{2}i\sigma^n \xi^* \partial_n \phi + \sqrt{2}F\xi \qquad \delta_\xi \psi^\dagger = \sqrt{2}i\xi^T c\sigma^n \partial_n \phi^* + \sqrt{2}\xi^\dagger F^*$$

$$\delta_\xi F = -\sqrt{2}i\xi^\dagger \overline{\sigma}^m \partial_m \psi \qquad \delta_\xi F^* = \sqrt{2}i\partial_m \psi^\dagger \overline{\sigma}^m \xi$$

This multiplet contains a complex field ϕ a left-handed fermion field ψ , and a complex field F. Its particle content is the scalar particle ϕ and its antiparticle, and the chiral fermion ψ and its antiparticle $-$ thus 4 Bose and 4 Fermi fields, yielding 2 Bose and 2 Fermi particles. The kinetic term invariant under SUSY is:

$$\mathcal{L} = |\partial_m \phi|^2 + \psi^\dagger i\overline{\sigma}^m \partial_m \psi + |F|^2$$

To give a mass or some nonlinear dynamics to these fields, we need to add terms with no derivatives. These are also constrained by SUSY. The general form is

$$\mathcal{L} = \left(F\frac{dW}{d\phi} - \frac{1}{2}\psi^T c\psi \ \frac{d^2W}{d\phi^2} \right) + h.c.$$

where $W(\phi)$ is an analytic function of ϕ. $W(\phi)$ is called the superpotential. This contruction generalizes to many fields, with W an analytic function of the ϕ_a.

The simplest example is: $W = \frac{1}{2}m\phi^2$. Then

$$|\partial_m\phi|^2 + \psi^\dagger i\overline{\sigma}^m\partial_m\psi + |F|^2 + [mF\phi - \frac{1}{2}m\psi^T c\psi] + h.c.$$

F obeys the simple equation $F^* + m\phi = 0$ and we can eliminate it. This gives

$$\mathcal{L} = |\partial_m\phi|^2 - m^2|\phi|^2 + \psi^\dagger i\overline{\sigma}^m\partial_m\psi - \frac{1}{2}m(\psi^T c\psi - \psi^\dagger c\psi)$$

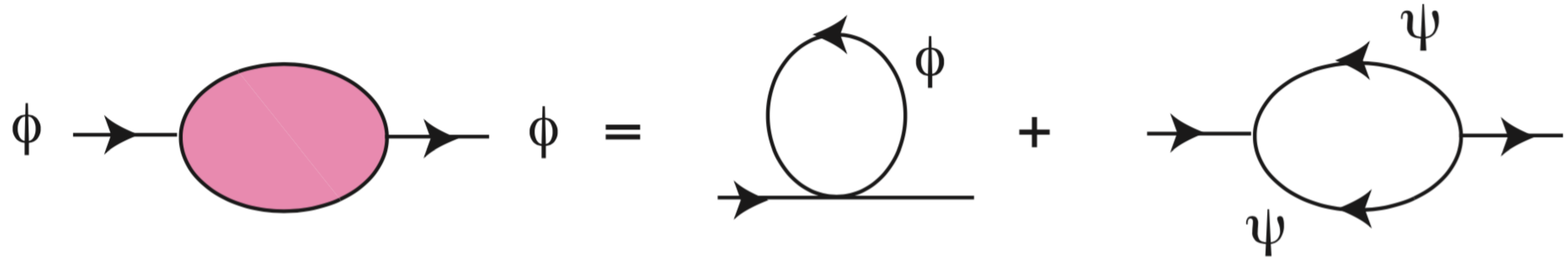
a theory of a massive boson and a massive Majorana fermion, both with mass m.

Another simple example is $W = g\phi^3/3$. This yields a nonlinear theory

$$\mathcal{L} = |\partial_m \phi|^2 + \psi^\dagger i\overline{\sigma}^m \partial_m \psi + |F|^2 + [gF\phi^2 - g\phi\,\psi^T c\psi] + h.c.$$
$$= |\partial_m \phi|^2 + \psi^\dagger i\overline{\sigma}^m \partial_m \psi - g^2|\phi^2|^2 - g\phi[\psi^T c\psi - \psi^\dagger c\psi^*]$$

with a relation between the Yukawa and the ϕ^4 coupling.

The ψ field cannot receive a mass due to chiral symmetry. However, it is not obvious that the ϕ cannot receive a quadratically divergent mass. So, compute the 1-loop diagrams.



boson loop: $$(-4ig^2)\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2} = 4g^2\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2}$$

fermion loop:

$$(-2ig)^2\cdot\frac{1}{2}\cdot(-1)\int \frac{d^4k}{(2\pi)^4}\mathrm{tr}\left[\frac{i\sigma\cdot k}{k^2}c\frac{[i\sigma\cdot(-k)]^T}{k^2}c\right]$$

$$= -2g^2\int \frac{d^4k}{(2\pi)^4}\frac{2}{k^2}$$

It can be shown that this cancellation persists to all orders.

In fact, there is a stronger result: Quite generally,
the superpotential recieves no additive renormalizations.

Now we have a supersymmetric theory of fermions and scalars. We can add vector fields and their fermionic partners, which come in vector supermultiplets. We can give masses to quarks and leptons by writing a Yukawa superpotential

$$W = y_e L_a H_{da} \overline{e} + y_d Q_a H_{da} \overline{d} + y_u Q_a \epsilon_{ab} H_{ub} \overline{u}$$

with $L = (\nu, e) \quad Q = (u, d)$

Note that, because of analyticity two Higgs fields are required, one with Y = +1/2, one with Y = -1/2. Then we can add

$$W = -\mu H_{da} \epsilon_{ab} H_{ub}$$

At this moment, the Higgs potential is

$$V = \mu^2 (|H_d|^2 + H_u|^2)$$

and there is no sign of SU(2)xU(1) breaking.

This theory is not yet realistic, because it gives equal mass to each Standard Model particle and its SUSY partner.

To make a realistic theory, we must break SUSY. The simplest way is to add soft perturbations — dimension 2 and 3 operators that give mass to the SUSY partners.

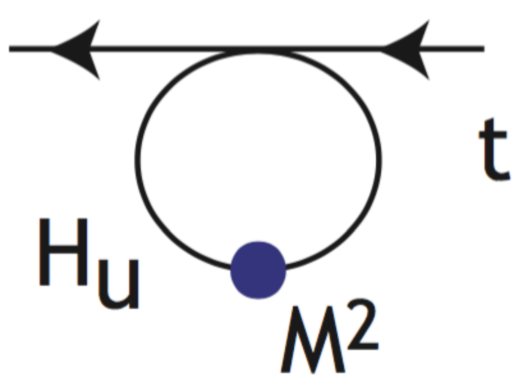
If SUSY is spontaneously broken by particles of mass M, integrating out these particles will generate such soft mass terms.

The resulting theory is called the Minimal Supersymmetric Standard Model (MSSM). It has 105 parameters. (Not all are important.)

The MSSM has a complex phenomenology. Here, I would like to consider only one aspect of it, the effect on the Higgs potential of soft mass terms for the scalar fields t , $\bar{t}$, H_u :

$$\delta L = -M_t^2 |t|^2 - M_{\bar{t}}^2 |\bar{t}|^2 - M_H^2 |H_u|^2$$

Whatever we put for the original values of these mass terms, they receive loop corrections from the 4-scalar terms coming from the Yukawa superpotential. A typical term is



$$= -i y_t^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} (-i M_H^2) \frac{i}{k^2}$$

$$= y_t^2 M^2 \frac{i}{(4\pi)^2} \log \Lambda^2$$

Comparing to the t mass insertion $-iM_t |t|^2$, this is a negative contribution to M_t^2 . Similar contributions are generated from each scalar for each scalar.

Now we can write the renomalization group equations for $M_t^2, M_{\bar{t}}^2, M_H^2$. The corrections due to y_t all have the same structure, but the coefficients are different, due to the number of SU(2)xSU(3) quantum numbers flowing in the loop.

$$\frac{dM_t^2}{d\log Q} = \frac{2y_t^2}{(4\pi)^2} \cdot 1 \cdot [M_t^2 + M_{\bar{t}}^2 + M_H^2] - \frac{8}{3\pi}\alpha_s^2 + \cdots$$

$$\frac{dM_{\bar{t}}^2}{d\log Q} = \frac{2y_t^2}{(4\pi)^2} \cdot 2 \cdot [M_t^2 + M_{\bar{t}}^2 + M_H^2] - \frac{8}{3\pi}\alpha_s^2 + \cdots$$

$$\frac{dM_H^2}{d\log Q} = \frac{2y_t^2}{(4\pi)^2} \cdot 3 \cdot [M_t^2 + M_{\bar{t}}^2 + M_H^2] + \cdots$$

The sign is such that the M^2 values are driven smaller as we go to the infrared. M_H^2 is driven fastest.

When M_H^2 becomes negative, H acquires a vacuum expectation value, and SU(2)xU(1) is spontaneously broken.

This is the result I promised: The top quark Yukawa coupling drives an instability to electroweak symmetric breaking, with a contribution that is computable in terms of the parameters of the theory.

In the MSSM, there are other possible competing effects, but, typically, this contribution to the Higgs potential is the dominant one.

Now look at a realization of this idea in a very different kind of model, one in which the Higgs is a composite particle.

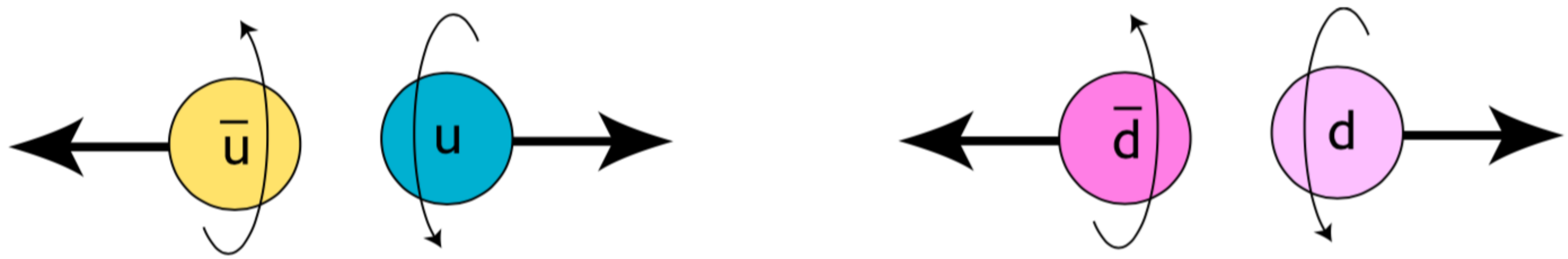
To begin, think about QCD with 3 generations of quarks (u, d, s). In the real world, these quarks have mass, but imagine setting their masses to zero. Then the QCD Lagrangian is

$$\mathcal{L} = -\frac{1}{4}(F^a_{mn})^2 + q^\dagger_{Lf} i\overline{\sigma}^m D_m q_{Lf} + q^\dagger_{Rf} i\sigma^m D_m q_{rf}$$

where $f = (u, d, s)$. This theory has the global symmetry

$$SU(3) \times SU(3)$$

If the symmetry $SU(3) \times SU(3)$ remained explicit, we would have a spectrum with massless hadrons, including massless baryons. However, this state is not energetically favored. Instead, the strong interactions bind spin 0, color singlet pairs such as



and these fill the vacuum (as e-e- pairs condense in the ground state of a superconductor). Note that q_{fL} pairs with $\overline{q}_{fL}$, which is the antiparticle of q_{fR}. Then the pair condensation corresponds to a state with

$$\langle \overline{q}_{fR} q_{fL} \rangle \neq 0$$

This links together the two $SU(3)$ symmetry groups and drives the symmetry breaking

$$SU(3) \times SU(3) \to SU(3)$$

This theory has a manifold of degenerate vacuum states. Its vacuum could be described by

$$\left\langle q_{fL} \cdot \overline{q}_{f'R} \right\rangle = -\Delta \; U_{ff'}$$

where $U_{ff'}$ is an SU(3) unitary matrix. Under an $SU(3) \times SU(3)$ transformation, this expectation value transforms as

$$\left\langle q_{fL} \cdot \overline{q}_{f'R} \right\rangle \to -\Delta \; V_L U V_R^\dagger$$

so all of the possible vacuum states are related by symmetry.

In this symmetry breaking, 8 symmetries are spontaneously broken, so we should find 8 Goldstone bosons. These are local rotations of the variable $U_{ff'}$.

We can write a field theory for the Goldstone bosons by generalizing U to a variable that depends on x and transforms under SU(3)xSU(3) as

$$U(x) \to V_L \; U(x) V_R^\dagger$$

U(x) is a unitary matrix, so we can parametrize it as

$$U(x) = e^{2i\Pi^a(x) t^a / f_\pi}$$

where t^a is a generator of SU(3). This introduces a dimensionful constant f_π, which turns out to be the pion decay constant

$$f_\pi = 93 \text{ MeV}$$

The simplest Lagrangian for U is

$$\mathcal{L} = \frac{1}{4} f_\pi^2 \ \mathrm{tr}[\partial_\mu U^\dagger \partial^\mu U]$$

Expanding, this is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Pi^a)^2 + \cdots$$

Note that it is not possible to write a mass term for Π^a consistent with SU(3)xSU(3) symmetry. A possible candidate is

$$\delta\mathcal{L} = m^2 \mathrm{tr}[U^\dagger U] = \mathrm{const}$$

This is in line with Goldstone's theorem; the Π^a fields must be massless.

The 8 Goldstone bosons have the quantum numbers of

$$\pi^+, \pi^-, \pi^0, K^+, K^0, \overline{K}^0, K^-, \eta$$

If we had $m_u = m_d = m_s = 0$ (and ignored electromagnetism, which breaks the chiral symmetry), these states would be exactly massless. With nonzero masses, as in real QCD, we still understand why these states are the lightest hadrons, and we can compute the 8 masses in terms of 3 parameters.

The nonlinear terms in the Lagrangian for U correctly predict the low energy interactions of pions and kaons.

It is very suggestive that we should use chiral symmetry breaking in QCD as a mechanism for SU(2)xU(1) breaking. The setup would be the following: Postulate a copy of QCD at high energies, with 2 massless quarks (U,D) coupled to SU(2)xU(1) in the same manner as the familiar quarks. We can write a nonlinear Lagrangian based on a 2x2 unitary matrix field U. Its covariant derivative under SU(2)xU(1) is

$$
\begin{aligned}
D_\mu U &= \partial_\mu U - igA^a_\mu \frac{\sigma^a}{2} U - ig' B_\mu \frac{1}{6} U + ig' B_\mu U \begin{pmatrix} 2/3 & \\ & -1/3 \end{pmatrix} \\
&= \partial_\mu U - igA^a_\mu \frac{\sigma^a}{2} U + ig' B_\mu U \frac{\sigma^3}{2}
\end{aligned}
$$

Write the nonlinear Lagrangian for this new theory

$$\mathcal{L} = \frac{1}{4} F^2 \; \mathrm{tr}[\partial_\mu U^\dagger \partial^\mu U]$$

couple it to SU(2)xU(1) gauge fields,

$$\mathcal{L} = \frac{1}{4} F^2 \; \mathrm{tr}[D_\mu U^\dagger D^\mu U]$$

and expand about U = 1:

$$\mathcal{L} = \frac{1}{4} F^2 \; \mathrm{tr}[(g A^a_\mu \frac{\sigma^a}{2} - g' B_\mu \frac{\sigma^3}{2})^2]$$

$$= \frac{1}{8} F^2 \left\{ (g A^1_\mu)^2 + (g A^2_\mu)^2 + (g A^3_\mu - g' B_\mu)^2 \right\}$$

This gives exactly the masses of the vector bosons of the Standard Model

$$m_W^2 = \frac{g^2 F^2}{4} \qquad m_Z^2 = \frac{(g^2 + g'^2) F^2}{4}$$

for $F = v = 246 \ \mathrm{GeV}$

So, if we introduce a new set of QCD-like interactions, with $F = 246 \text{ GeV}$, $m_\rho \approx 2 \text{ TeV}$.

and couple it to SU(2)xU(1), we get a dynamical model of electroweak symmetry breaking. This model is called technicolor; it was introduced in 1978 by Weinberg and Susskind.

Today, we see that the model has a fatal flaw. QCD has no light 0^+ state. So, this model contains no boson with the properties of the 125 GeV Higgs boson.

However, there is a different way that we could use new strong interactions to make a dynamical theory of SU(2)xU(1) breaking. We could identify the Higgs boson scalar doublet with a set of Goldstone bosons. Then Goldstone's theorem will protect the Higgs boson from obtain a quadratically divergent mass correction.

This idea is called "Little Higgs" (Arkani-Hamed, Cohen, Katz, Nelson).

Here is a concrete realization: Consider a theory with 3 flavors and QCD scale of about 10 TeV, so that the pion decay constant is $F \sim 1 \text{ TeV}$. Embed SU(2)xU(1) in the final unbroken symmetry SU(3). Then the 8 Goldstone bosons have the quantum numbers

$$\Pi = \begin{pmatrix} \Phi_0 + i\Phi_a\sigma^a & \varphi \\ \varphi^\dagger & -2\Phi_0 \end{pmatrix}$$

In particular, φ is an SU(2) complex doublet, and we can try to identify these Goldstone bosons with the Higgs doublet.

Couple this to SU(2)xU(1) by

$$D_\mu U = \partial_\mu U - i\Big[(gA_\mu^a \begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix} + ig' B_\mu \begin{pmatrix} 0 & \\ & 1/2 \end{pmatrix}), U\Big]$$

Then the vacuum state U = 1 gives zero masses for W and Z. However, if we can induce a further symmetry breaking to

$$\varphi = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$$

we will obtain the standard W and Z masses.

So far, this theory has no dynamics that drives this symmetry breaking. But, if we introduce couplings that break the SU(3)xSU(3) chiral symmetry, we might find such effects.

Try to use these ingredients to write a top quark Yukawa interactions.

To do this, introduce a triplet of left-handed quarks and two right-handed singlets

$$\chi_L = \begin{pmatrix} t \\ b \\ \hat{T} \end{pmatrix}_L \qquad \hat{t}_R \quad \hat{T}_R$$

and write the Yukawa terms

$$\delta\mathcal{L} = -y_1 F \; \bar{\hat{t}}_R U^\dagger_{3i} \chi_{iL} - y_2 F \bar{\hat{T}}_R \hat{T}_L + h.c.$$

In a more complete theory, these should have a dynamical origin, with composite t quarks.

This is the simplest structure possible, but it has some nontrivial features.

The first term preserves $SU(3)_L$ but breaks $SU(3)_R$.

The second term breaks $SU(3)_L$ but preserves $SU(3)_R$.

Notice that if either $SU(3)_L$ or $SU(3)_R$ is an exact symmetry, Goldstone's theorem requires that the should be massless. Thus, the Higgs mass is protect up to terms of order $y_1^2 y_2^2$.

Compute the Higgs mass explicitly in perturbation theory. For U = 1, one heavy quark gets a mass, and the mass eigenstates are

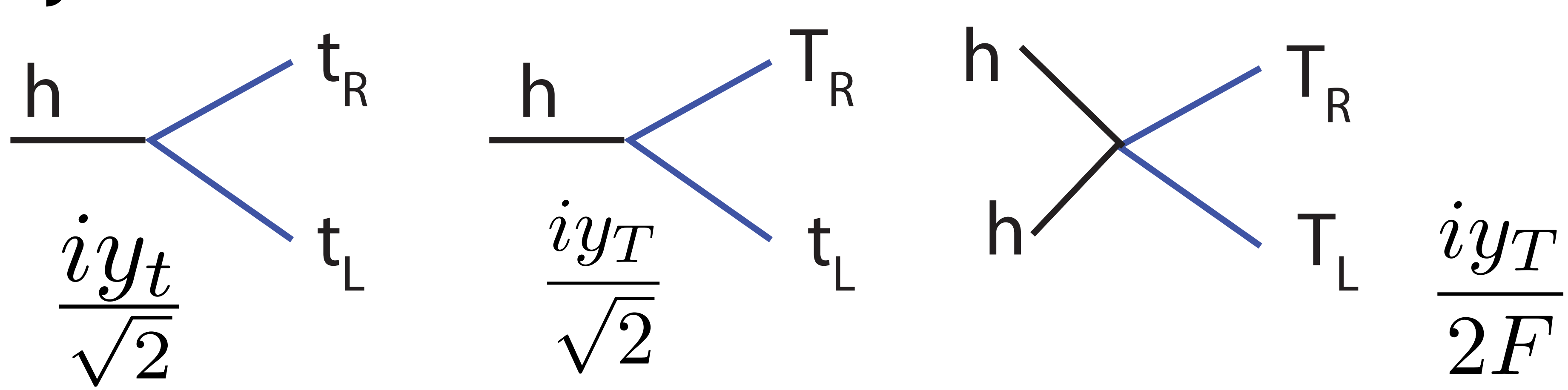
$$T_L = \hat{T}_L \qquad T_R = \frac{y_1 \hat{t}_R + y_2 \hat{T}_R}{\sqrt{y_1^2 + y_2^2}} \qquad m_T = \sqrt{y_1^2 + y_2^2}\; F$$

$$t_L = t_L \qquad t_R = \frac{y_2 \hat{t}_R - y_1 \hat{T}_R}{\sqrt{y_1^2 + y_2^2}} \qquad m_t = 0$$

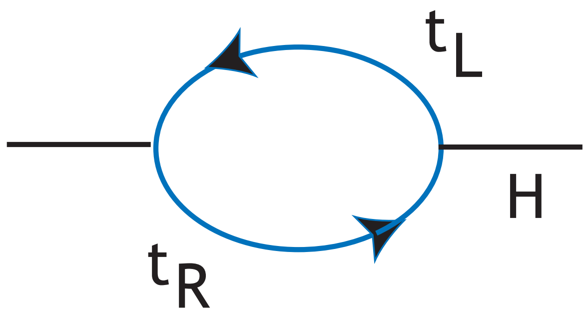
If we had set $v \neq 0$, we would also have a mass for t

$$m_t = \frac{y_t v}{\sqrt{2}} \qquad y_t = \frac{y_1 y_2}{\sqrt{y_1^2 + y_2^2}} \qquad y_T = \frac{y_1^2}{\sqrt{y_1^2 + y_2^2}}$$

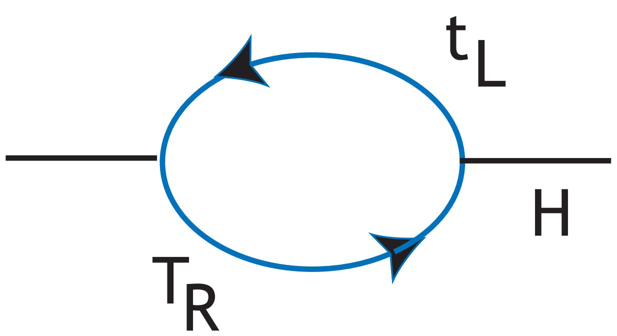
The Feynman rules are



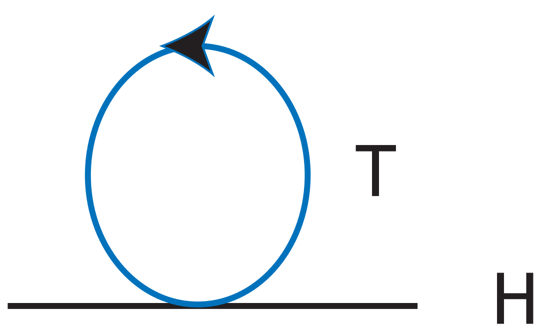
Three diagrams contribute to the Higgs mass term



$$= -6y_t^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}$$



$$= -6y_T^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_T^2}$$



$$= +6\frac{y_T}{F} \int \frac{d^4k}{(2\pi)^4} \frac{m_T}{k^2 - m_T^2}$$

The sum is

$$= -6\frac{y_1 y_2}{y_1^2 + y_2^2} \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2} - \frac{1}{k^2 - m_T^2} \right]$$

The final result simplifies to

$$\delta\mu^2 = -\frac{3}{8\pi^2} y_1^2 y_2^2 F^2 \log\frac{\Lambda^2}{m_T^2}$$

$$\delta\mu^2 = -\frac{3}{8\pi^2} y_t^2 m_T^2 \log\frac{\Lambda^2}{m_T^2}$$

It is free of quadratic UV divergences. It has the sign of the top quark loop, that is, a negative contribution to μ^2 .

This is plausibly the largest radiative correction that contributes to the Higgs potential.

Thus, here again, it is the top quark Yukawa coupling that drives the instability to SU(2)xU(1) breaking.

I hope that these two examples give you a taste of how to build dynamical models of electroweak symmetry breaking that might be relevant to the real world.

It is suggestive that the top quark plays an important role. Certainly, there is much about this heaviest quark that we do not yet understand.