

Topics in Electroweak Symmetry Breaking

1. The Goldstone Boson Equivalence Theorem

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In this lecture, I will describe the properties of the weak interactions at energies much greater than m_W, m_Z .

Some important conceptual issues arise here, especially when we search for new physics beyond the Standard Model. In particular, how do we parametrize possible deviations of the W, Z properties from the Standard Model predictions? There are dangers if you do this in the wrong way.

I would like to recommend a skeleton key for thinking about these issues, called the Goldstone Boson Equivalence Theorem.

Let's begin with the following question, which was one of the most difficult aspects to understand about spontaneously broken gauge theories:

In the rest frame, a massive vector boson has 3 polarization states

$$\epsilon_{\pm}^{\mu} = (1/\sqrt{2})(0, 1, \pm i, 0)^{\mu}$$

$$\epsilon_0^{\mu} = (0, 0, 0, 1)^{\mu}$$

representing the 3 possible states of a spin 1 particle with $J^3 = \pm 1, 0$.

Now boost along the 3 axis to high energy. The boosts of the polarization vectors are

$$\epsilon_{\pm}^{\mu} = (1/\sqrt{2})(0, 1, \pm i, 0)^{\mu}$$

$$\epsilon_0^{\mu} = \left(\frac{p}{m}, 0, 0, \frac{E}{m}\right)^{\mu}$$

Note that, as E becomes large, the components of ϵ_0^μ grow without bound; in fact,

$$\epsilon_0^\mu \rightarrow \frac{p^\mu}{m}$$

Another way to express this is that the polarization sum is

$$\sum \epsilon_i^\mu \epsilon_j^\nu = - \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2} \right)$$

and the second term on the right has unbounded matrix elements.

This potentially leads to very large contributions to vector boson amplitudes, even threatening violation of unitarity.

For example, the amplitude for production of a scalar in e^+e^- annihilation is

$$\mathcal{M}(e^+e^- \rightarrow \phi^+\phi^-) = i\frac{e^2}{s}(2E)\sqrt{2}\epsilon_- \cdot (k_+ - k_-)$$

In $e^+e^- \rightarrow W^+W^-$, we might expect

$$\mathcal{M} \sim i\frac{e^2}{s}(2E)\sqrt{2}\epsilon_- \cdot (k_+ - k_-) \epsilon^*(W^-) \cdot \epsilon^*(W^+)$$

But, for longitudinally polarized W bosons, this extra factor becomes

$$\frac{k_- \cdot k_+}{m_W^2} = \frac{s - 2m_W^2}{2m_W^2}$$

and this really does violate unitarity at high energy.

So, the question is: When are these enhancements from the form of ϵ_0^μ real — always, sometimes, or never?

The answer is given by the **Goldstone Boson Equivalence Theorem** of Cornwall and Tiktopoulos and Vayonakis:

In the Higgs mechanism, **a massive W boson acquired its longitudinal component by absorbing a Goldstone boson from the Higgs sector.** When the W is at rest, it is not so clear which polarization state comes from the original vector boson and which comes from the Higgs boson. However, for a highly boosted W, there is a clear distinction between the transverse and longitudinal polarization states. Then,

$$\mathcal{M}(X \rightarrow Y + W_0^+(p)) = \mathcal{M}(X \rightarrow Y + \pi^+(p)) (1 + \mathcal{O}(m_W/E_W))$$

The proof is too complicated to give in this lecture; an excellent reference is Chanowitz and Gaillard, Nucl. Phys. B261, 379 (1985). **The important point is the proof makes essential use of gauge invariance.**

I will discuss three examples that probe different aspects of the application of this theorem.

The first is the theory of W polarization in top quark decay.

The matrix element is

$$\mathcal{M}(t \rightarrow bW^+) = i \frac{g}{\sqrt{2}} u_L^\dagger(b) \bar{\sigma}^\mu u_L(t) \epsilon_\mu^*(W)$$

It is a good approximation to ignore the b quark mass. I will use coordinates in which the t is at rest with spin and the W moves in the 3 direction. Then

$$u_L(b) = \sqrt{2E_b} \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad u_L(t) = \sqrt{m_t} \xi$$

For a W_{-}^{+} , $\bar{\sigma}^{\mu} \epsilon_{-\mu}^{*} = \frac{1}{\sqrt{2}} (\sigma^1 + i\sigma^2) = \sqrt{2}\sigma^{+}$

and the amplitude is : $\mathcal{M} = -ig\sqrt{2m_t E_b} \xi_2$

For a W_{+}^{+} , $\bar{\sigma}^{\mu} \epsilon_{+\mu}^{*} = \frac{1}{\sqrt{2}} (\sigma^1 - i\sigma^2) = \sqrt{2}\sigma^{-}$

and the amplitude is : $\mathcal{M} = 0$

For a W_0^{+} , $\bar{\sigma}^{\mu} \epsilon_{0\mu}^{*} = -\left(\frac{p + E\sigma^3}{m_W}\right)$

and the amplitude is : $\mathcal{M} = ig\sqrt{2m_t E_b} \left(\frac{m_t}{m_W}\right) \xi_1$

Averaging over the t spin direction and integrating over phase space, we find

$$\Gamma(t \rightarrow bW_{-}^{\pm}) = \frac{1}{2m_t} \frac{1}{16\pi} \frac{2p}{m_t} \cdot g^2(2pm_t)$$

Using the kinematic relation $2pm_t = m_t^2 - m_W^2$ we then find

$$\Gamma(t \rightarrow bW_{-}^{\pm}) = \frac{\alpha_w}{8} m_t \left(1 - \frac{m_W^2}{m_t^2}\right)^2$$

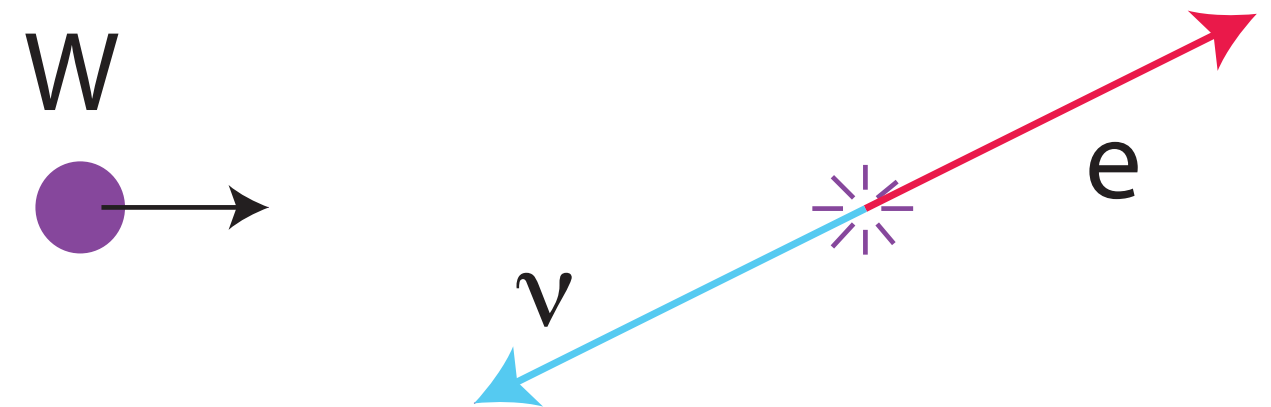
and similarly $\Gamma(t \rightarrow bW_{+}^{\pm}) = 0$

$$\Gamma(t \rightarrow bW_0^{\pm}) = \frac{\alpha_w}{8} m_t \left(1 - \frac{m_W^2}{m_t^2}\right)^2 \cdot \frac{m_t^2}{2m_W^2}$$

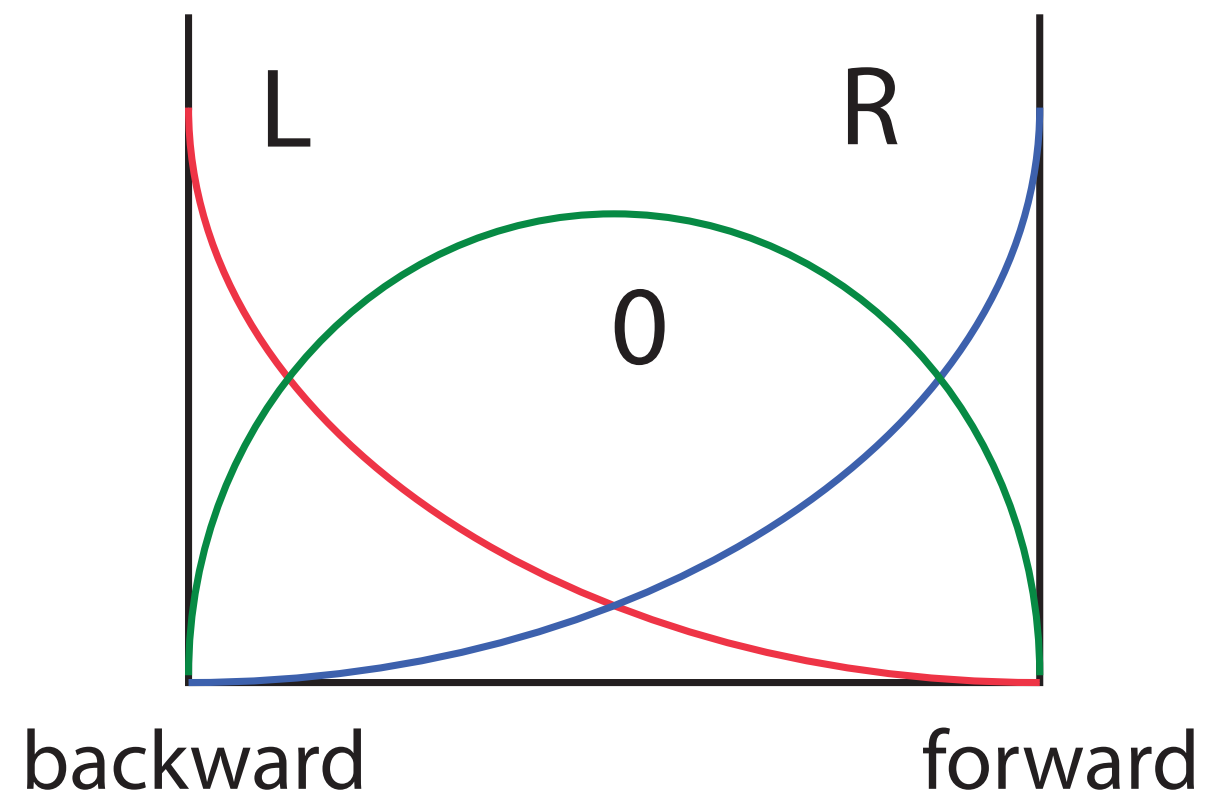
so the enhancement of the longitudinal polarization state is really predicted.

$$\frac{\Gamma(t \rightarrow bW_0^{\pm})}{\Gamma(t \rightarrow bW^{\pm})} = \frac{m_t^2/2m_W^2}{1 + m_t^2/2m_W^2} \approx 70\%$$

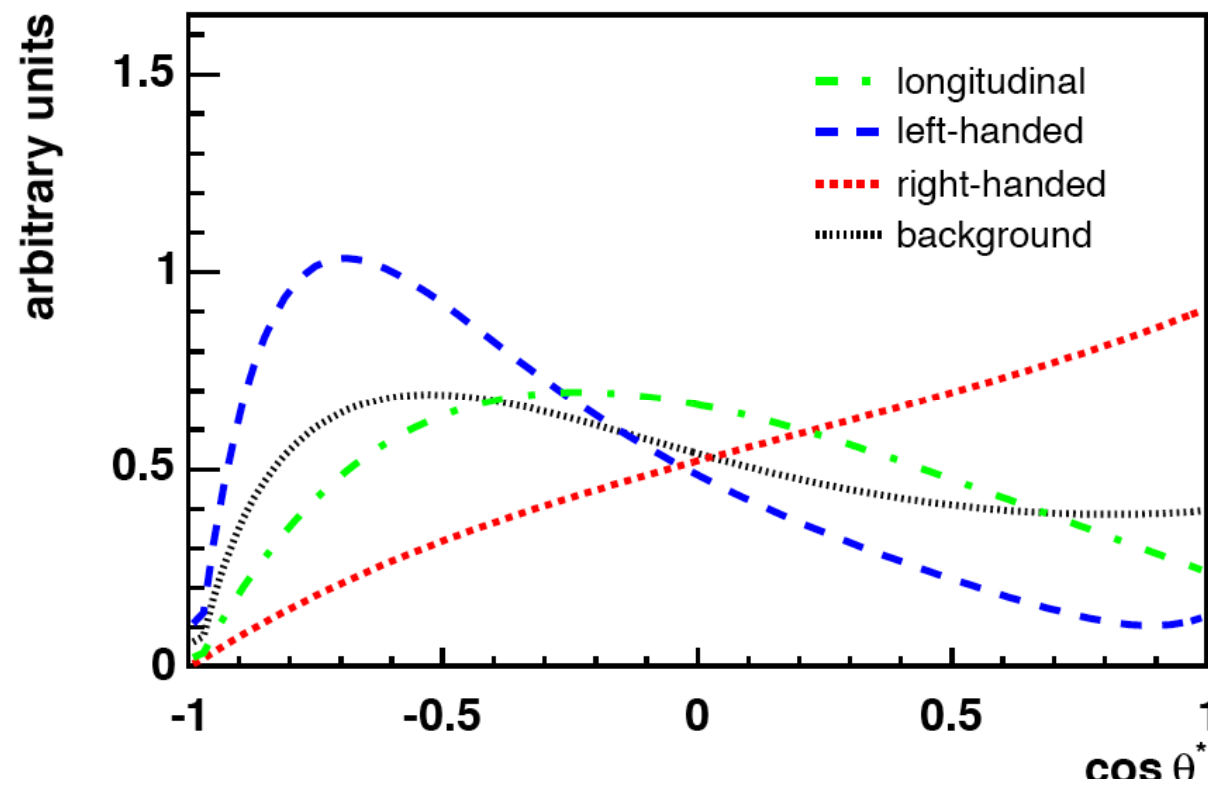
What does experiment say ? We can test this by reconstructing $pp \rightarrow t\bar{t} \rightarrow \ell\nu 4j$ and measuring the angular distribution of the W decay products



$$\frac{d\Gamma}{d\cos\theta} \sim \begin{cases} (1 + \cos\theta)^2 & + \\ \sin^2\theta/2 & 0 \\ (1 - \cos\theta)^2 & - \end{cases}$$

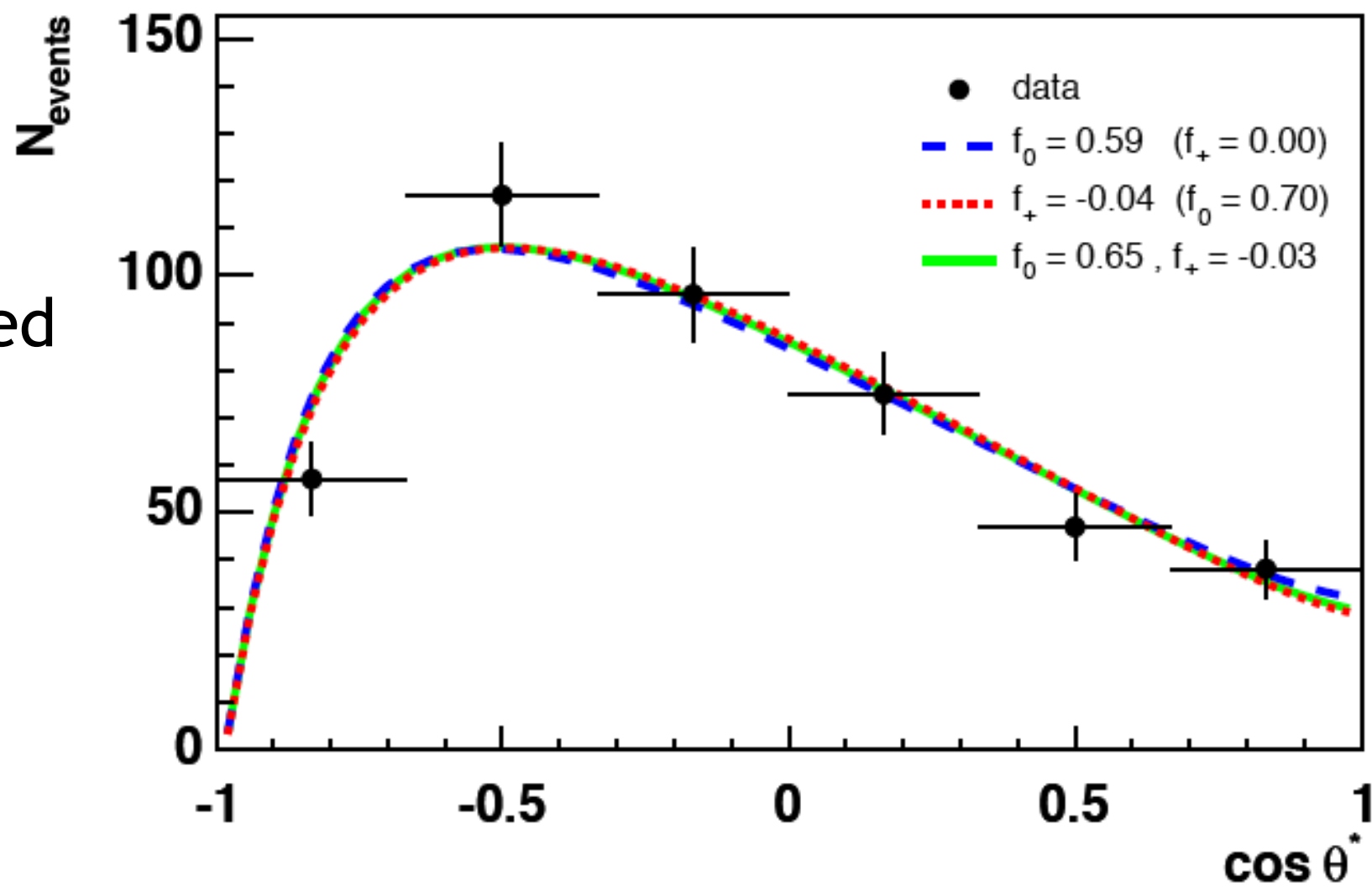


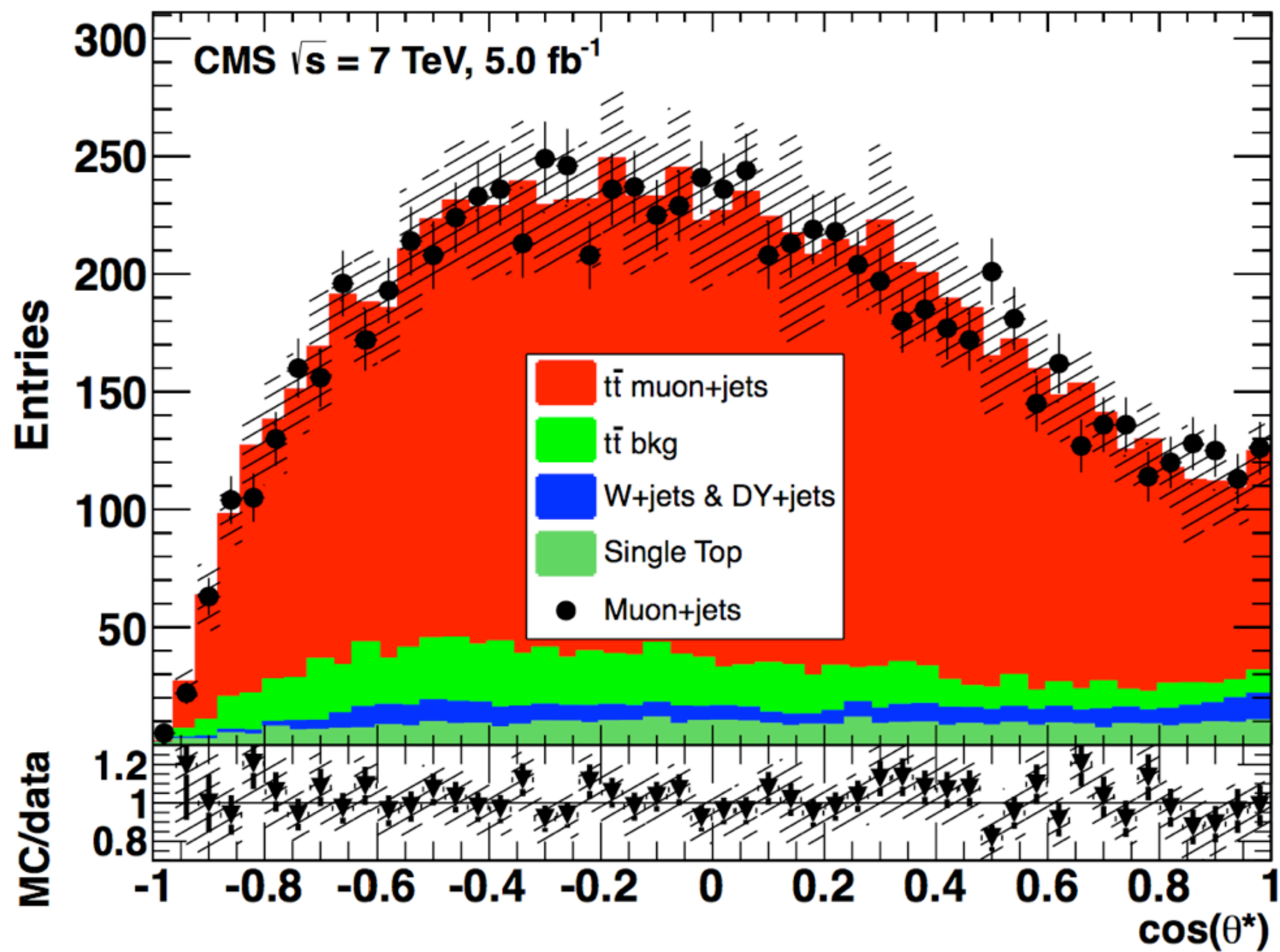
templates



CDF
experiment

observed





What does the GBET have to say ?

According to the GBET, we should have

$$\mathcal{M}(t \rightarrow bW_0^+) \rightarrow \mathcal{M}(t \rightarrow b\pi^+)$$

The amplitude for emission of a Higgs boson should be proportional to the top quark Yukawa coupling, given by

$$m_t = \frac{y_t v}{\sqrt{2}}$$

So, the W_0^+ amplitude should be larger by the factor

$$\frac{y_t^2}{g^2} = \frac{2m_t^2/v^2}{4m_W^2/v^2} = \frac{m_t^2}{2m_W^2}$$

which is exactly what we found.

Turn next to the process

$$e^+ e^- \rightarrow W_0^- W_0^+$$

I argued earlier that the apparently enhancement in this process is probably spurious, since it violates unitarity.

The GBET says:

$$\mathcal{M}(e^+ e^- \rightarrow W_0^- W_0^+) \rightarrow \mathcal{M}(e^+ e^- \rightarrow \pi^+ \pi^-)$$

This implies (using also the high energy limit of
 $SU(2) \times U(1)$ and the Higgs quantum nos $(I, Y) = (\frac{1}{2}, \frac{1}{2})$)

for $e_R^- e_L^+$:

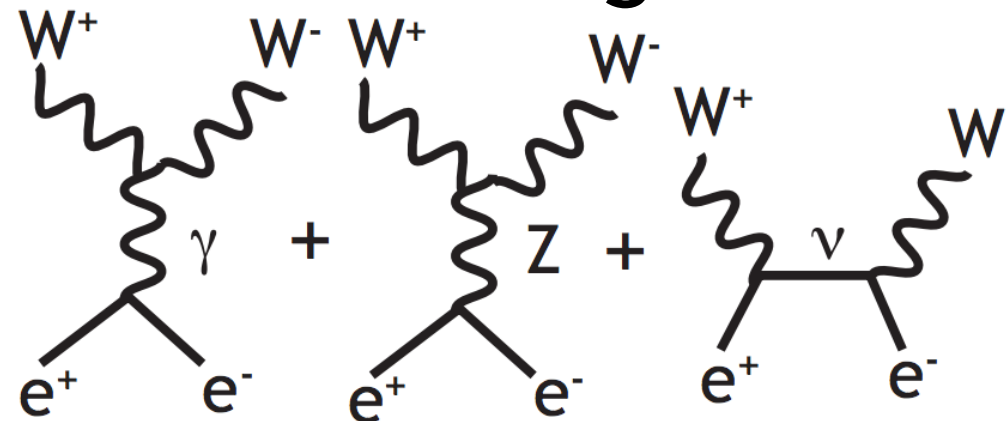
$$\mathcal{M} = -i(2E)\sqrt{2}\epsilon_+ \cdot (k_- - k_+) \cdot \frac{e^2}{2c_w^2} \frac{1}{s}$$

for $e_L^- e_R^+$:

$$\mathcal{M} = -i(2E)\sqrt{2}\epsilon_- \cdot (k_- - k_+) \cdot \left(\frac{e^2}{4c_w^2} \frac{1}{s} + \frac{e^2}{4s_w^2} \frac{1}{s} \right)$$

so some cancellation of the effect discussed above must occur.

In fact, at leading order in the Standard Model, the amplitude is the sum of 3 diagrams.



Work this out carefully for $e_R^- e_L^+$, where the ν diagram is absent.

$$i\mathcal{M} = (-ie)(ie)2E\sqrt{2}\epsilon_{+\mu} \left[\frac{-i}{s} + \frac{-s_w^2}{s_w c_w} \frac{c_w}{s_w} \frac{-i}{s - m_Z^2} \right] \\ \cdot \left[\epsilon_-^* \epsilon_+^* (k_- - k_+)^{\mu} + \epsilon_-^{*\mu} (-q - k_-) \cdot \epsilon_+^* + \epsilon_+^{*\mu} (q + k_+) \cdot \epsilon_-^* \right]$$

where $q = k_- + k_+$ and, in the 2nd line, ϵ_-^* and ϵ_+^* are the W polarizations. Send

$$\epsilon_-^* \rightarrow \frac{k_-}{m_W} \quad \epsilon_+^* \rightarrow \frac{k_+}{m_W}$$

Then the second term in brackets becomes

$$\frac{1}{m_W^2} \left[2k_- \cdot k_+ (k_- - k_+)^{\mu} + k_-^{\mu} (-2k_- \cdot k_+) + k_+^{\mu} (2k_+ \cdot k_-) \right]$$

$$= -\frac{k_+ \cdot k_-}{m_W^2} (k_- - k_+)^{\mu} = -\frac{s - 2m_W^2}{2m_W^2} (k_- - k_+)^{\mu}$$

On the other hand, the first term in brackets becomes

$$\left[\frac{-i}{s} - \frac{-i}{s - m_Z^2} \right] = \frac{im_Z^2}{s(s - m_Z^2)}$$

Assembling the pieces and using $m_Z^2/m_W^2 = 1/c_w^2$,
we find

$$i\mathcal{M} = i(e^2)2E\sqrt{2}\epsilon_{+\mu}(k_- - k_+)^{\mu} \left(\frac{1}{s(s - m_Z^2)} \right) \left(-\frac{s - 2m_W^2}{2c_w^2} \right)$$

which gives the predicted expression at high energy.

For $e_L^- e_R^+$, we must work a little harder. The first two diagrams contribute

$$i\mathcal{M} = (-ie)(ie)2E\sqrt{2}\epsilon_{-\mu} \left[\frac{-i}{s} + \frac{(1/2 - s_w^2)}{s_w c_w} \frac{c_w}{s_w} \frac{-i}{s - m_Z^2} \right] \\ \cdot \left[\epsilon_-^* \epsilon_+^* (k_- - k_+)^{\mu} + \epsilon_-^{*\mu} (-q - k_-) \cdot \epsilon_+^* + \epsilon_+^{*\mu} (q + k_+) \cdot \epsilon_-^* \right]$$

After the reductions described above, there is a term that does not cancel its high energy behavior

$$i\mathcal{M} = (-ie^2)2E\sqrt{2}\epsilon_{-\mu} \left[\frac{1}{2s_w^2} \frac{1}{s} \right] \left(-\frac{s}{2m_W^2} (k_- - k_+)^{\mu} \right) \\ = \left(\frac{ie^2}{4s_w^2} \right) 2E\sqrt{2}\epsilon_{-\mu} \frac{1}{m_W^2} (k_- - k_+)^{\mu}$$

However, now we must add the ν diagram, which contributes

$$i\mathcal{M} = \left(i\frac{g}{\sqrt{2}}\right)^2 v_R^\dagger \bar{\sigma} \cdot \epsilon_+^* \frac{i\sigma \cdot (p - k_-)}{(p - k_-)^2} \bar{\sigma} \cdot \epsilon_-^* u_L(p)$$

Substitute $\epsilon_-^* \rightarrow k_-/m_W$ and simplify

$$\frac{\sigma \cdot (p - k_-)}{(p - k_-)^2} \bar{\sigma} \cdot \frac{k_-}{m_W} u(p) = \frac{\sigma \cdot (p - k_-)}{(p - k_-)^2} \bar{\sigma} \cdot \frac{(k_- - p)}{m_W} u(p) = -\frac{1}{m_W} u(p)$$

Also $\epsilon_+^* \rightarrow k_+/m_W$ and

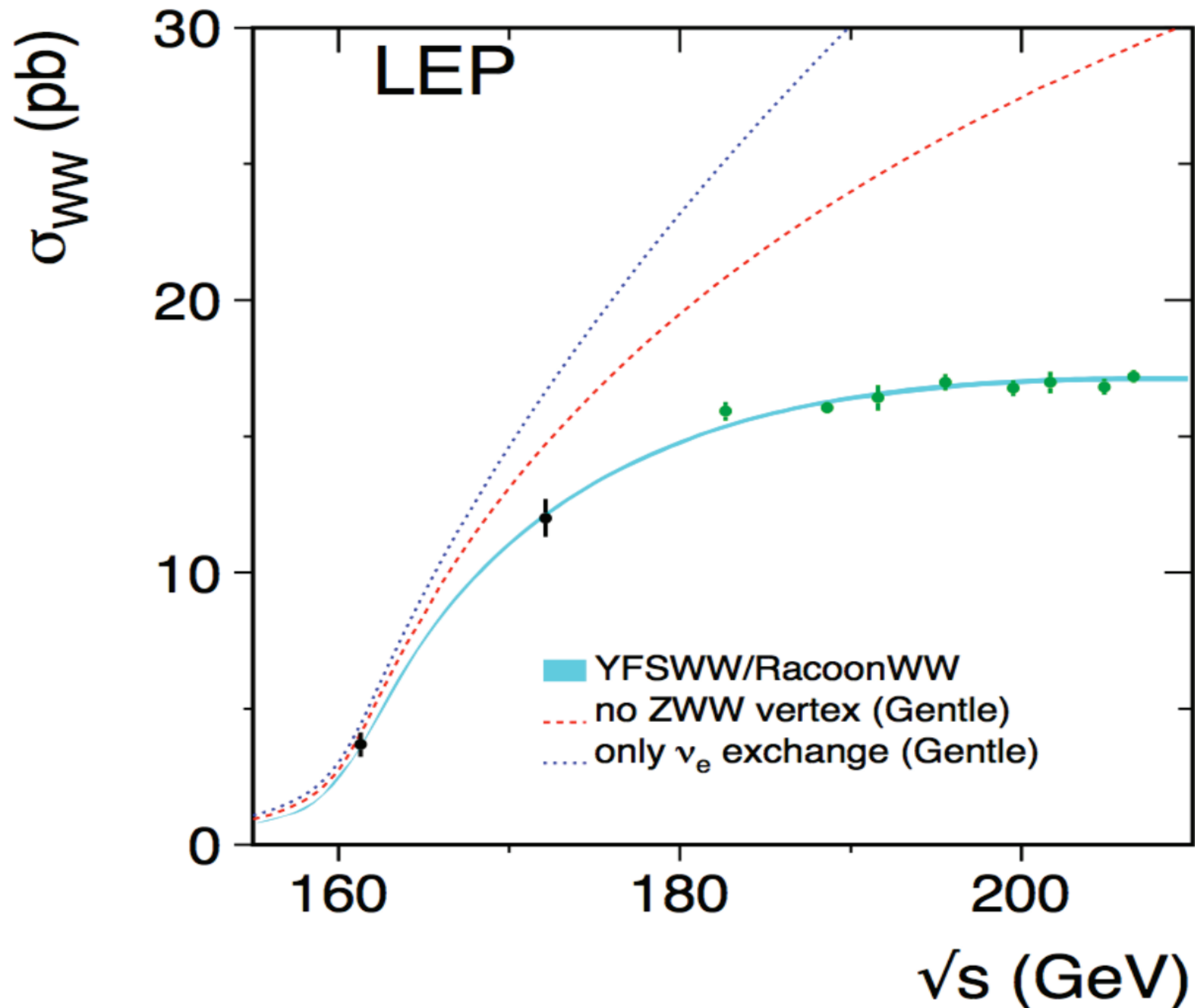
$$k_+ = (k_+ - k_- + (p + \bar{p}))/2$$

so, finally, we find

$$i\mathcal{M} = -i \frac{e^2}{2s_w^2} (2E) \sqrt{2} \epsilon_{-\mu} \frac{1}{2} (k_- - k_+)^{\mu} \frac{1}{m_W^2}$$

which indeed cancels the term on the previous page. In the end, these cancellations gives the GBET prediction.

The cross section for $e^+e^- \rightarrow W^+W^-$ was measured by the LEP experiments, with this result:



Before these LEP measurements were made, theorists tried to predict how deviations from the Standard Model might show up. One idea was to extend the 3-vector boson vertex by adding terms:

$$\delta\mathcal{L} = e[i g_{1\gamma} A_\mu (W_\mu^- W^{+\mu\nu} - W_\mu^+ W^{-\mu\nu}) + i \kappa_\gamma W_\mu^- W_\nu^+ A_{\mu\nu} + i \frac{\lambda_\gamma}{m_W^2} W_{\lambda\mu}^- W^{+\mu\nu} A_\nu^\lambda]$$

and similarly for Z. Here $V_{\mu\nu} = (\partial_\mu V_\nu - \partial_\nu V_\mu)$. Setting

$$g_{1\gamma} = \kappa_\gamma = 1$$

we have the Standard Model Lagrangian; any deviations are “extra”. If CP conservation is relaxed, more terms can be added.

It was quickly realized that the extra terms in the Lagrangian imply extra terms in the amplitudes enhanced by the factor s/m_W^2 . This would seem to imply high sensitivity to the new terms.

We now understand the origin of these terms. Modifying the Yang-Mills vertices breaks gauge invariance. Then the GBET does not apply, and the delicate cancellations that is requires do not happen. Probably, the enhanced sensitivity is completely spurious.

However, the discussion can be given in a more modern context. Now we assume that the weak interactions are a gauge theory of $SU(2) \times U(1)$. However, new particles from beyond the Standard Model can generate new terms in the effective Lagrangian describing Standard Model particles. These terms must be gauge-invariant, but they may be of higher dimension. If the new particles have mass M , their coefficients will contain inverse powers of M .

For the fields of the Standard Model with 1 generation, we can write 84 independent dimension 6 operators. These contribute to anomalous W vertices, anomalous Higgs vertices, and fermion-fermion contact interactions, among other effects. I will not try to discuss these operators systematically here.

Consider, though, adding to the renormalizable Standard Model the $SU(2) \times U(1)$ invariant terms

$$\delta\mathcal{L} = \frac{1}{\Lambda^2} \left[igc_W \Phi^{a\mu} D^\nu A_{\mu\nu}^a + (g'/2)c_B \Phi^\mu \partial^\nu B_{\mu\nu} \right. \\ \left. + 2gg'c_W c_B (\varphi^\dagger \frac{\sigma^a}{2} \varphi) A_{\mu\nu}^a B^{\mu\nu} + \frac{g}{6} c_{3W} \epsilon^{abc} A_{\lambda\mu}^a A^{b\mu\nu} A_\nu^c \right]$$

where $A_{\mu\nu}^a$ is the $SU(2)$ field strength and

$$\Phi^\mu = \varphi^\dagger D^\mu \varphi - (D^\mu \varphi)^\dagger \varphi \\ \Phi^{a\mu} = \varphi^\dagger \frac{\sigma^a}{2} D^\mu \varphi - (D^\mu \varphi)^\dagger \frac{\sigma^a}{2} \varphi$$

After the Higgs field obtains a vacuum expectation value, these operators contribute gauge boson interaction terms. For example

$$g' \Phi^\mu \rightarrow -ig' \sqrt{g^2 + g'^2} v^2 Z^\mu = -i4m_Z^2 c_w Z^\mu$$

Then, after some reduction, one finds

$$\kappa_\gamma - 1 = \frac{4m_W^2}{\Lambda^2} c_{WB} \quad \lambda_\gamma = -\frac{m_W^2}{\Lambda^2} c_{3W}$$

$g_{1\gamma}$ is not shifted; this is the electric charge of the W;
however

$$g_{1Z} - 1 = -\frac{m_Z^2}{\Lambda^2} c_W$$

There are relations

$$\kappa_Z = g_{1Z} - \frac{s_w^2}{c_w^2} (\kappa_\gamma - 1) \quad \lambda_Z = \lambda_\gamma$$

that are broken only by the inclusion of dimension 8 operators

The same operators contribute to precision electroweak observables

$$\alpha S = \frac{4m_Z^2 c_w^2 s_w^2}{\Lambda^2} (4c_{WB} + c_W + c_B)$$

The formulation of deviations from the Standard Model in terms of gauge-invariant dimension 6 operators is theoretically firm and no more complex than it needs to be.

Effects of dimension 6 operators are enhanced at high energy by the factor s/Λ^2

so the enhancement described earlier as a violation of the GBET is present here in a more sensible way.

However, dimension 8 operators are also potentially present, and these contribute terms of order s^2/Λ^4

So, an analysis purely in terms of dimension 6 operators makes sense only when $s/\Lambda^2 < 1$

In particular, I would like to call your attention to a Devil's bargain that arises in analyses at hadron colliders.

There is always a region of phase space where the parton-parton \hat{s} can become as large as possible.

This region gives the strongest sensitivity to anomalous gauge interactions. It is tempting to use cuts to emphasize this region.

However, it is also the region in which your analysis is most likely to be invalidated by corrections from operators of still higher dimension.

Be careful. I do not know a strict rule to decide when you enter the Devil's region.

For the final topic in this lecture, I will describe an analysis in which the GBET might be expected to apply, but actually it does not. This is in collinear W radiation from a quark line.

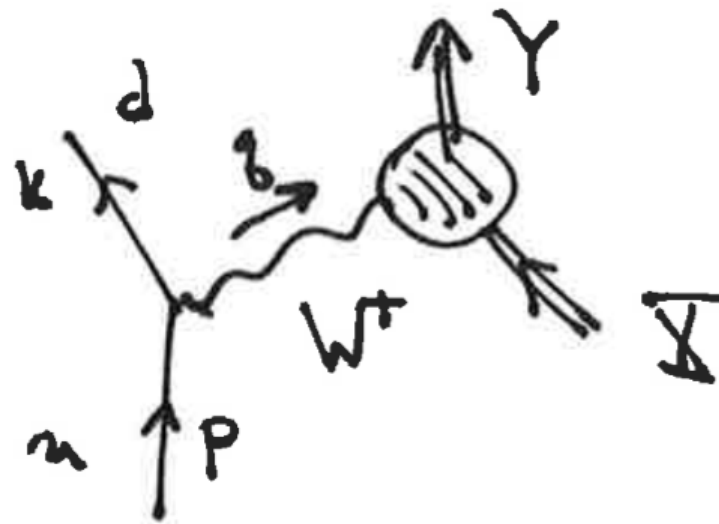
In QCD, quarks easily radiate photons and gluons in collinear directions, giving rise to initial- and final-state radiation described by the Altarelli-Parisi equations. This is the mechanism by which quarks become jets at the LHC.

At high energy, W bosons can also be radiated. An interesting question is: can the W_0^+ be radiated? By the GBET, this is a Higgs boson state that does not couple to light quarks.

Consider the almost-collinear radiation

$$u(p) \rightarrow d(k) + W^+(q)$$

This can yield a W parton distribution in the proton, allowing W-induced reactions at the LHC.



An important one is WW fusion: $W^+W^- \rightarrow h$

To produce the Higgs boson, we would like to have W partons in states of longitudinal polarization.

Write the momentum vectors of u, d, W for $p_T \ll E$, with u, d on shell and W off shell

$$p = (E, 0, 0, E)$$

$$k = ((1 - z)E, -p_T, 0, (1 - z)E - \frac{p_T^2}{2(1 - z)E})$$

$$q = (zE, p_T, 0, zE + \frac{p_T^2}{2(1 - z)E})$$

The denominator of the W propagator is

$$q^2 - m_W^2 = -p_T^2 - \frac{z}{(1 - z)}p_T^2 - m_W^2 = -(\frac{p_T^2}{(1 - z)} + m_W^2)$$

Now we must compute the matrix elements for W emission

$$i\mathcal{M} = ig \, u_L^\dagger(k) (\bar{\sigma} \cdot \epsilon_W^*) u_L(p)$$

to first order in $(p_T, m_W)/E$. Use the explicit spinors

$$u_L(k) = \sqrt{2(1-z)E} \begin{pmatrix} p_T/2(1-z) \\ 1 \end{pmatrix} \quad u_L(p) = \sqrt{2E} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The W polarization vectors are

$$\begin{aligned} \epsilon_\pm^{*\mu} &= (0, 1, \mp i, -p_T/zE)^\mu / \sqrt{2} \\ \epsilon_0^{*\mu} &= (q, p_T, 0, zE)^\mu / m_W \end{aligned}$$

with

$$q = [(zE)^2 - m_W^2]^{1/2} = zE - \frac{m_W^2}{2zE}$$

Then

$$\begin{aligned} \bar{\sigma} \cdot \epsilon_+^* &= \frac{1}{\sqrt{2}} \begin{pmatrix} -p_T/zE & 0 \\ 2 & p_T/zE \end{pmatrix} \\ \bar{\sigma} \cdot \epsilon_-^* &= \frac{1}{\sqrt{2}} \begin{pmatrix} -p_T/zE & 2 \\ 0 & p_T/zE \end{pmatrix} \end{aligned} \quad \bar{\sigma} \cdot \epsilon_0^* = \frac{1}{m_W} \begin{pmatrix} q + zE & p_T \\ p_T & q - zE \end{pmatrix}$$

With these ingredients, it is straightforward to work out the matrix elements

$$i\mathcal{M} = ig \begin{cases} \frac{\sqrt{1-z} \, p_T}{z(1-z)} & - \\ \frac{\sqrt{1-z} \, p_T}{z} & + \\ -\frac{1}{\sqrt{2}} \frac{\sqrt{1-z} \, m_W}{z} & 0 \end{cases}$$

The first two lines here are exactly what one finds in the derivation of the Altarelli-Parisi equations, with the substitution $g_s t^a \rightarrow g/\sqrt{2}$. The last line is new for a massive vector boson.

Now let's embed these results into the formula for the cross section. We begin from

$$\sigma(uX \rightarrow dY) = \frac{1}{2s} \int \frac{d^3k}{(2\pi)^3 2k} \int d\Pi_Y (2\pi)^4 \delta^{(4)}(p + p_X - k - p_Y) \left| \mathcal{M}(u \rightarrow W^+ d) \frac{1}{q^2 - m_W^2} \mathcal{M}(W^+ X \rightarrow Y) \right|^2$$

Using the collinear kinematics,

$$\frac{1}{2s} \int \frac{d^3k}{(2\pi)^3 2k} = \frac{1}{2(\hat{s}/z)} \int \frac{dz E d^2p_T}{16\pi^3 E(1-z)} = \frac{1}{2\hat{s}} \int \frac{dz dp_T^2 \pi}{16\pi^3} \frac{z}{(1-z)}$$

Then

$$\sigma(uX \rightarrow dY) = \int dz \int \frac{dp_T^2}{(4\pi)^2} \frac{z}{(1-z)} |\mathcal{M}(u \rightarrow W^+ d)|^2 \frac{1}{(p_T^2/(1-z) + m_W^2)^2} \cdot \frac{1}{2\hat{s}} \int d\Pi_Y (2\pi)^4 \delta^{(4)}(q + p_X - p_Y) |\mathcal{M}(W^+ X \rightarrow Y)|^2$$

The last line is $\sigma(W^+(q)X \rightarrow Y)$.

So we have an expression for the cross section in the form

$$\sigma(uX \rightarrow dY) = \int dz \, f_{W \leftarrow u}(z) \, \sigma(W^+(q)X \rightarrow Y)$$

where

$$f_{W \leftarrow u}(z) = \int \frac{dp_T^2}{(4\pi)^2} \frac{z}{(1-z)} \frac{(1-z)^2}{(p_T^2 + (1-z)m_W^2)^2} |\mathcal{M}(u \rightarrow W^+ d)|^2$$

We can evaluate this using the formulae for the matrix element given on a previous slide.

The result is:

$$f_{W-}(z) = \frac{\alpha_w}{4\pi} \int \frac{dp_T^2 p_T^2}{[p_T^2 + (1-z)m_W^2]^2} \frac{1}{z}$$

$$f_{W+}(z) = \frac{\alpha_w}{4\pi} \int \frac{dp_T^2 p_T^2}{[p_T^2 + (1-z)m_W^2]^2} \frac{(1-z)^2}{z}$$

$$f_{W0}(z) = \frac{\alpha_w}{8\pi} \int \frac{dp_T^2 m_W^2}{[p_T^2 + (1-z)m_W^2]^2} \frac{(1-z)^2}{z}$$

For the transverse W polarizations, we find a result very similar to the Altarelli-Parisi splitting functions.

Integrating over p_T ; we find

$$f_{WT}(z) = \frac{\alpha_w}{4\pi} \log \frac{Q^2}{m_W^2} \frac{1 + (1-z)^2}{z}$$

For a longitudinal W, the p_T integral is not divergent. The emission is restricted to an interval of p_T where the W can be thought to be approximately at rest in a collinearly moving frame. Here, despite the GBET, we get a nonzero answer

$$f_{W0}(z) = \frac{\alpha_w}{8\pi} \frac{1-z}{z}$$

with the W having characteristic $p_T \sim m_W$. This formula (due to Dawson) is the basis for the analysis of W fusion processes at the LHC.