Introduction to Composite Higgs Models

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Outline

Lecture 1:

- Motivation
- Review of the Standard Model and the roles of the Higgs multiplet
  - Standard Model: particles, symmetries and interactions
  - breaking of electroweak symmetry and gauge boson masses
  - quark masses
  - the Higgs particle
  - the hierarchy problem

Lecture 2:

- Gedankenexperiment: SM without a Higgs – are gauge bosons massless?
- Composite Higgs Models: main ideas, tools and the simplest model

Lecture 3: Applications: quark masses, a Higgs potential from the top sector

Lecture 4: Phenomenology of quark partners, special topics, outlook

Motivation

- ☺ Atlas and CMS found a Higgs-like resonance with a mass $m_h \sim 126$ GeV and couplings to $\gamma\gamma$, WW, ZZ, bb, and $\tau\tau$ compatible with the standard model Higgs.
- ☹ The standard model suffers from the hierarchy problem.

$\Rightarrow$ We need to search for an SM extension with a Higgs-like state which provides an explanation for why $m_h, v \ll M_{pl}$.

Possible solutions:

- The hierarchy problem is not a problem.
  There is only the standard model (at the EW scale).
- The hierarchy problem is a problem.
  There is a symmetry which protects the quadratic terms in the Higgs potential from quadratically divergent loop corrections.
  - supersymmetry?
  - Higgs as a pseudo goldstone boson (PGB) of a global symmetry.$\leftarrow$ our topic

The SM Higgs doublet fulfills several tasks:

- it generates of the W and Z masses via EWSB,
- it generates quark and lepton masses via Yukawa terms in the action,
- it provides a physical scalar degree of freedom and predicts its couplings (consistent with the newly observed 126 GeV particle).

$\Rightarrow$ In a composite Higgs setup, these beneficial features of the SM Higgs are most efficiently mimicked if the whole Higgs multiplet is realized as PGBs.

Simplest realization:
The minimal composite Higgs model (MCHM) Agashe, Contino, Pomarol [2004]
Based on $SO(5)/SO(4)$.

This (and similar) model(s) predict deviations of the SM Higgs couplings *and* additional particles (top-partners) at the TeV scale.
They can be searched for at the LHC.

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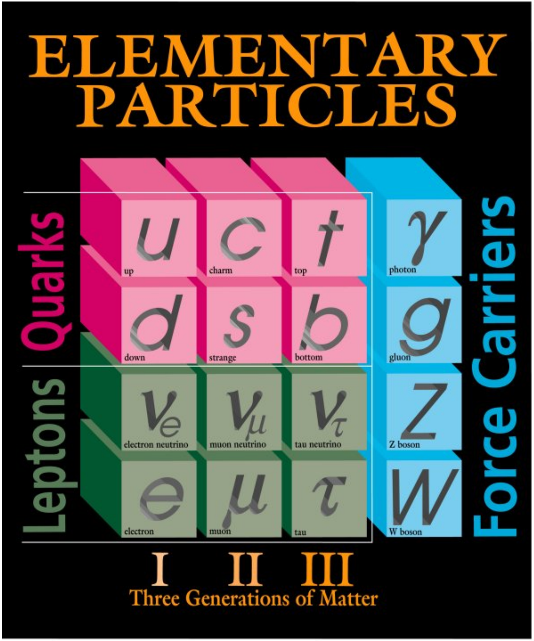
Standard Model: particles and symmetries



Fermilab 95-759

In more detail:

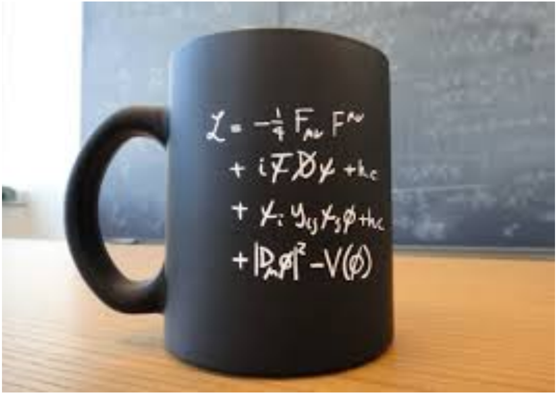
"Force carriers" are the gauge bosons of the SM symmetry group
$SU(3)_c \times SU(2)_L \times U(1)_Y$.
$SU(3)_c$: $G_\mu = G^a_\mu T^a$ 8 gluons (T^a are the 8 generators of $SU(3)$)
$SU(2)_L$: $W_\mu = W^i_\mu T^i$ 3 weak gauge bosons (T^i are the 3 generators of $SU(2)$)
$U(1)_Y$: B_μ 1 hyper charge boson

Leptons:
$e_R,\ \mu_R,\ \tau_R$: right handed leptons with charges $(1,1)_{-1}$
$\begin{pmatrix} \nu^e_L \\ e_L \end{pmatrix}, \begin{pmatrix} \nu^\mu_L \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu^\tau_L \\ \tau_L \end{pmatrix}$: left handed leptons with charges $(1,2)_{-1/2}$

Quarks:
$u_R,\ c_R,\ t_R$: right handed quarks with charges $(3,1)_{2/3}$
$d_R,\ s_R,\ b_R$: right handed quarks with charges $(3,1)_{-1/3}$
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$: left handed quarks with charges $(3,2)_{1/6}$

... and the Higgs multiplet
$H = \begin{pmatrix} \chi^+ \\ (h + v + i\chi_3)/\sqrt{2} \end{pmatrix}$ with charge $(1,2)_{1/2}$.

Standard Model: Lagrangian



Gauge field Lagrangian:

$$\mathcal{L}_g = -\frac{1}{2} Tr[G_{\mu\nu} G^{\mu\nu}] - \frac{1}{2} Tr[W_{\mu\nu} W^{\mu\nu}] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

where

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu] \\ G_{\mu\nu} &= \partial_\mu G_\nu - \partial_\nu G_\mu - ig_s[G_\mu, G_\nu] \end{aligned}$$

Fermion Lagrangian:

$$\mathcal{L}_f = i\overline{\psi} \not{D} \psi,$$

where ψ is any of the left- or right-handed fermions, $\not{D}\psi = \gamma^\mu D_\mu \psi$, and

$$D_\mu \psi = \left(\partial_\mu + ig_s G_\mu + igW_\mu + ig' Y_\psi B_\mu \right) \psi,$$

where the 2nd (3rd) term is only present if ψ is and $SU(3)$ triplet (an $SU(2)$ doublet), and Y_ψ is the hyper charge of ψ.

Higgs Lagrangian:

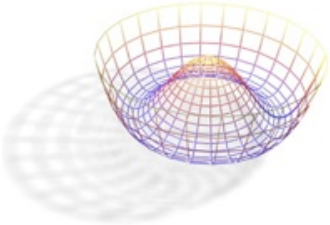
$$\mathcal{L}_H = (D_\mu H)^\dagger D^\mu H - \lambda V(H)$$

with

$$V(H) = -\mu^2 |H|^2 + |H|^4$$

Note:
The Lagrangian is invariant under $SU(2) \times U(1)$.
BUT:

The potential minimum lies at
$|\langle H \rangle| = \sqrt{\mu^2/2\lambda} \neq 0$.
W.l.o.g. one can choose
$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \Leftrightarrow v = \sqrt{\mu^2/\lambda}$.
$\langle H \rangle$ is NOT invariant under $SU(2) \times U(1)$.
Only a residual $U(1)_{em}$ is conserved.
$\Rightarrow$ 3 Goldstone bosons $\chi^\pm, \chi^3$ (eaten by $W^\pm, Z$, which become massive.
The only massive mode in H is h with $m_h = \sqrt{2}\mu$.



Standard Model: gauge boson masses

To see gauge boson masses explicitly, write out the kinetic term, expanded around v:

$$D_\mu H^\dagger D^\mu H \supset \frac{v^2}{4}\left(g^2 W^+ W^- + \frac{1}{2}\left(-gW^3_\mu + g' B_\mu)(-gW^{3\mu} + g' B^\mu\right)\right)$$

Exercise for students:

Verify the above, find all other terms contained in $D_\mu H^\dagger D^\mu H$ and interpret them.

The first term is a mass term for the $W^\pm$ with $m_W^2 = \frac{g^2 v^2}{4}$.
The second term is a mass for one linear combination of W^3 and B.
Diagonalizing the mass matrix (in (W^3, B) space) yields

$$\begin{aligned} Z_\mu &= \cos\theta_w W^3_\mu - \sin\theta_w B_\mu \quad \text{with} \quad m_Z^2 = \frac{(g^2 + g'^2)v^2}{4} \\ A_\mu &= \cos\theta_w W^3_\mu + \sin\theta_w B_\mu \quad \text{with} \quad m_A^2 = 0 \end{aligned}$$

where $\tan\theta_w \equiv g'/g$.

Note:

- $m_{W,Z} \propto v \propto \mu$. μ is the only dimensionful input parameter in the SM.
- The SM predicts (at classical level): $m_W = \cos\theta_w m_Z$.
- The mass eigenstates Z_μ, A_μ are linear combinations of the gauge eigenstates W^3_μ, B_μ. To obtain Feynman rules in the mass eigenbasis, we have to rewrite our simple interaction terms in $\mathcal{L}_g$ and $\mathcal{L}_f$ in terms of Z_μ, A_μ.

The final part of the Lagrangian:

$$\mathcal{L}_{Yuk} = -\lambda^u_{ij}\overline{Q}_i\tilde{H}u_j - \lambda^d_{ij}\overline{Q}_iHd_j - \lambda^e_{ij}\overline{L}_iHe_j + \text{h.c.},$$

where $\tilde{H} = i\sigma_2 H^*$, Q and L are the quark and lepton $SU(2)$ doublets, and i, j are family indices.

Expanding H around its vacuum expectation value yields fermions mass matrices $M^{u,d,e}_{ij} = \lambda^{u,d,e}_{ij}\frac{v}{\sqrt{2}}$ terms and interactions of h to the quarks.

The Yukawa matrices (and therefore the mass matrices) are complex 3×3 matrices. They can be diagonalized by bi-unitary transformations:

$$\begin{array}{ccc} \tilde{u}^i_L = S^{ij}_u u^j_L & , & \tilde{u}^i_R = T^{ij}_u u^j_R \\ \tilde{d}^i_L = S^{ij}_d d^j_L & , & \tilde{d}^i_R = T^{ij}_d d^j_R \\ \tilde{e}^i_L = S^{ij}_e e^j_L & , & \tilde{e}^i_R = T^{ij}_e e^j_R \end{array}$$

To obtain the Feynman rules in the mass eigenbasis, one has to again rewrite the gauge eigenstates of the fermions in terms of mass eigenstates. However, most of the terms are invariant under the above field redefinitions.
Example:

$$\begin{aligned} \mathcal{L} \supset i\overline{u}_R \not{D} u_R &= i\overline{u}_R T^\dagger_u T_u \not{D} T^\dagger_u T_u u_R \\ &= i\overline{u}_R T^\dagger_u \not{D} T_u u_R = i\overline{\tilde{u}}_R \not{D} \tilde{u}_R \end{aligned}$$

Standard Model: quark masses

Only exception (apart from Yukawa terms themselves) are the fermion interactions with W bosons:

$$\begin{aligned} \overline{u}_L \not{W}^+ d_L &= \overline{u}_L S_u^\dagger S_u \not{W}^+ S_d^\dagger S_d d_L \\ &= \overline{\tilde{u}}_L S_u S_d^\dagger \not{W}^+ \tilde{d}_L \\ &\equiv \overline{\tilde{u}}_L V_{CKM} \not{W}^+ \tilde{d}_L \end{aligned}$$

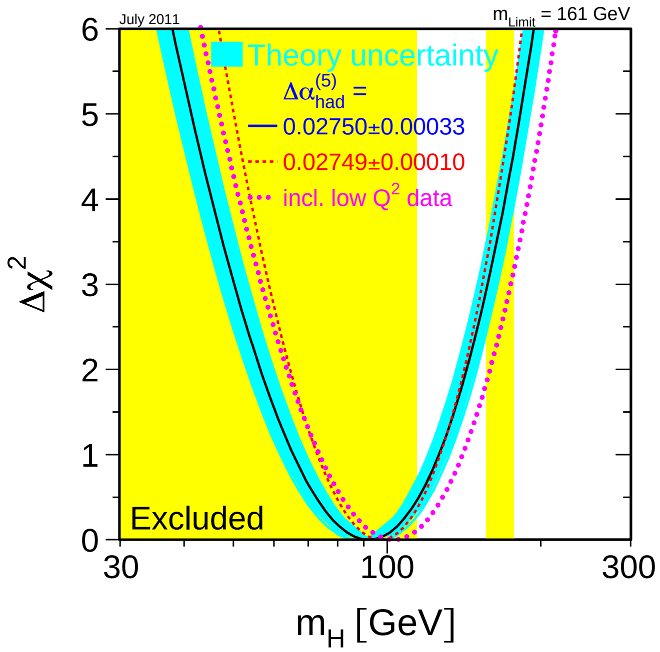
Summary on Yukawa part:

- The only physical degrees of freedom hidden in the Yukawa matrices (which cannot be absorbed by field re-definitions) are the quark and lepton masses (9 parameters), and three mixing angles as well as one phase in the CKM matrix.
- Only W bosons have flavor mixing interactions with quarks.
- $m_q \propto v$. Again the Higgs vev is responsible for mass generation.

All of what we so far discussed (electroweak symmetry breaking, gauge boson masses, fermion masses) are consequences of the VEV of the Higgs multiplet, but they did not depend directly on the Higgs particle h.

Before LHC there was indirect information on m_h:

- LEP and Tevatron did not find the Higgs: $m_h \gtrsim 114\,\mathrm{GeV}$. (for the SM Higgs!)
- Perturbative unitarity: When calculating the WW scattering amplitude without including h, one finds that it diverges at high energies. When the higgs is included and lighter than $\mathcal{O}(3\,\mathrm{TeV})$, its contributions to the scattering amplitude cancel the divergent part.
- Our discussion above was only at the classical level. At loop-level, the Higgs contributes to many electroweak processes, some of which are very precisely measured by LEP. Fitting all these measurements results in a best-fit value for m_h.



[LEP/TEV EW WG Plots for Summer 2011]

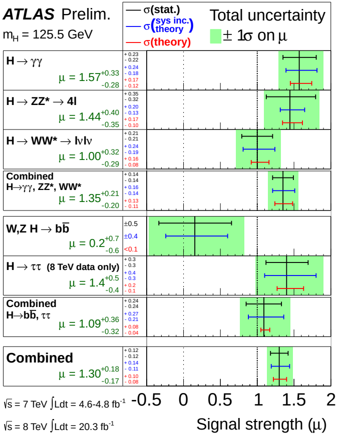
The current status:

A higgs-like particle has been found at LHC:

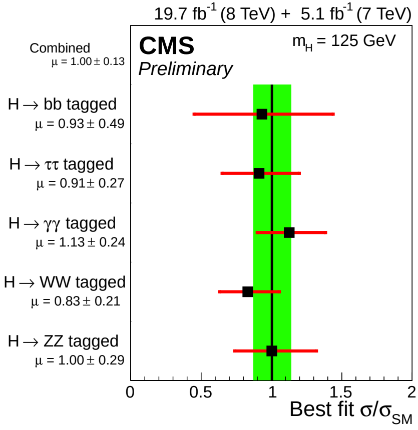
ATLAS: $m_h = 125.36 \pm 0.37(stat) \pm 0.18(syst)\,\mathrm{GeV}$ [ATLAS-HIGG-2013-12]

CMS: $m_h = 125.03\ ^{+0.26}_{-0.27}\ (stat.)\ ^{+0.13}_{-0.15}\ (syst.)\,\mathrm{GeV}$ [CMS-PAS-HIG-14-009]

ATLAS and CMS also determined the values of various Higgs couplings:

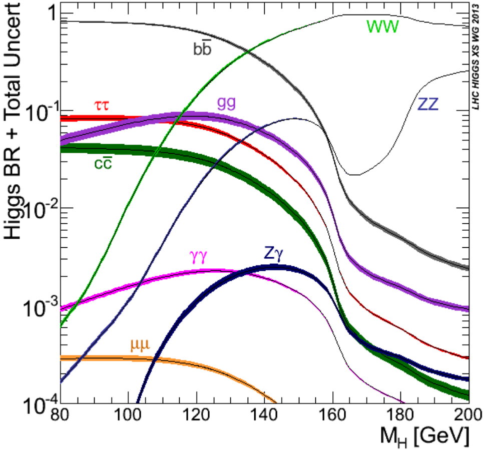


[ATLAS-CONF-2014-009]



[CMS-PAS-HIG-14-009]

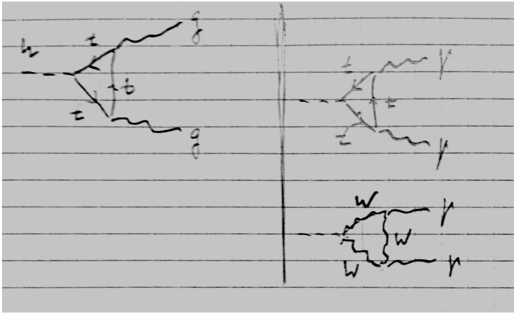
The Higgs particle is in a particularly interesting mass range. There are many decay channels available for $m_h = 125\,\mathrm{GeV}$.



[LHC Higgs Cross Section Working Group]

Note:

From our earlier tree-level discussion of the Higgs, the Higgs couples to *WW*, *ZZ*, and fermions, but at loop level, also couplings to *gg* and $\gamma\gamma$ are induced.



These couplings are potentially sensitive to BSM particles which are colored/charge and couple to the Higgs as then, they yield an additional loop contribution.

Standard Model: the hierarchy problem

With the discovery of the Higgs, the SM seems complete, and the measured couplings are over all in good agreement with the SM predictions. So why looking for SM extensions?

The hierarchy problem:

All mass scales in the SM are $\propto \mu$, $\mu \sim \mathcal{O}(100\,\mathrm{GeV})$, and the Higgs mass squared term is negative. Can we understand that?

- The SM is a renormalizable QFT which is perturbative up to the Planck scale (even beyond), but at the Planck scale we expect something more general, which simultaneously includes gravity. So one expects the natural cutoff scale of the SM to be (at most) M_{pl}.
- μ is a dimensionful parameter, and there is no reason (symmetry) why $\mu \ll \Lambda \leq M_{pl}$ (small μ is unnatural).
- Even if we write a tree level Lagrangian with a μ tuned small: loop contributions to μ^2 are quadratically divergent, so the Higgs mass obtains corrections of $\mathcal{O}(\Lambda)$.

Note: this does not mean that the SM is inconsistent as a QFT; it just means that it is very fine tuned.

...and the SM provides us with examples how to create or stabilize mass hierarchies:

- Do we understand why the up quark is so much lighter than the bottom which is much lighter than the top?

Standard Model: solutions to other hierarchy problems within the SM

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- Do we understand why the up quark is so much lighter than the bottom which is much lighter than the top?
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- Do we understand why $\Lambda_{QCD} \ll M_{pl}$?
  Yes!
  Gauge couplings run logarithmically, for $SU(3)$ decreases with energy.

Lecture 1:

- Motivation
- Review of the Standard Model and the roles of the Higgs multiplet

Lecture 2:

- Gedankenexperiment: SM without a Higgs – are gauge bosons massless?
- Technicolor and composite Higgs models: main ideas and generic challenges
- Tools: The CCWZ formalism
- The minimal composite Higgs model

Lecture 3: Applications: generating quark masses, a Higgs potential from the top sector

Lecture 4: Phenomenology of quark partners, special topics, outlook

Gedankenexperiment: Weinberg, Susskind
Standard Model without a Higgs – are gauge boson massless?

SM with one family:

$$\mathcal{L}_f = \overline{q}^i_L i \not{D} q^i_L + \overline{q}^i_R i \not{D} q^i_R \text{ with } q^i = u, d$$

First, ignore EW gauge interactions.
Above Λ_{QCD} quarks are free. Below they condensate:

$$\langle \overline{q}^j_L q^i_R \rangle = \Delta \delta^{ij} \neq 0.$$

The Lagrangian has a symmetry $SU(2)_L \times SU(2)_R \times U(1)$ which the vacuum breaks to $SU(2) \times U(1)$.
$\Rightarrow$ 3 Goldstone bosons.

$$\Sigma = e^{i\pi^i T^i / f_\pi}$$

where π^i are the Goldstone bosons (pions), T^i are the broken generators, $f_\pi = 93\text{MeV}$ is the pion decay constant.
The Lagrangian of the pions is (more on this later)

$$\mathcal{L}_\pi = \frac{f_\pi^2}{2} Tr \left[(\partial_\mu \Sigma)^\dagger \partial^\mu \Sigma \right]$$

Gedankenexperiment: Standard Model without a Higgs – are gauge boson massless?

Now, gauge the $SU(2)_L$ and T^3 of the $SU(2)_R$:

$$\partial_\mu \Sigma \to D_\mu \Sigma = \left(\partial_\mu - igW_\mu^a T^a + ig' B T^3\right)\Sigma.$$

Putting this into the Lagrangian:

$$\begin{aligned}\mathcal{L}_\pi &\supset \frac{f^2}{2} Tr \left| \frac{g}{\sqrt{2}} \left(W_\mu^+ T^+ + W_\mu^- T^-\right) + gW_\mu^3 T^3 - g' B_\mu T^3 \right|^2 \\ &= \frac{f_\pi^2}{4}\left[g^2 W_\mu^+ W^{-\mu} + \frac{1}{2}\left(gW_\mu^3 - g' B_\mu\right)\left(gW^{3\mu} - g' B^\mu\right)\right]\end{aligned}$$

so we obtain masses $m_W^2 = \frac{g^2 f_\pi^2}{4}$ and $m_Z^2 = \frac{(g^2+g'^2) f_\pi^2}{4}$.

- ☺ That's EWSB without a hierarchy problem!
- ☹☹ ...but $f_\pi = 93\,\mathrm{MeV}$ and not $250\,\mathrm{GeV}$
- ☹ also, the pions come as an $SU(2)$ triplet. Where is the Higgs particle?

Technicolor:

Main idea: Scale up this QCD toy model:

- Introduce a new gauge group (Technicolor) which gets strongly coupled at $\Lambda_{TC} \sim \mathrm{TeV}$.
- Introduce fermions which are charged under this group and form which condensate.
  Requirements:
  - The fermion Lagrangian must have a global symmetry group $\mathcal{G}$.
  - The techniquark condensate must break $\mathcal{G}$ to $\mathcal{H}$, with $\mathcal{G} \supset SU(2) \times U(1)$ and $\mathcal{H} \supset U(1)_{em}$
  - The technipions are the Goldstone bosons of this breaking which (like in the toy model) have to give masses to W and Z.

The main questions to ask in a Technicolor talk:

- How are the quark masses generated?
  (The vev now comes from a techniquark condensate. To give masses to quarks, this needs to couple to SM quarks.
  This can be done in extended technicolor models, but it requires additional structure.)
- If quark masses are realized, are FCNCs under control?
  (In the SM, we saw that at tree level, there are no flavor changing neutral currents. In the SM, even loop corrections induce only small FCNCs (GIM mechanism), but this is often hard to realize in models with extended quark sector.)
- What about electroweak precision constraints?
  (This is a two-fold question: i) what unitarizes the EW gauge boson scattering amplitudes? and b) how large are effects of the technicolor sector on loop corrections?)

And after the LHC discovery of the higgs-like particle:
What is taking over the roles of the higgs particle:

- Unitarizing WW scattering (already mentioned above)
- What is the particle found at the LHC at $m_h = 125\,\mathrm{GeV}$
- How does it couple to WW, ZZ, $b\bar{b}$, $\tau\bar{\tau}$, gg, and $\gamma\gamma$.
  (C.f. last lecture: These couplings are being tested already at LHC, and in the SM, they have very different origins; EW or Yukawa-terms, tree-level or loop-induced)

Composite Higgs Models:

CH models are in a way a subclass of Technicolor models.

The proposed solution to the hierarchy problem is the same:
Extend the SM with a sector which via running of the couplings becomes strongly coupled at $\Lambda \ll M_{pl}$ (natural) with a global symmetry which gets broken due to a condensate.

One main difference:
Build the model such that the Goldstone sector includes *the whole Higgs multiplet* with quantum numbers of the SM Higgs.

The main questions to ask in a composite Higgs talk:

- The same as for Technicolor.
  (but note: CH models mimic the SM higgs multiplet, so some answers are easier.)
- If the whole Higgs multiplet is a Goldstone boson multiplet, how is the Higgs potential generated?
- What is the UV completion of the model?
  (Composite Higgs models are often discussed in terms of a linear sigma model, the low-energy effective theory. Full UV completions are currently not known. But there is some recent progress [Gherghetta *et al.* ; Feretti *et al.*].)

The CCWZ formalism is a generic way to parameterize Goldstone bosons which arise from a $\mathcal{G}/\mathcal{H}$ symmetry breaking, and to systematically construct G invariant Lagrangians.

First, lets split the generators of $\mathcal{G}$ into
T^a: Generators of the subgroup $\mathcal{H}$, and
T^i: Broken generators (they span $\mathcal{G}/\mathcal{H}$).

Now, define the "Goldstone boson matrix":

$$U(\pi) = e^{i\frac{\sqrt{2}}{f}\pi_i T^i}.$$

CCWZ:
Under a general group transformation, its transformation can be written as $U(\pi) \xrightarrow{g} gUh^{-1}$. where $h \in \mathcal{H}$ is a function of the π^i.

Now look at:

$$\begin{aligned} U^\dagger i\partial_\mu U &\xrightarrow{g} hU^\dagger g^{-1} i\partial_\mu g U h^{-1} \qquad \text{(assume } g \text{ global)} \\ &= hU^\dagger i\partial_\mu U h^{-1} \\ &= h(U^\dagger i\partial_\mu U)h^{-1} + hi\partial_\mu h^{-1} \end{aligned}$$

Decompose the expression into components along T^i and T^a:

$$U^\dagger i\partial_\mu U \equiv -d^i_\mu T^i - e^a_\mu T^a$$

$$\Rightarrow (*) \begin{cases} e_\mu = e^a_\mu T^a \to h(e_\mu - i\partial_\mu)h^{-1} \\ d_\mu = d^i_\mu T^i \to h d_\mu h^{-1} \end{cases}$$

e_μ transforms like a connection (will be relevant for quark kinetic terms later)

With d_μ one can build a $\mathcal{G}$ invariant which is a function of derivatives and the pions:

$$\mathcal{L}_{kin} = \frac{f^2}{2} Tr[d_\mu d^\mu]$$

Note: This works for general (compact, connected, semi-simple) Lie Groups, but the explicit form of d_μ and e_μ depends on the specific groups and can be complicated.

We need one more ingredient: Above we assumed that the group $\mathcal{G}$ is global, but we want to gauge a subgroup of it (our $SU(2) \times U(1)$).
To do this, introduce sources for these generators which transform inhomogeniously:

$$A_\mu = A_\mu^A T^A \xrightarrow{g} g(A_\mu + i\partial_\mu)g^{-1}$$

Exercise for students:

Repeat the calculation on the last slide , but now allow the transformation g to be local (in the gauged directions T^A).
Show that the transformations of e_μ and d_μ are still the same as in $(*)$.

The minimal composite Higgs model: $SO(5)/SO(4)$ [Agashe, Contino, Pomarol]

"Consider a strongly coupled theory with a global symmetry $SO(5)(\times U(1)_X)$ which is spontaneously broken to $SO(4)(\times U(1)_X)$ at a scale f."

- The Goldstone bosons live in $SO(5)/SO(4) \to$ 4 d.o.f.
- $SO(4) \simeq SU(2)_L \times SU(2)_R \to$
  Gauging $SU(2)_L$ yields an $SU(2)_L$ goldstone doublet.
  (Later: Gauging $Y = T^3_R + X$ allows for fermion embeddings with consistent $U(1)_Y$ charge.)

The CCWZ objects for $SO(5)/SO(4)$: c.f. Appendix of [Rattazzi etal, 1211.5663]

Unbroken Generators: $(T^a_{LR})_{IJ} = -\frac{i}{2}\left[\frac{1}{2}\epsilon^{abc}\left(\delta^b_I \delta^c_J - \delta^b_J \delta^c_I\right) \pm \left(\delta^a_I \delta^4_J - \delta^a_J \delta^4_I\right)\right]$
Broken Generators: $T^i_{IJ} = -\frac{i}{\sqrt{2}}\left(\delta^i_I \delta^5_J - \delta^i_J \delta^5_I\right)$
$(I = 1..5,\ i = 1..4,\ a = 1..3)$,

Goldstone Matrix:

$$U(\Pi) = \exp\left(\frac{i}{f}\Pi_i T^i\right) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos \overline{h}/f & \sin \overline{h}/f \\ 0 & 0 & 0 & -\sin \overline{h}/f & \cos \overline{h}/f \end{pmatrix},$$

where $\Pi = (0,0,0,\overline{h})$ with $\overline{h} =< h > +h$.

Note: Π of course contains also $\chi^{\pm,3}$. The above expression is in unitary gauge.

c.f. Appendix of [Rattazzi etal, 1211.5663] for the general expressions

Definition of d and e symbols:

$$d^i_\mu = \sqrt{2}\left(\frac{1}{f} - \frac{\sin \Pi/f}{\Pi}\right)\frac{\vec{\Pi}\cdot\nabla_\mu\vec{\Pi}}{\Pi^2}\Pi^i + \sqrt{2}\frac{\sin \Pi/f}{\Pi}\nabla_\mu\Pi^i$$

$$e^a_\mu = -A^a_\mu + 4\,i\,\frac{\sin^2(\Pi/2f)}{\Pi^2}\vec{\Pi}^t t^a \nabla_\mu\vec{\Pi}$$

d_μ symbol transforms as a fourplet under the unbroken $SO(4)$ symmetry, while e_μ belongs to the adjoint representation.
$\nabla_\mu\Pi$ is the "covariant derivative" of the Goldstone field Π

$$\nabla_\mu\Pi^i = \partial_\mu\Pi^i - iA^a_\mu\left(t^a\right)^i{}_j\Pi^j\,,$$

A_μ: gauge fields of the gauged subgroup of $SO(4) \simeq SU(2)_L \times SU(2)_R$

$$A_\mu = \frac{g}{\sqrt{2}}W^+_\mu\left(T^1_L + iT^2_L\right) + \frac{g}{\sqrt{2}}W^-_\mu\left(T^1_L - iT^2_L\right) + g\left(c_w Z_\mu + s_w A_\mu\right)T^3_L + g'\left(c_w A_\mu - s_w Z_\mu\right)T^3_R\,.$$

Explicit form in unitary gauge:

$$\begin{cases} e_L^{1,2} = -\cos^2\left(\frac{\overline{h}}{2f}\right) W_L^{1,2} \\ e_L^3 = -\cos^2\left(\frac{\overline{h}}{2f}\right) W^3 - \sin^2\left(\frac{\overline{h}}{2f}\right) B \end{cases}, \begin{cases} e_R^{1,2} = -\sin^2\left(\frac{\overline{h}}{2f}\right) W_L^{1,2} \\ e_R^3 = -\cos^2\left(\frac{\overline{h}}{2f}\right) B - \sin^2\left(\frac{\overline{h}}{2f}\right) W^3 \end{cases}$$

and

$$\begin{cases} d_\mu^{1,2} = -\sin(\overline{h}/f)\frac{W_\mu^{1,2}}{\sqrt{2}} \\ d_\mu^3 = \sin(\overline{h}/f)\frac{B_\mu - W_\mu^3}{\sqrt{2}} \\ d_\mu^4 = \frac{\sqrt{2}}{f}\partial_\mu h, \end{cases} .$$

Kinetic term for the “Higgs”:

$$\mathcal{L}_\Pi = \frac{f^2}{4} d^i_\mu d^{i\mu} = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{4} f^2 \sin^2\left(\frac{\overline{h}}{f}\right)\left(W_\mu W^\mu + \frac{1}{2c_w^2} Z_\mu Z^\mu\right)$$

So we see explicitly that we get the canonical kinetic term for the Higgs.

Setting $\overline{h} = \langle \overline{h} \rangle$ yields the mass term for W and Z

$$m_W^2 = c_w^2 m_Z^2 = \frac{g^2}{4} f^2 \sin^2\left(\frac{< \overline{h} >}{f}\right)$$

$$\Rightarrow v = 246 \text{ GeV } = f \sin\left(\frac{< \overline{h} >}{f}\right) \equiv f \sin(\epsilon).$$

Expanding to the n^{th} power in h yields the couplings of WW and ZZ to h^n.

Exercise for students:

Calculate the couplings of hWW and $hhWW$ in this model.

The SM couplings are $g_{hWW} = \frac{g^2 v}{2}$ and $g_{hhWW} = g^2/2$.

How much do the CH couplings deviate from the SM couplings if $f = 750\,\mathrm{GeV}$?

The minimal composite Higgs model: $SO(5)/SO(4)$

...so we calculated the couplings of the higgs particle to EW gauge bosons ☺.

Is this enough to do phenomenology (Higgs, EW, new LHC signatures)?

The minimal composite Higgs model: $SO(5)/SO(4)$

...so we calculated the couplings of the higgs particle to EW gauge bosons ☺.

Is this enough to do phenomenology (Higgs, EW, new LHC signatures)?

No ☹.

- For Higgs production, the main process is gluon fusion, and that depends on top- (and top-partner) loops.
- For Higgs decay rates, we also need to know the (possibly modified) Yukawa couplings of top and bottom.
- For EW physics, tops are also important.
- And for new collider signatures we need to understand the spectrum of new particles.

For all this, we need to understand how quark masses are generated.

Also note: in the above we were *assuming*, that the Higgs obtains a potential (and develops a vev). But where does it come from?

Those are the topics for tomorrow.

Outline

Lecture 1:

- Motivation
- Review of the Standard Model and the roles of the Higgs multiplet

Lecture 2:

- Gedankenexperiment: SM without a Higgs – are gauge bosons massless?
- Technicolor and composite Higgs models: main ideas and generic challenges
- Tools: The CCWZ formalism
- The minimal composite Higgs model

Lecture 3:

- Generating quark masses
- a Higgs potential from the top sector
- Electroweak bounds
- Higgs physics in CH models

Lecture 4: Phenomenology of quark partners, special topics, outlook

Generating quark masses

In the minimal composite Higgs model (MCHM), the Higgs multiplet is realized as bound states of a strongly coupled theory with a global symmetry $SO(5)$ which is spontaneously broken to $SO(4)$ (and parts of the $SO(4)$ are gauged). To write down Yukawa couplings, we need to couple the SM quarks to the Higgs, in an $SO(5)$ invariant way.

This does not work with two elementary fermions and the Higgs multiplet which is encoded in the Goldstone matrix.

Idea [Kaplan]: Take elementary fermions q in a linear (incomplete) $SO(5)$ representation and composite fermionic operator of the strongly coupled theory, which mix via linear interactions

$$\mathcal{L}_{mix} = y\bar{q}^{\alpha}\Delta_{\alpha I_{\mathcal{O}}}\mathcal{O}^{I_{\mathcal{O}}} + \text{h.c.} \equiv y\bar{q}_{I_{\mathcal{O}}}\mathcal{O}^{I_{\mathcal{O}}} + \text{h.c.}$$

where $\mathcal{O}$ is an operator of the strongly coupled theory in the rep. $I_{\mathcal{O}}$.

$\Rightarrow$ Choose which quarks are elementary, and their embedding $\Delta_{\alpha I_{\mathcal{O}}}$ into $SO(5)$.

Example: If q is embedded in the **5** we need $\mathcal{O}$ in the **5**. Now, composite fermionic resonances transform non-linearly under $SO(5)$, so that $\mathcal{O}$ cannot be a fermionic resonance, but it can be the combination $U(\Pi)_{IJ}Q^{J}$, coupling the elementary quark to the Goldstone matrix and a composite quark.

Generating quark masses

Problem: Hypercharge:
We so far identified T_R^3 with the $U(1)_Y$. But we want to have $Y = 1/6, 2/3, -1/3$ for the elementary quarks. This cannot be realized from the diagonal subgroup of $SU(2)$.

Solution:
Extend the global symmetry group and braking to $(SO(5) \times U(1)_X / SO(4) \times U(1)_X)$ and realize hyper charge as $Y = T_R^3 + X$.
The $U(1)_X$ does not affect the Goldstone bosons (it is unbroken), so they are neural under it, and our previous discussion of the Higgs and gauge boson sector is unaltered.

One simple choice (partially composite quarks in the **5**):

$$\bar{q}_L^5 = \frac{1}{\sqrt{2}}\Big(-i\bar{d}_L\,,\bar{d}_L\,,-i\bar{u}_L\,,-\bar{u}_L\,,0\Big)\,,$$
$$\bar{u}_R^5 = (0\,,0\,,0\,,0\,,\bar{u}_R)\,,$$

This fixes composite partner quarks to be embedded as **5** reps. of $SO(5)$:

$$\psi = \begin{pmatrix} Q \\ \tilde{U} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} iD - iX_{5/3} \\ D + X_{5/3} \\ iU + iX_{2/3} \\ -U + X_{2/3} \\ \sqrt{2}\tilde{U} \end{pmatrix}.$$

The down-type sector can be realized similarly.

Generating quark masses

BSM particle content:

	U	$X_{2/3}$	D	$X_{5/3}$	$\tilde{U}$
$SO(4)$	**4**	**4**	**4**	**4**	**1**
$SU(3)_c$	**3**	**3**	**3**	**3**	**3**
$U(1)_X$ charge	$2/3$	$2/3$	$2/3$	$2/3$	$2/3$
EM charge	$2/3$	$2/3$	$-1/3$	$5/3$	$2/3$

Fermion Lagrangian:

$$\mathcal{L}_{comp} = i\,\overline{Q}(D_\mu + ie_\mu)\gamma^\mu Q + i\overline{\tilde{U}}\not{D}\tilde{U} - M_4\overline{Q}Q - M_1\overline{\tilde{U}}\tilde{U} + \left(ic\overline{Q}^i\gamma^\mu d^i_\mu \tilde{U} + \text{h.c.}\right),$$

$$\mathcal{L}_{el,mix} = i\,\overline{q}_L\not{D}q_L + i\,\overline{u}_R\not{D}u_R - y_L f\overline{q}^5_L U_{gs}\psi_R - y_R f\overline{u}^5_R U_{gs}\psi_L + \text{h.c.},$$

- $M_{4,1}$ are the masses of the strongly coupled fermionic resonances.
- c is a coupling constant purely arising from the strong sector.
- $y_{L,R}$ are pre-yukawa couplings which parameterize the mixing between the elementary and composite sector.

Note: The Lagrangian is written in a $SO(5)$ invariant way, but the mixing terms break $SO(5)$ (and even $SO(4)$) explicitly, because q_L and u_R are in *incomplete* representations of $SO(5)$.

Generating quark masses

So how do we get a Yukawa coupling from this Lagrangian?

- q_L mixes with ψ_R states ($\propto y_L f$).
- The ψ_R states are Dirac fermions. They "mix" with ψ_L states via $M_{4,1}$.
- ψ_L states mix with q_R ($\propto y_R f$).

Diagonalizing the quark mass matrix, one finds

$$m_q = \frac{v}{\sqrt{2}} \frac{|M_1 - M_4|}{f} \frac{y_L f}{\sqrt{M_4 + y_L^2 f^2}} \frac{y_R f}{\sqrt{|M_1|^2 + y_R^2 f^2}} + \mathcal{O}(\sin^3(v/f))$$

So the SM Yukawa coupling is proportional to the product of the pre-yukawa couplings.

Note:

- In the above model, there are already several parameters $(f, M_4, M_1, \lambda_L, \lambda_R, c)$ which in the low-energy theory are free parameters which could only be calculated (in principle) if the UV description of the model is known.
- There are many other consistent combinations of representations for q_L, q_R and ψ embeddings.
  $\Rightarrow$ many models with different quark partner particle content, which can be tested at LHC.
- The model above generates a mass for *ONE* up-type quark. To truly embed the SM quarks, we need partners for *ALL* quarks (and leptons) to reproduce their masses *AND* the CKM matrix.

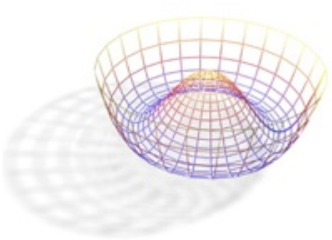
A Higgs potential from the top sector

In MCHM, the Higgs multiplet is identified as the Goldstone bosons of the breaking $SO(5) \to SO(4)$.

If the Higgs multiplet is an *exact* Goldstone multiplet, it cannot have a potential.

The Lagrangian is invariant under shift symmetries $\pi^i \to \pi^i + a^i$, and therefore π^i can only occur in derivative interactions in the Lagrangian – not in potential terms.

Another way to see this:
The Goldstone bosons span the vacuum manifold of the model.
In the picture on the right, this is the circle (a $U(1)$) at the bottom of V.
In the MCHM, this "circle" is a 4-dim. space spanned by $\pi^{\pm,3}$ *and* h.
(Now, h is *not* the radial direction in the picture.)
To give a potential to h, we have to "tip the potential to the side a bit" in the h direction.



There is one source of explicit breaking of the $SO(5)$ (and even of $SO(4)$) in our model, however:

The elementary quarks are embedded as *incomplete* $SO(5)$ multiplets.
$\Rightarrow$ Loop contributions including quarks and quark partners can induce a potential for the Higgs.
(this contribution of course depends on the embedding of the quarks)
Note: The Higgs couples to the elementary fermions via the pre-Yukawa couplings $y^q_{L,R}$, and $m_q \propto y^q_L y^q_R$, so typically, top loops play the dominant role.

There is another source of explicit breaking of $SO(5)$ (and even of $SO(4)$):

The subgroup $SU(2)_L \times U(1)_Y$ is gauged while the other directions are not.
$\Rightarrow$ Loop contributions including EW gauge bosons also contribute to the Higgs potential.

To see this more explicitly (from [Contino, 1005.4269])
To match to our notation: start from the Lagrangian

$$\begin{aligned}\mathcal{L} &\supset i\overline{q}_L \not{D} q_L + i\bar{t}_R \not{D} t_R + \overline{T}\left(i\not{D} - M_T\right) T \\ &= y_L \overline{q}_L U_{44} T_R - y_R \bar{t}_R U_{54} T_L + \text{h.c.}\end{aligned}$$

Now when we are at a scale below M_T, we can integrate out the top partner, and the Lagrangian for the top in momentum space reads:

$$\begin{aligned}\mathcal{L} &= \overline{q}_L \not{p} \left(\Pi_0^q(p) + \Pi_1^q(p)\cos(\overline{h}/f)\right) q_L \\ &\quad \bar{t}_R \not{p} \left(\Pi_0^t(p) - \Pi_1^t(p)\cos(\overline{h}/f)\right) t_R \\ &\quad + \sin(\overline{h}/f) M_1^t(p) \overline{q}_L \frac{H}{|\overline{h}|} t_R,\end{aligned}$$

In the above, $\Pi_{0,1}^{q,t}$ and $M_1^u(p)$ are the self-energy and mass contributions in momentum space.
E.g.: the mass of the top is

$$m_t \approx \frac{\langle h \rangle}{f} \frac{M_1^t(0)}{\sqrt{(\Pi_0^q + \Pi_1^q)(\Pi_0^t - \Pi_1^t)}}.$$

Now, integrating out also the top, one can calculate the one-loop Coleman-Weinberg potential.

[If you have not seen 1-loop effective potentials, Chapter 5 of “Aspects of Symmetries" by Coleman gives a wonderful introduction]

Result:

$$\begin{aligned} V(\bar{h}) &= -6\int \frac{d^4p}{(2\pi)^4}\left\{2\ln\left(1+\frac{\Pi_1^q}{\Pi_0^q}\cos(\bar{h}/f)\right)+\ln\left(1-\frac{\Pi_1^t}{\Pi_0^t}\cos(\bar{h}/f)\right)\right. \\ &\quad \left. +\ln\left(1-\frac{(M_1^t\sin(\bar{h}/f))^2}{p^2(\Pi_0^q+\cos(\bar{h}/f)\Pi_1^q)(\Pi_0^t+\cos(\bar{h}/f)\Pi_1^t)}\right)\right\} \\ &\simeq \alpha\cos(\bar{h}/f)-\beta\sin(\bar{h}/f) \end{aligned}$$

where

$$\begin{aligned} \alpha &= 6\int \frac{d^4p}{(2\pi)^4}\left(\frac{\Pi_1^t}{\Pi_0^t}-2\frac{\Pi_1^q}{\Pi_0^q}\right) \\ \beta &= 6\int \frac{d^4p}{(2\pi)^4}\frac{(M_1^t)^2}{-p^2(\Pi_0^q+\Pi_1^q)(\Pi_0^t-\Pi_1^t)} \end{aligned}$$

A Higgs potential from the top sector c.f. [Contino, 1005.4269]

$$V(\bar{h}) = \alpha \cos(\bar{h}/f) - \beta \sin(\bar{h}/f)$$

has a minimum at

$$\xi \equiv \sin^2(\langle h \rangle)/f) = 1 - (\alpha/2\beta)^2$$

if $\alpha < 2\beta$, and the 2nd derivative of the potential at this point (aka m_h^2) is

$$m_h^2 = 2\beta\xi/f^2$$

One can relate the factor β to the top mass and the mass of the lightest top resonance $m_* \sim M_T$ which yields c.f. [Contino, 1005.4269]

$$m_h^2 = \frac{3}{4\pi}\frac{m_t^2}{v^2} m_*^2 \xi$$

Upshots:

- $\sin(v/f)$ is a measure for the fine-tuning of the model. Natural: $v \sim f$.
- To obtain a light Higgs, the lightest top partner should not be too heavy.

A Higgs potential from the top sector c.f. [Contino, 1005.4269]

Note:

- $\sin(v/f)$ is a measure for the fine-tuning of the model. Natural: $v \sim f$.
- To obtain a light Higgs, the lightest top partner should not be too heavy.

Remarks:

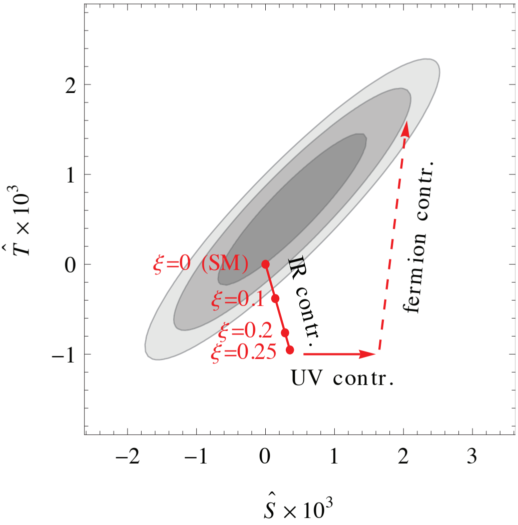
- The above calculation does not include the gauge contribution to the Higgs potential.
  C.f. [Contino, 1005.4269] for a more detailed discussion.
- The form of the potential ($V(\bar{h}) = \alpha \cos(\bar{h}/f) - \beta \sin(\bar{h}/f)$) depends on the representation of $SO(4)$ in which q_L, t_R, T are embedded into.
  Here, t_R is a singlet, and q_L, T are in the **4**.

A qualitative discussion:

There are several sources of electroweak corrections:

- UV contributions:
  The composite Higgs particle is similar but not identical to the SM Higgs
  In the SM, h *exactly* unitarizes the gauge boson scattering amplitudes.
  In CHMs , h does this only partially (relaxing but not solving the problem).
  The missing part has to be done by other, heavy resonances of the strong sector. $\Rightarrow$ UV sensitivity.
- Higgs loop contributions:
  The Higgs also contributes to EWPO in loops, but the couplings to EW gauge bosons are slightly modified.
- top partner loop contributions:
  The top partners couple to the EW gauge bosons and therefore add to loop corrections to gauge boson propagators.
  These corrections depend on the realization of the top-partner sector.
- Corrections to $Zb\bar{b}$: Top- or bottom partners also contribute to the (strongly constraint) $Zb\bar{b}$ vertex corrections.
  Again, these corrections depend on the realization of the quark-partner sector.

Electroweak precision constraints



[Grojean, Matsedonskyi, Panico, 1306.4655]

Typical bound : $\xi \lesssim 0.1 \Leftrightarrow f \gtrsim 750\,\mathrm{GeV}$
(for a top-partner scale $M_T \sim 1\,\mathrm{TeV}$)

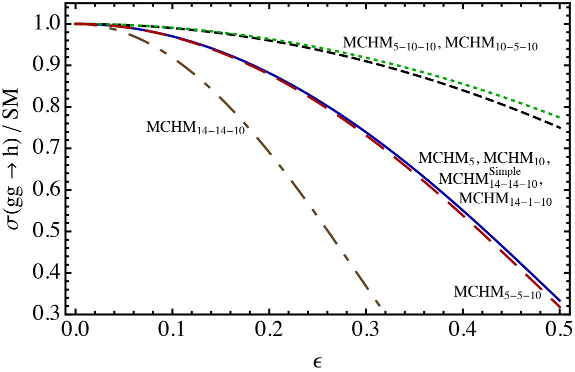
Electroweak precision constraints

Remarks:

- The detailed constraint on f depends on the implementation of the quark sector and on the quark partner masses. Generically, heavier top partners yield milder constraints on f.
- But remember that the Higgs mass (from the generated potential) is $\propto m_*^2 \xi$.
- This tension in the end leads to having a bound on $f \gtrsim 750\,\mathrm{GeV}$ and $m_* \sim \mathrm{TeV}$.

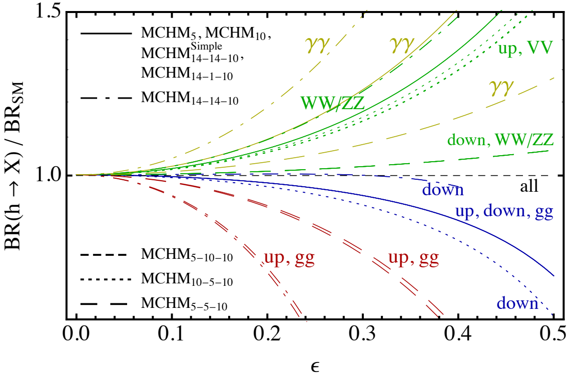
A qualitative discussion:

- We saw explicitly the in the MCHM, the coupling hWW is suppressed by a factor $\cos(\langle h \rangle / f)$.
- The couplings of the Higgs to fermions are also suppressed, but the suppression factor depends on the quark embedding.
- For Higgs production, this implies that the Higgs production rate (via gluon fusion and via vector boson fusion) is reduced.
  (But their relative strength can shift, depending on the quark representations chosen)
- The BR also shift and "typically" predict enhancement of decays into $\gamma\gamma$, WW^*, ZZ^* and reduction of qq and gg.
  The magnitude of change again depends on the quark implementation.



[Carena, Da Rold, Ponton, 1402.2987]

$(\epsilon = \sin(\langle h \rangle / f))$



[Carena, Da Rold, Ponton, 1402.2987]

$(\epsilon = \sin(\langle h \rangle / f))$

Summary of todays lecture

- We saw how quark masses can be realized in CHMs.
  This requires heavy quark partner states.
  There are many possibilities (representations) and several free parameters associated with the quark partner sector.
- We saw that a Higgs potential can be generated from the quark partners (in particular the top).
  Having a light Higgs requires to have top partners which are not too heavy.
- Electroweak precision observables yield a “generic” constraint: $f \gtrsim 750\,\mathrm{GeV}$.
  This implies a mild fine-tuning.
- The Higgs production rate and its branching ratios are modified.
  The modifications depend on the embedding of the quark sector.
  Taking the EW bound of $f \gtrsim 750\,\mathrm{GeV}$ into account, the predicted deviations from the SM production rate and BRs are in agreement with the current LHC data.
  But the predicted deviations of many models are large enough to tell a composite Higgs apart from the SM Higgs, eventually.