Flavor Physics, CP Violation and New Physics

Pyeong-Chang Summer Institute 2014
2014.08.24~2014.08.30

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imagine the impossible
KIAS KOREA INSTITUTE FOR ADVANCED STUDY

Outline

- Lecture 1
  - The Standard Model
  - New Physics beyond the SM
  - Minimal Supersymmetric Standard Model (MSSM)
- Lecture 2
  - Minimal Flavor Violation (MFV)
  - $(g-2)_\mu$, FCNC processes, EDM in the MSSM

Minimal Flavor Violation (MFV)

$$M(B_d-\bar{B}_d) \sim \frac{(y_t^2 V_{tb}^* V_{td})^2}{16\pi^2 m_t^2} + c_{NP}\frac{1}{\Lambda^2}$$

Isidori (2012)

c_{NP}:

- ~ 1 — tree/strong + generic flavor → $\Lambda \gtrsim 2\times10^4$ TeV [K]
- $\sim 1/(16\pi^2)$ — loop + generic flavor → $\Lambda \gtrsim 2\times10^3$ TeV [K]
- $\sim (y_t V_{ti}^* V_{tj})^2$ — tree/strong + “alignment” → $\Lambda \gtrsim 5$ TeV [K & B]
- $\sim (y_t V_{ti}^* V_{tj})^2/(16\pi^2)$ — loop + “alignment” → $\Lambda \gtrsim 0.5$ TeV [K & B]

- Can we build NP models where the alignment with the CKM is “natural”?

MFV

Minimal Flavour Violation

- Flavour symmetry:
  $U(3)^5$
  [global symmetry of the SM gauge sector]

- Symmetry-breaking terms: Y_U & Y_D
  [quark Yukawa couplings]

$$\mathscr{L}_{\mathrm{SM}} = \mathscr{L}_{\mathrm{gauge}} + \mathscr{L}_{\mathrm{Higgs}}$$

$$\mathscr{L}_{\mathrm{Higgs}} \longrightarrow \bar{Q}_L^{\,i}\, Y_U^{ij}\, U_R^{\,j}\, \phi + \bar{Q}_L^{\,i}\, Y_D^{ij}\, D_R^{\,j}\, \phi_c$$

This specific symmetry + symmetry-breaking pattern is responsible for the GIM suppression of FCNCs, the suppression of CPV,...
all the successful SM predictions in the quark flavour sector

MFV

However, we can (formally) promote this symmetry to be an exact symmetry, assuming the Yukawa matrices are the vacuum expectation values of appropriate auxiliary fields:

E.g.: $Y_D \sim (3,1,\bar{3})$ & $Y_U \sim (3,\bar{3},1)$ under $SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$

$$\mathscr{L}_{\text{Yukawa}} = \bar{Q}_L Y_D D_R \phi + \bar{Q}_L Y_U U_R \phi_c + \bar{L}_L Y_L e_R \phi + \text{h.c.}$$

$\bar{Q}_L$ ← $(\bar{3},1,1)$; Y_D ← $(3,1,\bar{3})$; D_R ← $(1,1,3)$

↓

$(1,1,1)$

MFV hypothesis: Y's are the only source of flavor violation also in BSM.

Typical FCNC dim.-6 operator: $\bar{Q}_L^{\,i} (Y_U Y_U^{+})_{ij} Q_L^{\,j} \times \bar{L}_L L_L$

$$\rightarrow \quad \overline{d'_L} V^\dagger \hat{Y}_u^2 V d'_L \rightarrow y_t^2 V_{32}^* V_{33} \overline{s'_L} b'_L$$

- NP modifies only the flavor-independent magnitude

Operator	Bound on Λ	Observables
$H^\dagger (\bar{D}_R Y^{d\dagger} Y^u Y^{u\dagger} \sigma_{\mu\nu} Q_L)(e F_{\mu\nu})$	6.1 TeV	$B \to X_s\gamma$, $B \to X_s \ell^+\ell^-$
$\frac{1}{2}(\bar{Q}_L Y^u Y^{u\dagger} \gamma_\mu Q_L)^2$	5.9 TeV	ε_K, Δm_{B_d}, Δm_{B_s}
$H_D^\dagger (\bar{D}_R Y^{d\dagger} Y^u Y^{u\dagger} \sigma_{\mu\nu} T^a Q_L)(g_s G^a_{\mu\nu})$	3.4 TeV	$B \to X_s\gamma$, $B \to X_s \ell^+\ell^-$
$(\bar{Q}_L Y^u Y^{u\dagger} \gamma_\mu Q_L)(\bar{E}_R \gamma_\mu E_R)$	2.7 TeV	$B \to X_s \ell^+\ell^-$, $B_s \to \mu^+\mu^-$

A few important comments:

I) MFV is not a theory of flavour

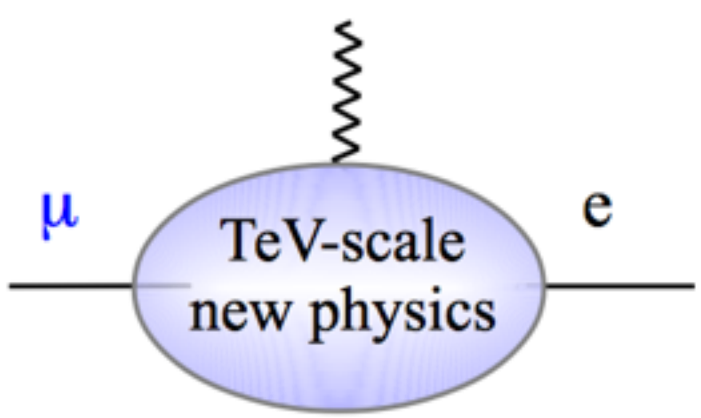
II) Despite its phenomenological success, MFV is far from being "verified"

III) Even within the "pessimistic" MFV hypothesis, we can still expect sizable deviations from the SM in various B physics observables...

Typical examples:

$B_{d,s} \to l^+ l^-$ — Large enhancements possible in models with an extended Higgs sector

... and, hopefully, spectacular NP effects in the charged lepton sector:



B(μ→eγ) could reach values in the 10^{-12} - 10^{-13} range (*within the reach of MEG*)

Muon g-2

- Although flavor conserving observable, it can give constraint on flavor models

$$\mathcal{L}_{\mathrm{MDM}} = \frac{e}{4m_\mu} a_\mu \bar{\mu} \sigma_{\alpha\beta} \mu F^{\alpha\beta}$$

- muon is ~40000 times more sensitive to NP than electron

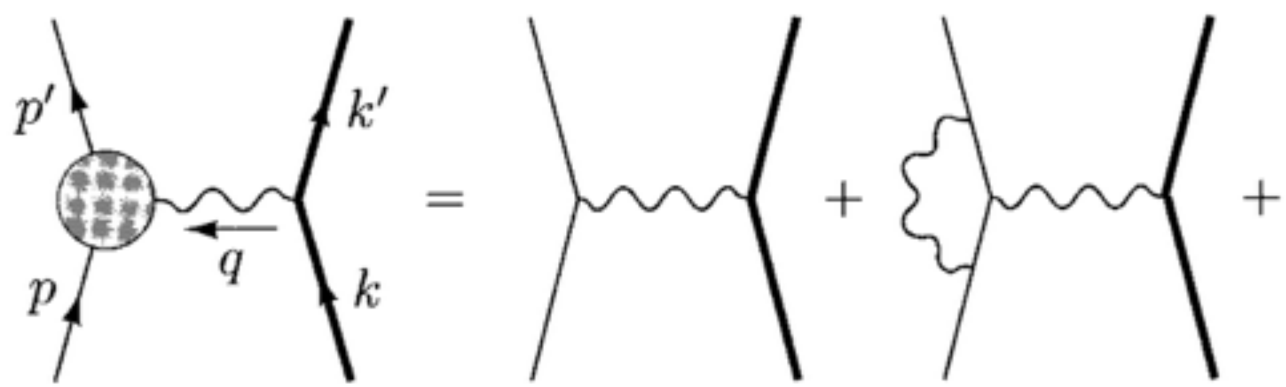
$$a_\mu \propto \frac{m_\mu^2}{\Lambda^2}$$

Basics of g-2

Scattering of electron with EM field

$$\Delta H_{\text{int}} = \int d^3x\, e A^{\text{cl}}_\mu j^\mu \qquad j^\mu(x) = \bar\psi(x)\gamma^\mu\psi(x)$$

$$i\mathcal{M}\,(2\pi)\delta(p^{0\prime} - p^0) = -ie\bar u(p')\gamma^\mu u(p)\cdot \widetilde{A}^{\text{cl}}_\mu(p'-p)$$



$$i\mathcal{M}\,(2\pi)\delta(p^{0\prime} - p^0) = -ie\bar u(p')\,\Gamma^\mu(p',p)\,u(p)\cdot \widetilde{A}^{\text{cl}}_\mu(p'-p)$$

From Lorentz, P invariance,

$$\Gamma^\mu = \gamma^\mu\cdot A + (p'^\mu + p^\mu)\cdot B + (p'^\mu - p^\mu)\cdot C.$$

Basics of g-2

Ward identity: $q_\mu \Gamma^\mu = 0$

Using Gordon identity $\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{p'^\mu + p^\mu}{2m} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$

$$\Gamma^\mu(p',p) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)$$

Tree level, $F_1 = 1 \text{ and } F_2 = 0$

Electron within external B-field $A_\mu^{\text{cl}}(x) = (0, \mathbf{A}^{\text{cl}}(\mathbf{x}))$

$$i\mathcal{M} = -i(2m) \cdot e\xi'^\dagger \left(\frac{-1}{2m}\sigma^k \left[F_1(0) + F_2(0)\right]\right)\xi\, \tilde{B}^k(\mathbf{q}),$$

$$\widetilde{B}^k(\mathbf{q}) = -i\epsilon^{ijk} q^i \widetilde{A}^j_{\text{cl}}(\mathbf{q})$$

We can interpret the scattering amp. as Born approx. to scattering of an electron from a potential well.

Basics of g-2

$$V(\mathbf{x}) = -\langle \boldsymbol{\mu} \rangle \cdot \mathbf{B}(\mathbf{x}),$$

$$\langle \boldsymbol{\mu} \rangle = \frac{e}{m}\left[F_1(0) + F_2(0)\right]\xi'^{\dagger}\frac{\boldsymbol{\sigma}}{2}\xi$$

$$\boldsymbol{\mu} = g\left(\frac{e}{2m}\right)\mathbf{S}$$

$$g = 2\big[F_1(0) + F_2(0)\big] = 2 + 2F_2(0)$$

electric charge of e=1

Anomalous magnetic dipole moment,

$$a = \frac{g-2}{2} = F_2(0)$$

$$a_e \equiv \frac{g-2}{2} = \frac{\alpha}{2\pi} \approx .0011614$$ Schwinger(1948)

Muon g-2

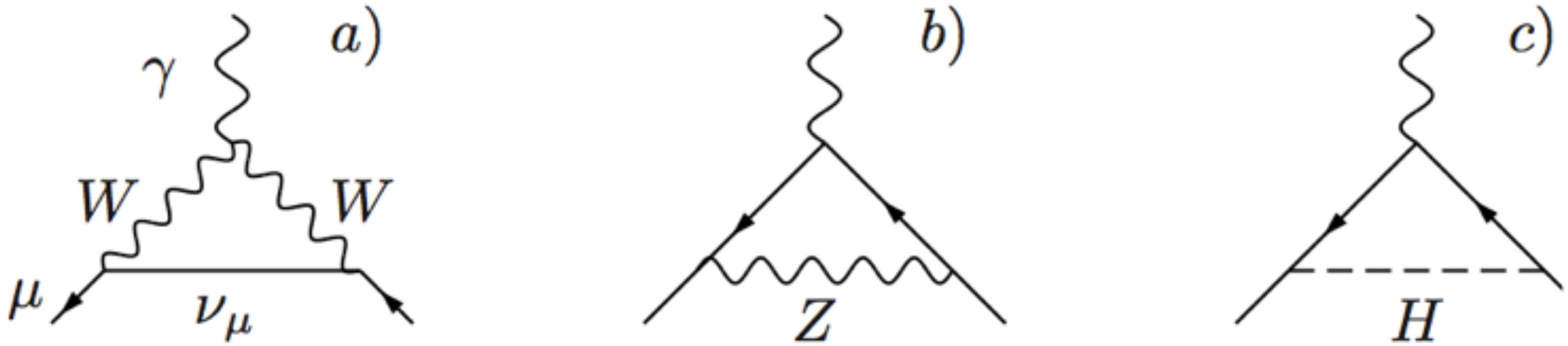
QED result up to 5 loops

Include contributions from all leptons (Aoyama et al. '12):

$$
\begin{aligned}
a_\mu^{\text{QED}} &= \frac{1}{2} \times \left(\frac{\alpha}{\pi}\right) \\
&+ 0.765\,857\,425 \underbrace{(17)}_{m_\mu/m_{e,\tau}} \times \left(\frac{\alpha}{\pi}\right)^2 \\
&+ 24.050\,509\,96 \underbrace{(32)}_{m_\mu/m_{e,\tau}} \times \left(\frac{\alpha}{\pi}\right)^3 \\
&+ 130.8796 \underbrace{(63)}_{\text{num. int.}} \times \left(\frac{\alpha}{\pi}\right)^4 \\
&+ 753.29 \underbrace{(1.04)}_{\text{num. int.}} \times \left(\frac{\alpha}{\pi}\right)^5 \\
&= 116\,584\,718.853 \underbrace{(9)}_{m_\mu/m_{e,\tau}} \underbrace{(19)}_{C_4} \underbrace{(7)}_{C_5} \underbrace{(29)}_{\alpha(a_e)} [36] \times 10^{-11}
\end{aligned}
$$

Muon g-2

Contributions from weak interaction

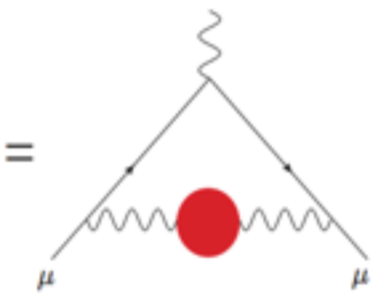


Total weak contribution:

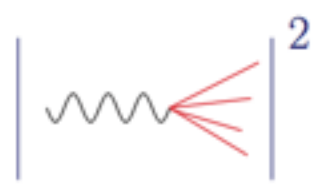
$$a_\mu^{\text{weak}} = (153.2 \pm 1.8) \times 10^{-11}$$

Muon g-2

Hadronic vacuum polarization

$a_\mu^{\text{had. v.p.}} =$ 

Optical theorem (from unitarity; conservation of probability) for hadronic contribution → dispersion relation:

Im  $\sim$  $\sim$ $\sigma(e^+e^- \to \gamma^* \to \text{hadrons})$

$$a_\mu^{\text{had. v.p.}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{ds}{s} K(s)\, R(s), \qquad R(s) = \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

[Bouchiat, Michel '61; Durand '62; Brodsky, de Rafael '68; Gourdin, de Rafael '69]

$K(s)$ slowly varying, positive function $\Rightarrow$ $a_\mu^{\text{had. v.p.}}$ positive. Data for hadronic cross section σ at low center-of-mass energies $\sqrt{s}$ important due to factor $1/s$: $\sim 70\%$ from $\pi\pi$ [$\rho(770)$] channel, $\sim 90\%$ from energy region below 1.8 GeV.

Muon g-2

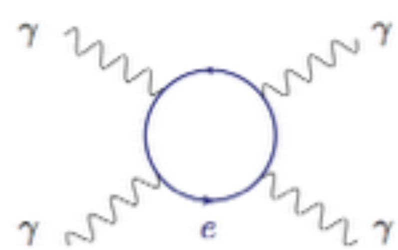
Hadronic vacuum polarization: some recent evaluations

Authors	Contribution to $a_\mu^{\text{had v.p.}} \times 10^{11}$
Jegerlehner '08; JN '09 (e^+e^-)	6903.0 ± 52.6
Davier et al. '09 (e^+e^-)	6955 ± 41
Davier et al. '09 ($e^+e^- + \tau$)	7053 ± 45
Teubner et al. '09 (e^+e^-)	6894 ± 40
Davier et al. '10 (e^+e^-)	6923 ± 42
Davier et al. '10 ($e^+e^- + \tau$)	7015 ± 47
Jegerlehner + Szafron '11 (e^+e^-)	6907.5 ± 47.2
Jegerlehner + Szafron '11 ($e^+e^- + \tau$)	6909.6 ± 46.5
Hagiwara et al. '11 (e^+e^-)	6949.1 ± 42.7
Benayoun at al. '12 ($e^+e^- + \tau$: HLS improved)	6877.2 ± 46.3

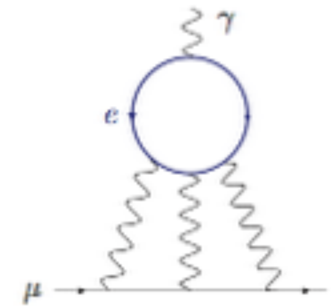
Muon g-2

Hadronic light-by-light scattering in the muon $g-2$

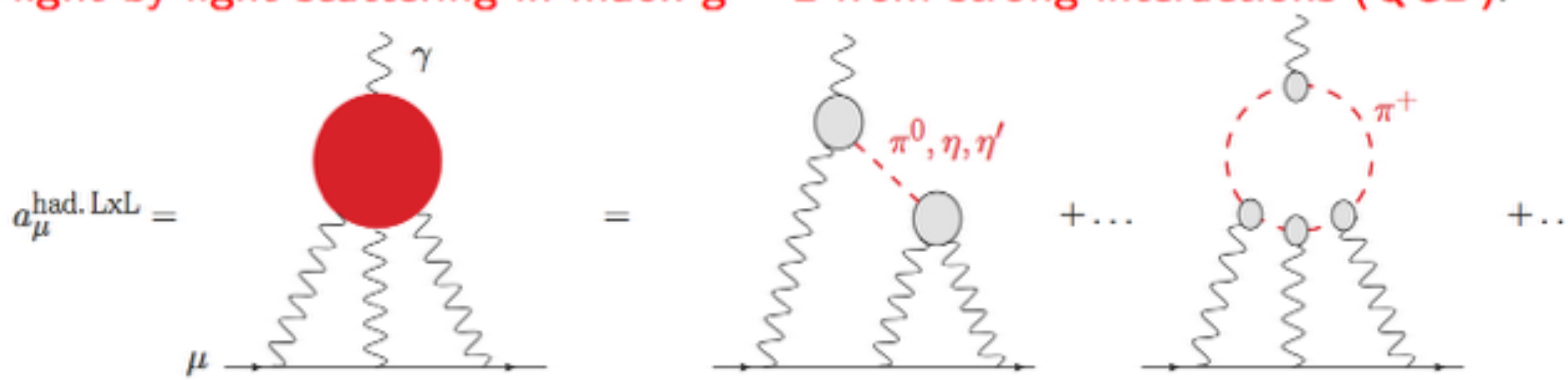
QED: light-by-light scattering at higher orders in perturbation series via lepton-loop:



In muon $g-2$:
$\Rightarrow$



Hadronic light-by-light scattering in muon $g-2$ from strong interactions (QCD):



- Currently often used estimates (neutral-pion exchange dominates numerically):

$a_\mu^{\text{had. LbyL}} = (105 \pm 26) \times 10^{-11}$ (Prades, de Rafael, Vainshtein '09)
(error estimate more progressive)

$a_\mu^{\text{had. LbyL}} = (116 \pm 40) \times 10^{-11}$ (Nyffeler '09; Jegerlehner, Nyffeler '09)
(error estimate more conservative)

Muon g-2

Muon $g-2$: current status

Summary of SM contributions to a_μ (based on various recent papers):

- Leptonic QED contributions: $a_\mu^{\text{QED}} = (116\ 584\ 718.853 \pm 0.036) \times 10^{-11}$
- Weak contributions: $a_\mu^{\text{weak}} = (153.2 \pm 1.8) \times 10^{-11}$
- Hadronic contributions:
  - Vacuum Polarization:
    $a_\mu^{\text{had. v.p.}}(e^+e^-) = (6907.5 \pm 47.2 - (100.3 \pm 2.2)) \times 10^{-11}$
  - Light-by-Light scattering: $a_\mu^{\text{LbyL}} = (116 \pm 40) \times 10^{-11}$
- Total SM contribution:

$$a_\mu^{\text{SM}} = (116\ 591\ 795 \pm \underbrace{47}_{\text{v.p.}} \pm \underbrace{40}_{\text{LbyL}} \pm \underbrace{1.8}_{\text{QED + EW}} \quad [\pm 62]) \times 10^{-11}$$

- "New" experimental value (shifted $+9.2 \times 10^{-11}$ due to new $\lambda = \mu_\mu / \mu_p$):

$$a_\mu^{\text{exp}} = (116\ 592\ 089 \pm 63) \times 10^{-11}$$

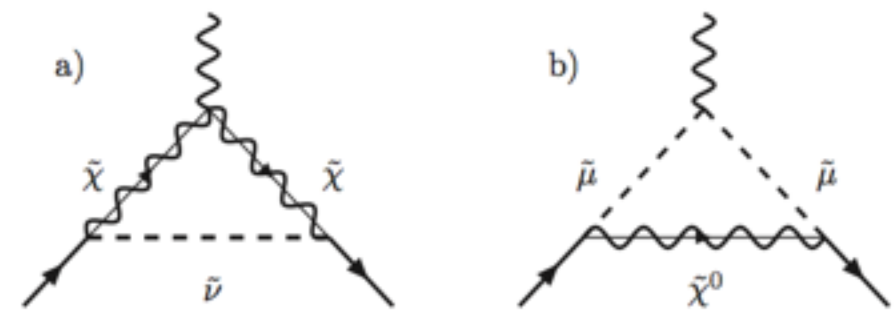
$$\Rightarrow a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (294 \pm 88) \times 10^{-11} \qquad [3.3\ \sigma]$$

Muon g-2 (MSSM)

a_μ: Supersymmetry

Supersymmetry for large $\tan\beta, \mu > 0$:

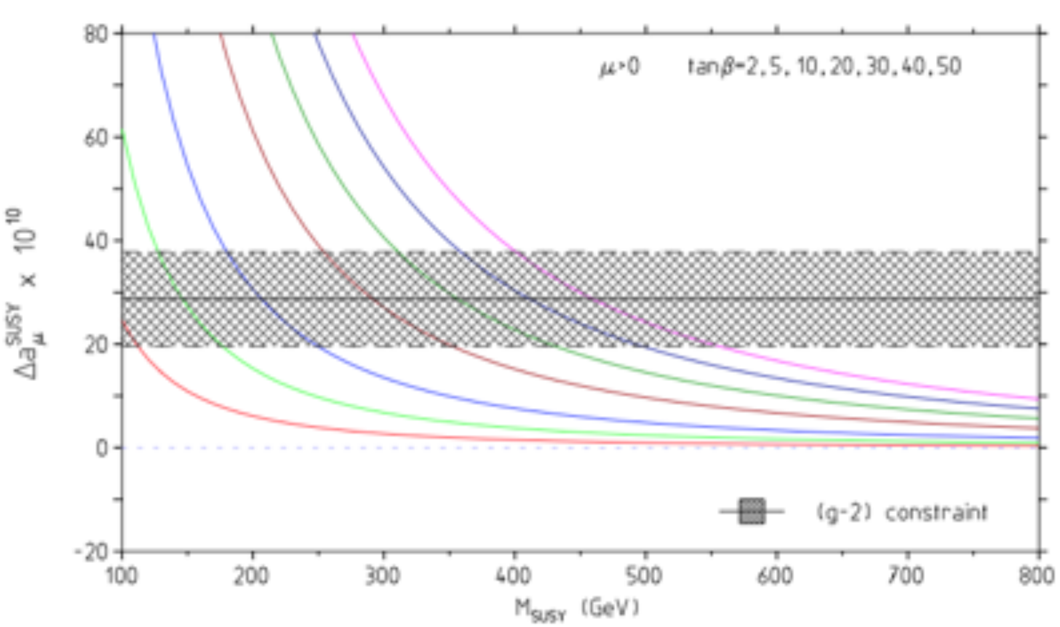
$$a_\mu^{\text{SUSY}} \approx 123 \times 10^{-11} \left(\frac{100\text{ GeV}}{M_{\text{SUSY}}}\right)^2 \tan\beta$$



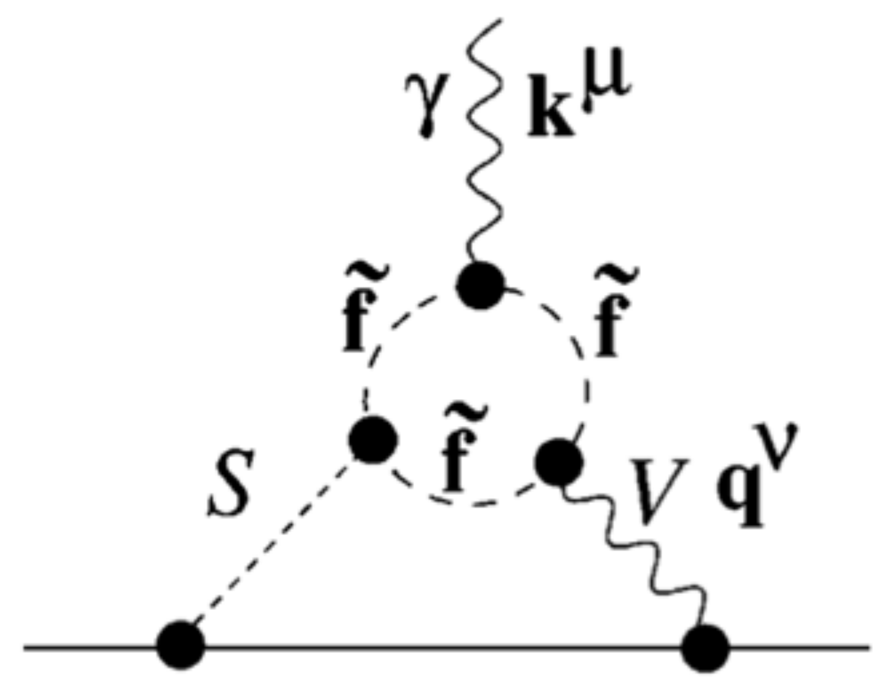
(Czarnecki, Marciano, 2001)

Explains $\Delta a_\mu = 290 \times 10^{-11}$ if $M_{\text{SUSY}} \approx (93 - 414)$ GeV $(2 < \tan\beta < 40)$.

In some regions of the parameter space, there can also be large two-loop contributions.

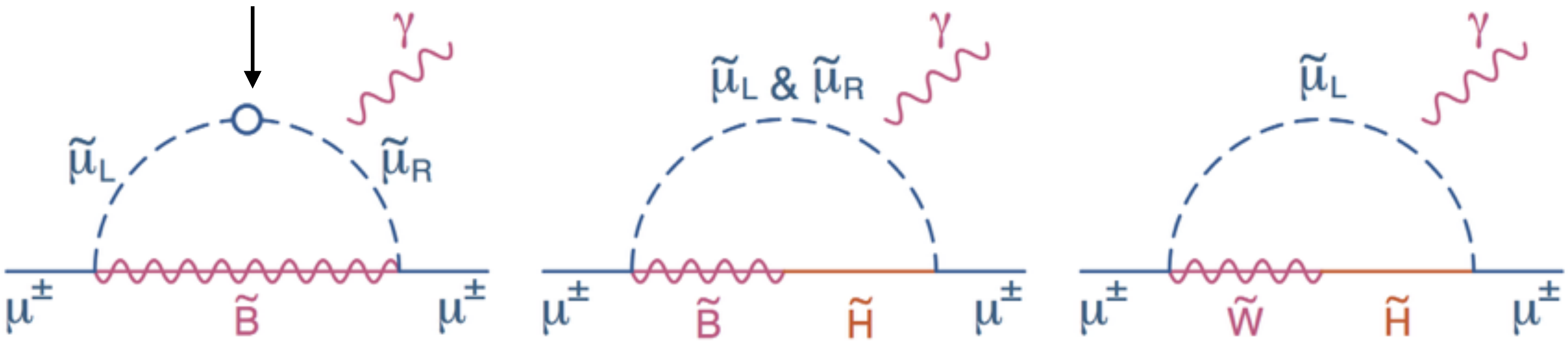


Constraint on large $\tan\beta$ SUSY contributions as a function of M_{SUSY}.



Muon g-2 (MSSM)

chirality flip can occur inside the loop



$\Rightarrow a_\mu^{(\mathrm{SUSY})}$ is approximately proportional to $\tan\beta$

Muon g-2 (Dark photon)

a_e, a_μ: Dark photon

In some dark matter scenarios, there is a relatively light, but massive "dark photon" A'_μ that couples to the SM through mixing with the photon:

$$\mathcal{L}_{\text{mix}} = \frac{\varepsilon}{2} F^{\mu\nu} F'_{\mu\nu}$$

$\Rightarrow$ A'_μ couples to ordinary charged particles with strength $\varepsilon \cdot e$.
$\Rightarrow$ additional contribution of dark photon with mass m_V to the $g-2$ of a lepton (electron, muon) (Pospelov '09):

$$\begin{aligned} a_\ell^{\text{dark photon}} &= \frac{\alpha}{2\pi}\varepsilon^2 \int_0^1 dx \frac{2x(1-x)^2}{\left[(1-x)^2 + \frac{m_V^2}{m_\ell^2}x\right]} \\ &= \frac{\alpha}{2\pi}\varepsilon^2 \times \begin{cases} 1 & \text{for } m_\ell \gg m_V \\ \frac{2m_\ell^2}{3m_V^2} & \text{for } m_\ell \ll m_V \end{cases} \end{aligned}$$

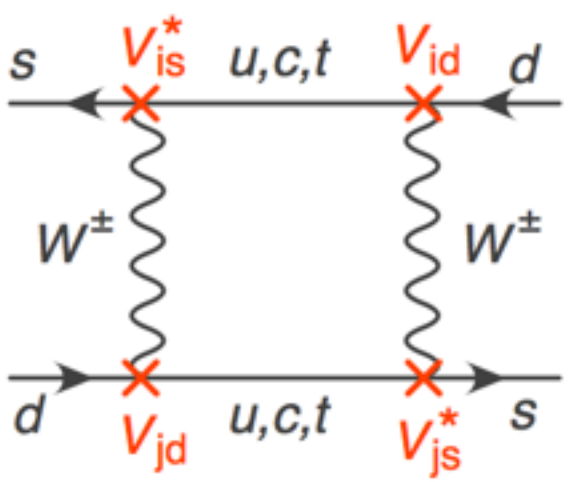
For values $\varepsilon \sim (1-2) \times 10^{-3}$ and $m_V \sim (10-100)$ MeV, the dark photon could explain the discrepancy $\Delta a_\mu = 290 \times 10^{-11}$.

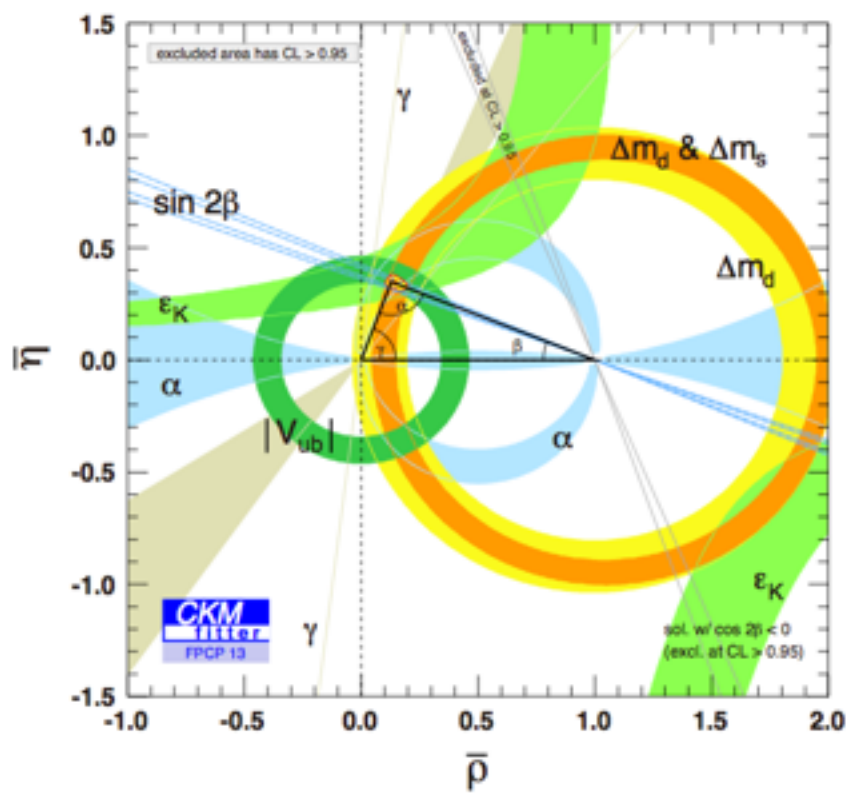
FCNC processes in MSSM

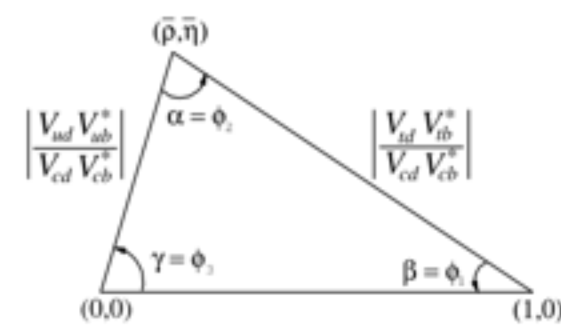
- Ref.
  - Gabbiani, Gabrielli, Masiero, Silvestrini, hep-ph/9604387

$K^0 - \overline{K}^0$ mixing

In the SM, K^0-$\bar{K}^0$ mixing originates from $W^{\pm}$-boson loop







[CKMfitter ('13)]

- $|\epsilon_K^{(\mathrm{exp})}| = (2.228 \pm 0.011) \times 10^{-3}$
- $|\epsilon_K^{(\mathrm{exp})}| - |\epsilon_K^{(\mathrm{SM})}| = (3.8 \pm 2.7) \times 10^{-4}$ $\Rightarrow |\epsilon_K^{(\mathrm{extra})}| < 9.2 \times 10^{-4}$ (2σ)

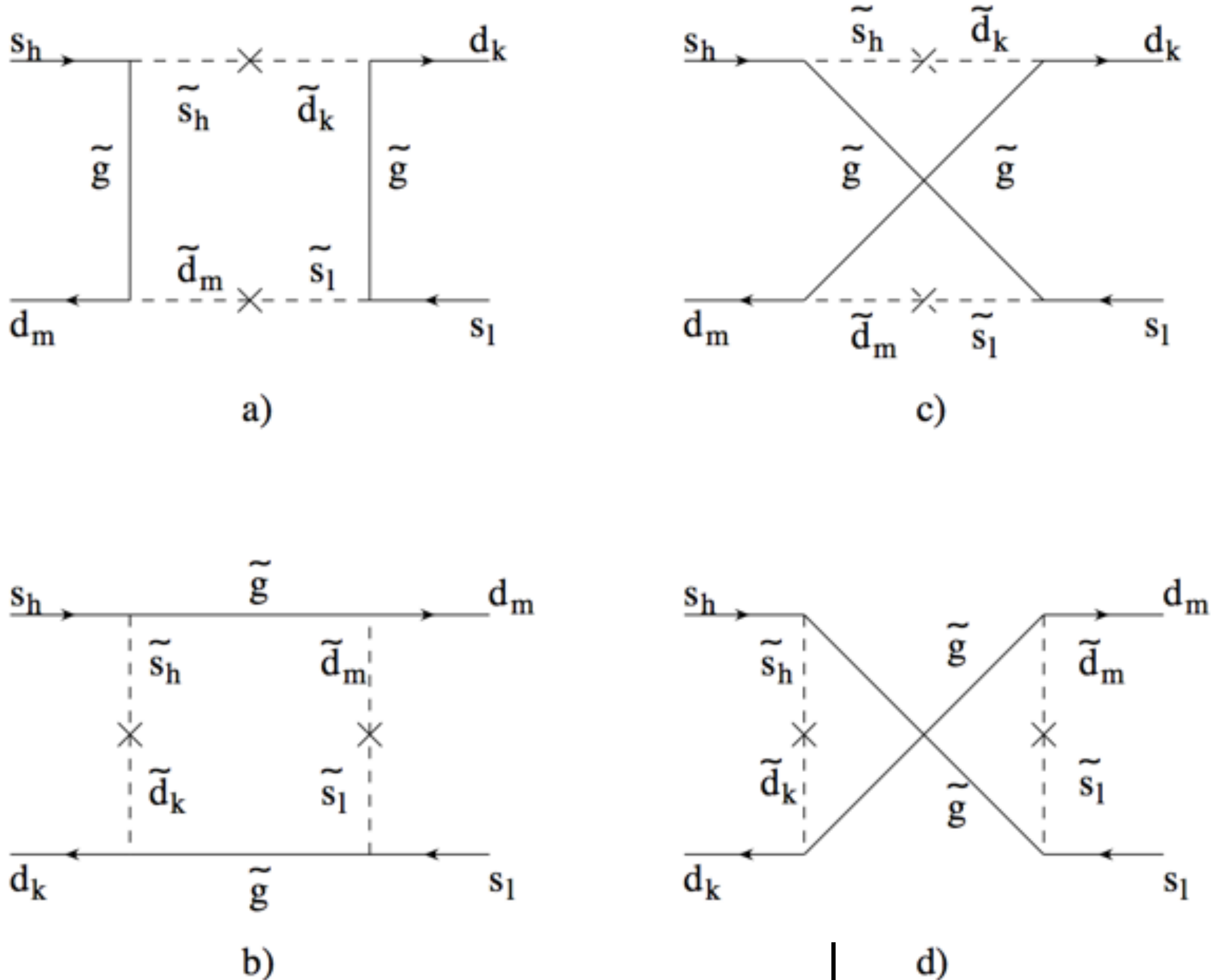


Figure 1: Feynman diagrams for $\Delta S = 2$ transitions, with $h, k, l, m = \{L, R\}$.

New topology not present in the SM

Effective operators for

$K^0 - \overline{K}^0$ mixing

$$\mathcal{H}_{\rm eff}^{\Delta S=2} = \sum_{i=1}^{5} C_i\, Q_i + \sum_{i=1}^{3} \tilde{C}_i\, \tilde{Q}_i\,,$$

$$\begin{aligned}
Q_1 &= \bar{d}_L^\alpha \gamma_\mu s_L^\alpha \bar{d}_L^\beta \gamma^\mu s_L^\beta\,,\\
Q_2 &= \bar{d}_R^\alpha s_L^\alpha \bar{d}_R^\beta s_L^\beta\\
Q_3 &= \bar{d}_R^\alpha s_L^\beta \bar{d}_R^\beta s_L^\alpha\\
Q_4 &= \bar{d}_R^\alpha s_L^\alpha \bar{d}_L^\beta s_R^\beta\\
Q_5 &= \bar{d}_R^\alpha s_L^\beta \bar{d}_L^\beta s_R^\alpha
\end{aligned}$$

In the SM, only Q1 is generated!

$$\begin{aligned}
C_1 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\left(24\, x\, f_6(x) + 66\, \tilde{f}_6(x)\right)\left(\delta_{12}^d\right)_{LL}^2\\
C_2 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\, 204\, x\, f_6(x)\left(\delta_{12}^d\right)_{RL}^2\\
C_3 &= \frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\, 36\, x\, f_6(x)\left(\delta_{12}^d\right)_{RL}^2\\
C_4 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\Big[\left(504\, x\, f_6(x) - 72\, \tilde{f}_6(x)\right)\left(\delta_{12}^d\right)_{LL}\left(\delta_{12}^d\right)_{RR}\\
&\qquad - 132\, \tilde{f}_6(x)\left(\delta_{12}^d\right)_{LR}\left(\delta_{12}^d\right)_{RL}\Big]\\
C_5 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\Big[\left(24\, x\, f_6(x) + 120\, \tilde{f}_6(x)\right)\left(\delta_{12}^d\right)_{LL}\left(\delta_{12}^d\right)_{RR}\\
&\qquad - 180\, \tilde{f}_6(x)\left(\delta_{12}^d\right)_{LR}\left(\delta_{12}^d\right)_{RL}\Big]\\
\tilde{C}_1 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\left(24\, x\, f_6(x) + 66\, \tilde{f}_6(x)\right)\left(\delta_{12}^d\right)_{RR}^2\\
\tilde{C}_2 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\, 204\, x\, f_6(x)\left(\delta_{12}^d\right)_{LR}^2\\
\tilde{C}_3 &= \frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\, 36\, x\, f_6(x)\left(\delta_{12}^d\right)_{LR}^2\,,
\end{aligned}$$

Hadronic matrix elements

$$\langle K^0|Q_1|\bar{K}^0\rangle_{\rm VIA} = \frac{1}{3}M_K f_K^2$$
$$\langle K^0|Q_2|\bar{K}^0\rangle_{\rm VIA} = -\frac{5}{24}\left(\frac{M_K}{m_s+m_d}\right)^2 M_K f_K^2$$
$$\langle K^0|Q_3|\bar{K}^0\rangle_{\rm VIA} = \frac{1}{24}\left(\frac{M_K}{m_s+m_d}\right)^2 M_K f_K^2$$
$$\langle K^0|Q_4|\bar{K}^0\rangle_{\rm VIA} = \left[\frac{1}{24}+\frac{1}{4}\left(\frac{M_K}{m_s+m_d}\right)^2\right] M_K f_K^2$$
$$\langle K^0|Q_5|\bar{K}^0\rangle_{\rm VIA} = \left[\frac{1}{8}+\frac{1}{12}\left(\frac{M_K}{m_s+m_d}\right)^2\right] M_K f_K^2\,,$$

Use

$$\boxed{\delta_{il}\delta_{kj} = \frac{1}{N}\delta_{kl}\delta_{ij} + 2t^a_{ij}t^a_{kl}}$$

to calculate the VIA

$$\langle \mathrm{K}|[\bar{\mathrm{d}}\gamma^\mu(1-\gamma_5)\mathrm{s}][\bar{\mathrm{d}}\gamma_\mu(1-\gamma_5)\mathrm{s}]|\bar{\mathrm{K}}\rangle = \frac{8}{3}\langle \mathrm{K}|\bar{\mathrm{d}}\gamma^\mu\gamma_5\mathrm{s}|0\rangle\langle 0|\bar{\mathrm{d}}\gamma_\mu\gamma_5\mathrm{s}|\bar{\mathrm{K}}\rangle = \frac{8}{3}\frac{f_\mathrm{K}^2 m_\mathrm{K}^2}{2m_\mathrm{K}} \quad (12.$$

$$\langle \bar{K}^0|\hat{Q}_1(\mu)|K^0\rangle = \frac{1}{3}M_K f_K^2 B_1(\mu)$$
$$\langle \bar{K}^0|\hat{Q}_2(\mu)|K^0\rangle = -\frac{5}{24}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K f_K^2 B_2(\mu)$$
$$\langle \bar{K}^0|\hat{Q}_3(\mu)|K^0\rangle = \frac{1}{24}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K f_K^2 B_3(\mu)$$
$$\langle \bar{K}^0|\hat{Q}_4(\mu)|K^0\rangle = \frac{1}{4}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K f_K^2 B_4(\mu)$$
$$\langle \bar{K}^0|\hat{Q}_5(\mu)|K^0\rangle = \frac{1}{12}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K f_K^2 B_5(\mu)\,,$$

we have used, for $\mu = 2$ GeV:

$$B_1(\mu) = 0.60(6)$$
$$B_2(\mu) = 0.66(4)$$
$$B_3(\mu) = 1.05(12)$$
$$B_4(\mu) = 1.03(6)$$
$$B_5(\mu) = 0.73(10)\,.$$

Mass difference and CPV parameter ε

$$\Delta M_K = 2\,\mathrm{Re}\langle K^0|\mathcal{H}_{\mathrm{eff}}^{\Delta S=2}|\bar{K}^0\rangle$$

$$\varepsilon = \frac{1}{\sqrt{2}\Delta M_K}\,\mathrm{Im}\langle K^0|\mathcal{H}_{\mathrm{eff}}^{\Delta S=2}|\bar{K}^0\rangle$$

Comparing with the exp., we get the constraint

Masiero, Vempati, Vives, 0711.2903

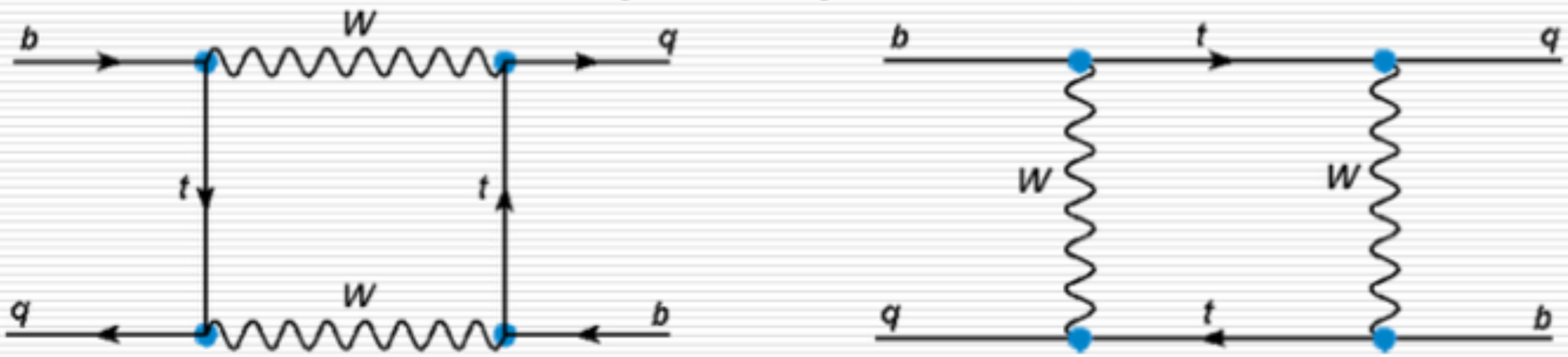
	$\sqrt{\lvert\Re(\delta^d_{12})^2_{LL}\rvert}$		$\sqrt{\lvert\Im(\delta^d_{12})^2_{LL}\rvert}$	
x	TREE	NLO	TREE	NLO
0.3	1.4×10^{-2}	2.2×10^{-2}	1.8×10^{-3}	2.9×10^{-3}
1.0	3.0×10^{-2}	4.6×10^{-2}	3.9×10^{-3}	6.1×10^{-3}
4.0	7.0×10^{-2}	1.1×10^{-1}	9.2×10^{-3}	1.4×10^{-2}
	$\sqrt{\lvert\Re(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$		$\sqrt{\lvert\Im(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$	
x	TREE	NLO	TREE	NLO
0.3	1.8×10^{-3}	8.6×10^{-4}	2.3×10^{-4}	1.1×10^{-4}
1.0	2.0×10^{-3}	9.6×10^{-4}	2.6×10^{-4}	1.3×10^{-4}
4.0	2.8×10^{-3}	1.3×10^{-3}	3.7×10^{-4}	1.8×10^{-4}
	$\sqrt{\lvert\Re(\delta^d_{12})^2_{LR}\rvert}$		$\sqrt{\lvert\Im(\delta^d_{12})^2_{LR}\rvert}$	
x	TREE	NLO	TREE	NLO
0.3	3.1×10^{-3}	2.6×10^{-3}	4.1×10^{-4}	3.4×10^{-4}
1.0	3.4×10^{-3}	2.8×10^{-3}	4.6×10^{-4}	3.7×10^{-4}
4.0	4.9×10^{-3}	3.9×10^{-3}	6.5×10^{-4}	5.2×10^{-4}

Table 1: Maximum allowed values for $\lvert\Re\left(\delta^d_{12}\right)_{AB}\rvert$ and $\lvert\Im\left(\delta^d_{12}\right)_{AB}\rvert$, with $A, B = (L, R)$ for an average squark mass $m_{\tilde{q}} = 500$ GeV and for different values of $x = m^2_{\tilde{g}}/m^2_{\tilde{q}}$. The bounds are given at tree level in the effective Hamiltonian and at NLO in QCD corrections as explained in the text. For different values of $m_{\tilde{q}}$ the bounds scale roughly as $m_{\tilde{q}}/500$ GeV.

$B^0 - \bar{B}^0$ mixing

Calculation of M_{12}

- ΔB=2 transitions need to go through a W loop



- As usual we write an effective Hamiltonian that captures the SD physics:

$$M_{12} = \frac{G_F^2 m_W^2}{12\pi^2} \eta_B m_B \left(f_B^2 \hat{B}_B\right) S_0\left(\frac{m_t^2}{m_W^2}\right) \left(V_{tb} V_{tq}^*\right)^2$$

$$\Delta m_B = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B m_B \left(f_B^2 \hat{B}_B\right) S_0\left(\frac{m_t^2}{m_W^2}\right) \left|V_{tb}^* V_{tq}\right|^2$$

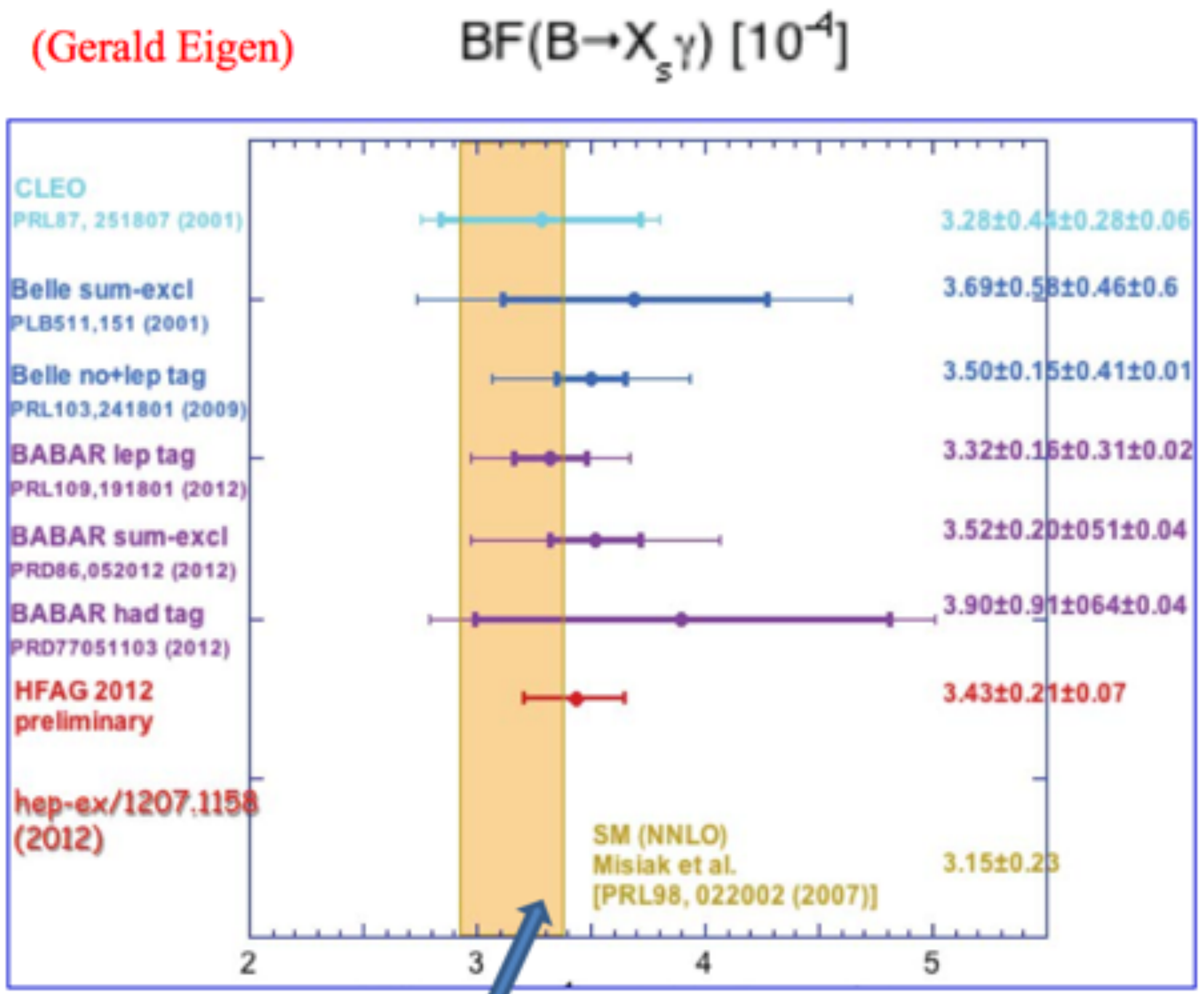
$$\frac{q}{p} = -\frac{(V_{tb}^* V_{tq})^2}{|V_{tb}^* V_{tq}|^2} = \begin{cases} -e^{-2i\beta} & \text{q=d} \\ -e^{-2i\beta_s} & \text{q=s} \end{cases}$$

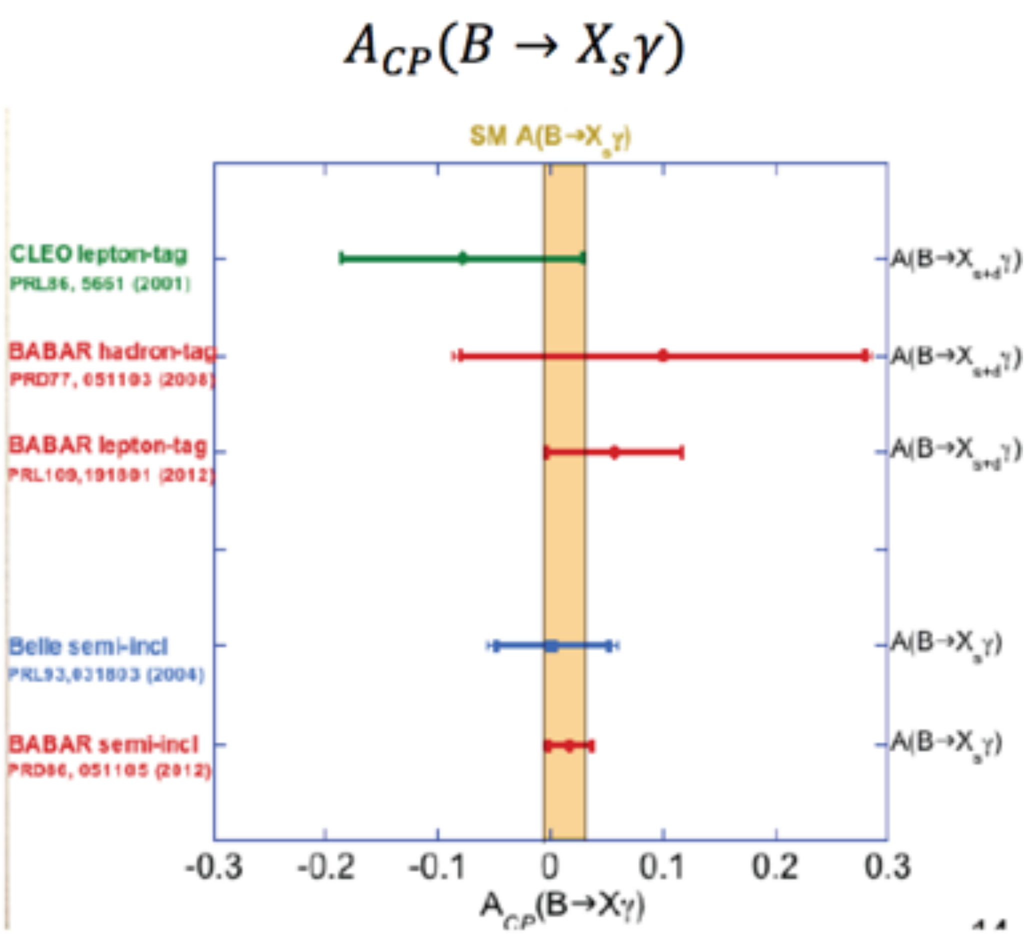
	$\lvert\Re(\delta^d_{13})_{LL}\rvert$		$\lvert\Re(\delta^d_{13})_{LL=RR}\rvert$	
x	TREE	NLO	TREE	NLO
0.25	4.9×10^{-2}	6.2×10^{-2}	3.1×10^{-2}	1.9×10^{-2}
1.0	1.1×10^{-1}	1.4×10^{-1}	3.4×10^{-2}	2.1×10^{-2}
4.0	6.0×10^{-1}	7.0×10^{-1}	4.7×10^{-2}	2.8×10^{-2}
	$\lvert\Im(\delta^d_{13})_{LL}\rvert$		$\lvert\Im(\delta^d_{13})_{LL=RR}\rvert$	
x	TREE	NLO	TREE	NLO
0.25	1.1×10^{-1}	1.3×10^{-1}	1.3×10^{-2}	8.0×10^{-3}
1.0	2.6×10^{-1}	3.0×10^{-1}	1.5×10^{-2}	9.0×10^{-3}
4.0	2.6×10^{-1}	3.4×10^{-1}	2.0×10^{-2}	1.2×10^{-2}
	$\lvert\Re(\delta^d_{13})_{LR}\rvert$		$\lvert\Re(\delta^d_{13})_{LR=RL}\rvert$	
x	TREE	NLO	TREE	NLO
0.25	3.4×10^{-2}	3.0×10^{-2}	3.8×10^{-2}	2.6×10^{-2}
1.0	3.9×10^{-2}	3.3×10^{-2}	8.3×10^{-2}	5.2×10^{-2}
4.0	5.3×10^{-2}	4.5×10^{-2}	1.2×10^{-1}	–
	$\lvert\Im(\delta^d_{13})_{LR}\rvert$		$\lvert\Im(\delta^d_{13})_{LR=RL}\rvert$	
x	TREE	NLO	TREE	NLO
0.25	7.6×10^{-2}	6.6×10^{-2}	1.5×10^{-2}	9.0×10^{-3}
1.0	8.7×10^{-2}	7.4×10^{-2}	3.6×10^{-2}	2.3×10^{-2}
4.0	1.2×10^{-1}	1.0×10^{-1}	2.7×10^{-1}	–

Table 2: Maximum allowed values for $|\Re\left(\delta^d_{13}\right)_{AB}|$ and $|\Im\left(\delta^d_{13}\right)_{AB}|$, with $A, B = (L, R)$ for an average squark mass $m_{\tilde{q}} = 500$ GeV and different values of $x = m^2_{\tilde{g}}/m^2_{\tilde{q}}$. with NLO evolution and lattice B parameters, denoted by NLO. The missing entries correspond to cases in which no constraint was found for $|\left(\delta^d_{ij}\right)_{AB}| < 0.9$.

b→s γ

(Gerald Eigen)





(Mikolaj Misiak)

Update of the SM prediction (preliminary)

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{SM}_{E_\gamma > 1.6\ \text{GeV}} = (3.14 \pm 0.22) \times 10^{-4}$$

All results consistent with the SM,
→ strong NP constraints

b→s γ

Relavant effective operators

- Chirality flip as in MDM op.
- Dominate in the SM

$$O_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R, \qquad \tilde{O}_7 = \frac{e}{16\pi^2} m_b \bar{s}_R \sigma_{\mu\nu} F^{\mu\nu} b_L,$$

$$O_8 = \frac{g_s}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} G_a^{\mu\nu} t_a b_R, \qquad \tilde{O}_8 = \frac{g_s}{16\pi^2} m_b \bar{s}_R \sigma_{\mu\nu} G_a^{\mu\nu} t_a b_L.$$

Wilson coefficients at mb scale

$$c_7(m_b) = \left[\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right]^{16/23} \left\{ c_7(M_W) - \frac{8}{3} c_8(M_W) \left[1 - \left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{2/23}\right] + \frac{232}{513}\left[1 - \left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{19/23}\right]\right\},$$

$$\frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\nu)} = \frac{6\alpha}{\pi\rho\lambda} |c_7(m_b)|^2$$

$b \to s\,\gamma$

- Gluino contribution

$$\mathrm{BR}(b \to s\gamma) = \frac{\alpha_s^2 \alpha}{81\pi^2 m_{\tilde{q}}^4} m_b^3 \tau_B \left\{ \left| m_b M_3(x) \left(\delta_{23}^d\right)_{LL} + m_{\tilde{g}} M_1(x) \left(\delta_{23}^d\right)_{LR} \right|^2 + L \leftrightarrow R \right\} .$$

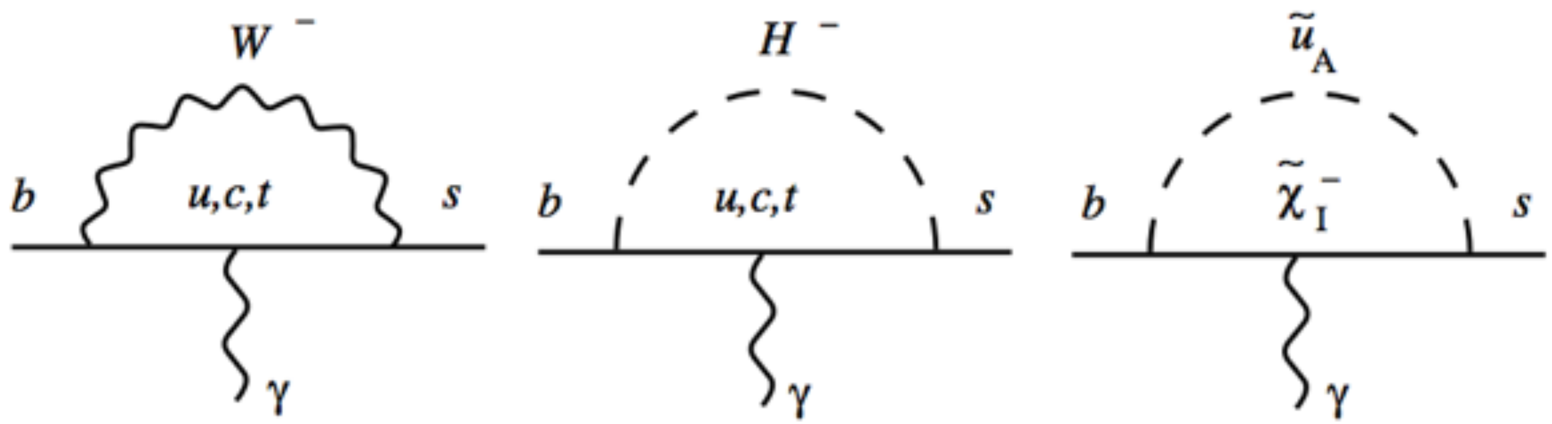
x	$\left\|\left(\delta_{23}^d\right)_{LL}\right\|$	$\left\|\left(\delta_{23}^d\right)_{LR}\right\|$
0.3	4.4	1.3×10^{-2}
1.0	8.2	1.6×10^{-2}
4.0	26	3.0×10^{-2}

LR is more strongly constrained

Table 6: Limits on the $\left|\delta_{23}^d\right|$ from $b \to s\gamma$ decay for an average squark mass $m_{\tilde{q}} = 500\mathrm{GeV}$ and for different values of $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$. For different values of $m_{\tilde{q}}$, the limits can be obtained multiplying the ones in the table by $(m_{\tilde{q}}(\mathrm{GeV})/500)^2$.

$b \to s\,\gamma$

SM + Charged Higgs + Chargino



・ Charged Higgs: constructive interference with the SM

・ Chargino: destructive if $\mu > 0$

※ $\mu > 0$ is preferred to explain $(g-2)_\mu$ anomaly

$B_s \to \mu^+ \mu^-$

chirality flip

Misiak (2014)

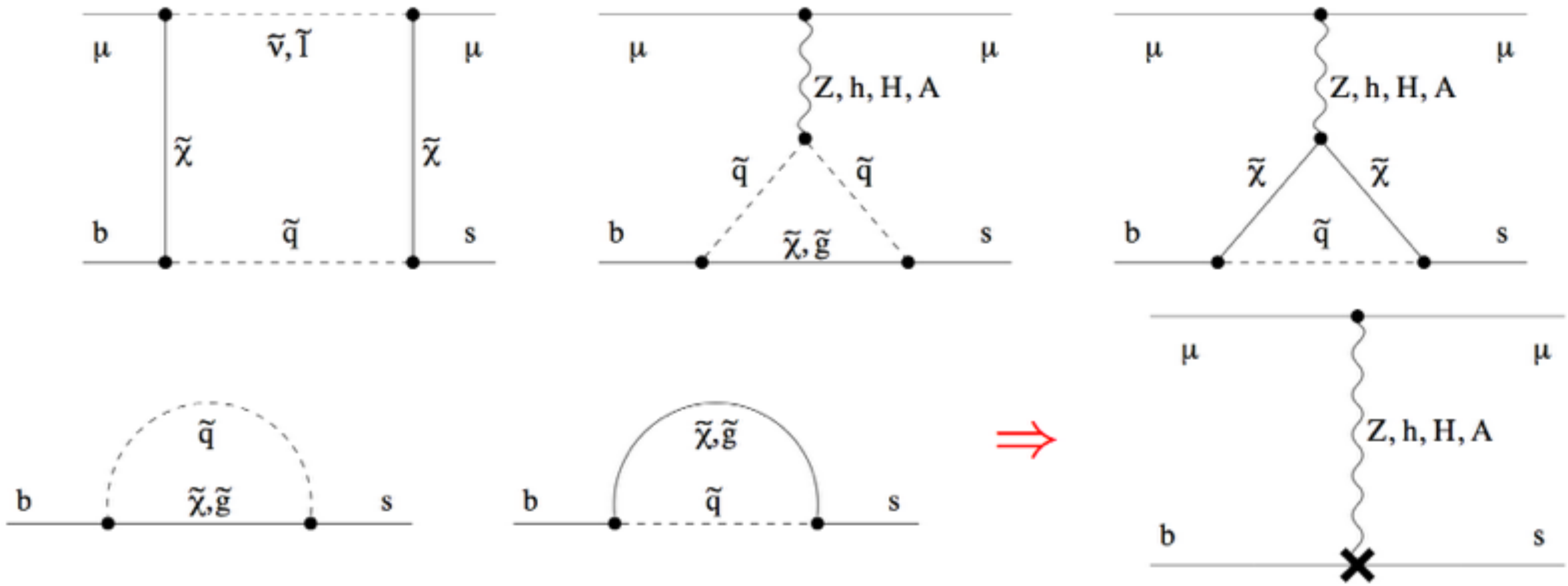
- It is a strongly suppressed, loop-generated process in the SM. Its average time-integrated branching ratio (with final-state photon bremsstrahlung included) reads:

$$\overline{\mathcal{B}}_{SM} = (3.65 \pm 0.23) \times 10^{-9}$$

[C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou and M. Steinhauser, Phys. Rev. Lett. 112 (2014) 101801]

- It is very sensitive to new physics even in models with Minimal Flavour Violation (MFV). Enhancements by orders of magnitude are possible even when constraints from all the other measurements are taken into account.

Misiak (2014)

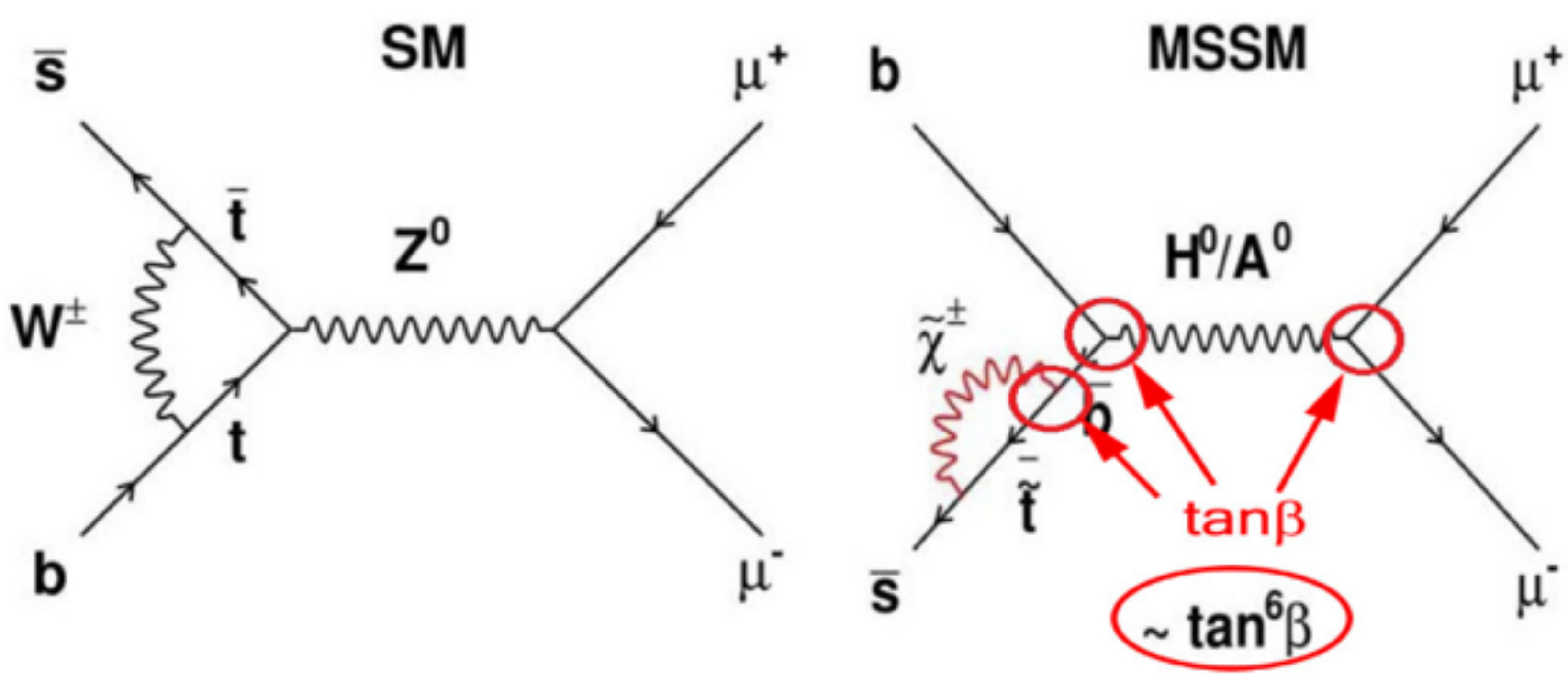


For large $\tan\beta$: $\mathcal{B}\left(B_s \to \mu^+\mu^-\right) \sim \frac{m_b^2 m_\mu^2}{M_A^4} \tan^6\beta$

K. S. Babu and C. F. Kolda, Phys. Rev. Lett. 84 (2000) 228.

$B_s \rightarrow \mu^+ \mu^-$

$$\mathcal{B}(B_s \to \mu^+\mu^-) = 8.75 \times 10^{-8} \left(\frac{|V_{ts}|}{0.040}\right)^2 \left(\frac{f_{B_s}}{210\mathrm{MeV}}\right)^2 \times \left\{ |M_{B_s} C_S|^2 + |M_{B_s} C_P + \frac{2m_\mu}{M_{B_s}} C_{10A}|^2 \right\}$$



$B_s \to \mu^+ \mu^-$

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \Big\{ \sum_{i=1}^{10} C_i(\mu) Q_i(\mu) + C_S(\mu) Q_S(\mu) + C_P(\mu) Q_P(\mu) + C_S'(\mu) Q_S'(\mu) + C_P'(\mu) Q_P'(\mu) \Big\}$$

$$Q_{9V} = \frac{e^2}{16\pi^2} (\bar{s}_\alpha \gamma_\mu P_L b_\alpha) (\bar{l} \gamma^\mu l)$$

$$Q_{10A} = \frac{e^2}{16\pi^2} (\bar{s}_\alpha \gamma_\mu P_L b_\alpha) (\bar{l} \gamma^\mu \gamma_5 l)$$

$$Q_{9V}' = \frac{e^2}{16\pi^2} (\bar{s}_\alpha \gamma_\mu P_R b_\alpha) (\bar{l} \gamma^\mu l)$$

$$Q_{10A}' = \frac{e^2}{16\pi^2} (\bar{s}_\alpha \gamma_\mu P_R b_\alpha) (\bar{l} \gamma^\mu \gamma_5 l)$$

$$Q_S = \frac{e^2}{16\pi^2} (\bar{s}_\alpha P_R b_\alpha) (\bar{l} l)$$

$$Q_P = \frac{e^2}{16\pi^2} (\bar{s}_\alpha P_R b_\alpha) (\bar{l} \gamma_5 l)$$

$$Q_S' = \frac{e^2}{16\pi^2} (\bar{s}_\alpha P_L b_\alpha) (\bar{l} l)$$

$$Q_P' = \frac{e^2}{16\pi^2} (\bar{s}_\alpha P_L b_\alpha) (\bar{l} \gamma_5 l)$$

$B_s \longrightarrow \mu^+ \mu^-$

Necessary non-perturbative input: $\langle 0|\bar{b}\gamma^\alpha\gamma_5 s|B_s(p)\rangle = ip^\alpha f_{B_s}$

Recent lattice determinations of the B_s-meson decay constant:

$$f_{B_s} = \begin{cases} 225.0(4.0)\ \text{MeV}, & \text{HPQCD (r)}, & \text{arXiv:1110.4510} \\ 224.0(5.0)\ \text{MeV}, & \text{HPQCD (nr)}, & \text{arXiv:1302.2644} \\ 234.0(6.0)\ \text{MeV}, & \text{ROME}, & \text{arXiv:1212.0301} \\ 242.0(9.5)\ \text{MeV}, & \text{FNAL/MILC}, & \text{arXiv:1112.3051} \\ 232.0(10)\ \text{MeV}, & \text{ETM}, & \text{arXiv:1107.1441} \\ 219.0(12)\ \text{MeV}, & \text{ALPHA}, & \text{arXiv:1210.6524} \\ 235.4(12)\ \text{MeV}, & \text{RBC/UKQCD}, & \text{arXiv:1404.4670} \\ 224.0(14)\ \text{MeV}, & \text{ALPHA}, & \text{arXiv:1404.3590} \end{cases}$$

Flavour Lattice Averaging Group (FLAG), arXiv:1310.8555 gives

$$f_{B_s} = 227.7(4.5)\ \text{MeV}.$$

$B_s \longrightarrow \mu^+ \mu^-$

- Recently measured branching ratios

$$\overline{\mathcal{B}}_{\text{exp}} = \begin{cases} (2.9\,^{+1.1}_{-1.0}) \times 10^{-9}, & \text{LHCb} \quad \text{[Phys. Rev. Lett. 111 (2013) 101805]} \\ (3.0\,^{+1.0}_{-0.9}) \times 10^{-9}, & \text{CMS} \quad \text{[Phys. Rev. Lett. 111 (2013) 101804]} \end{cases}$$

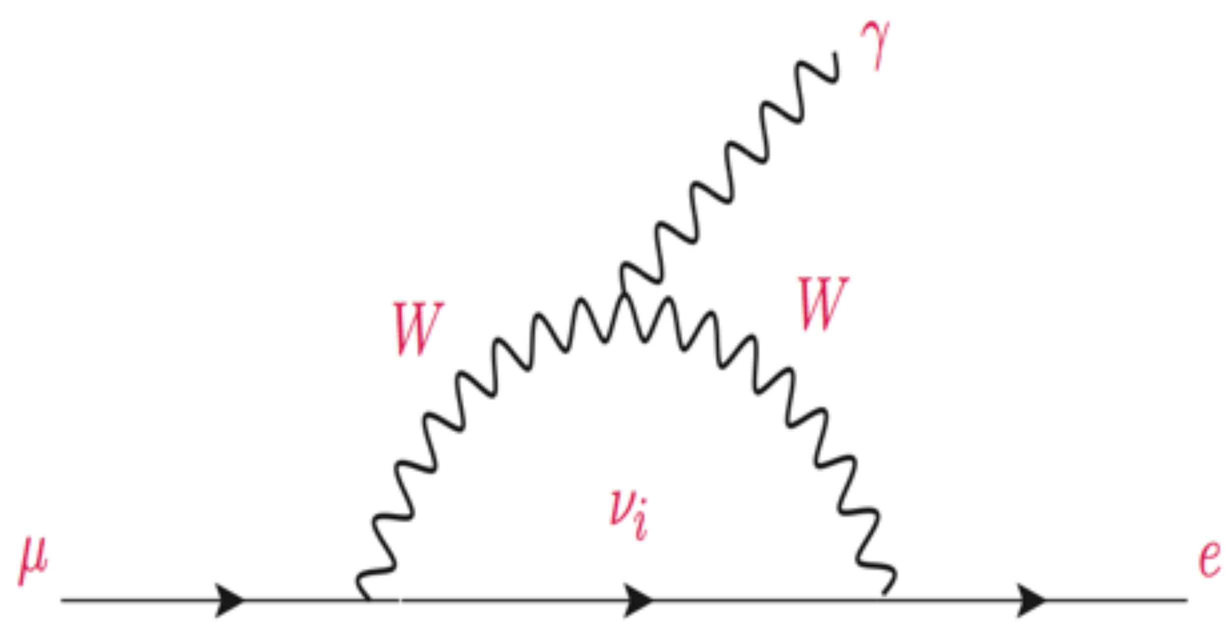
Combined: $\overline{\mathcal{B}}_{\text{exp}} = (2.9 \pm 0.7) \times 10^{-9}$ [CMS-PAS-BPH-13-007, LHCb-CONF-2013-012]

- ATLAS: $\overline{\mathcal{B}}_{\text{exp}} < 1.5 \times 10^{-8}$ @ 95%C.L.

Chared Lepton Flavor Violation (CLFV)

We have already seen LFV: neutrino oscillation

Oscillations experiments have shown that $m_\nu \neq 0$:



$\Rightarrow$ GIM suppressed by small neutrino masses

CLFV

$$\Gamma(\mu \to e\gamma) \approx \underbrace{\frac{G_F^2 m_\mu^5}{192\pi^3}}_{\mu-\text{decay}} \quad \underbrace{\left(\frac{\alpha}{2\pi}\right)}_{\gamma-\text{vertex}} \quad \underbrace{\sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2}{M_W^2}\right)}_{\nu-\text{oscillation}}$$

$$\approx \frac{G_F^2 m_\mu^5}{192\pi^3} \frac{3\alpha}{32\pi} \left(\frac{\Delta m_{23}^2 s_{13} c_{13} s_{23}}{M_W^2}\right)^2$$

relative probability ~ 10^{-54}

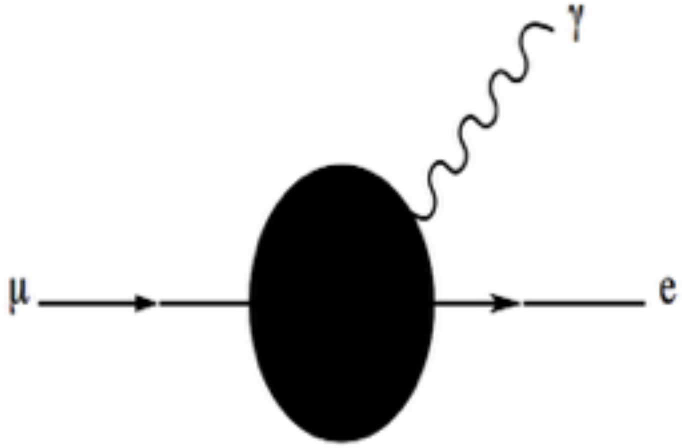
Experimental constraints

Process	90 % C.L. bound
$\mu \to e\gamma$	5.7×10^{-13}
$\tau \to e\gamma$	1.5×10^{-8}
$\tau \to \mu\gamma$	1.8×10^{-8}

MEG(2013)

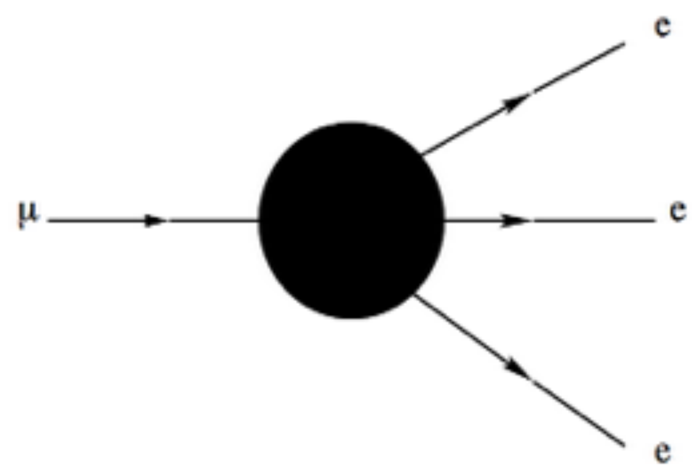
Other CLFV

$\mu \to e\gamma$:



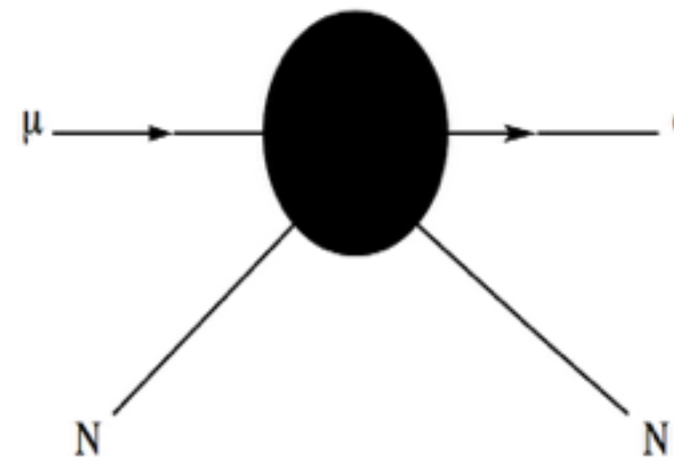
$< 5.7 \times 10^{-13}$

$\mu \to 3e$

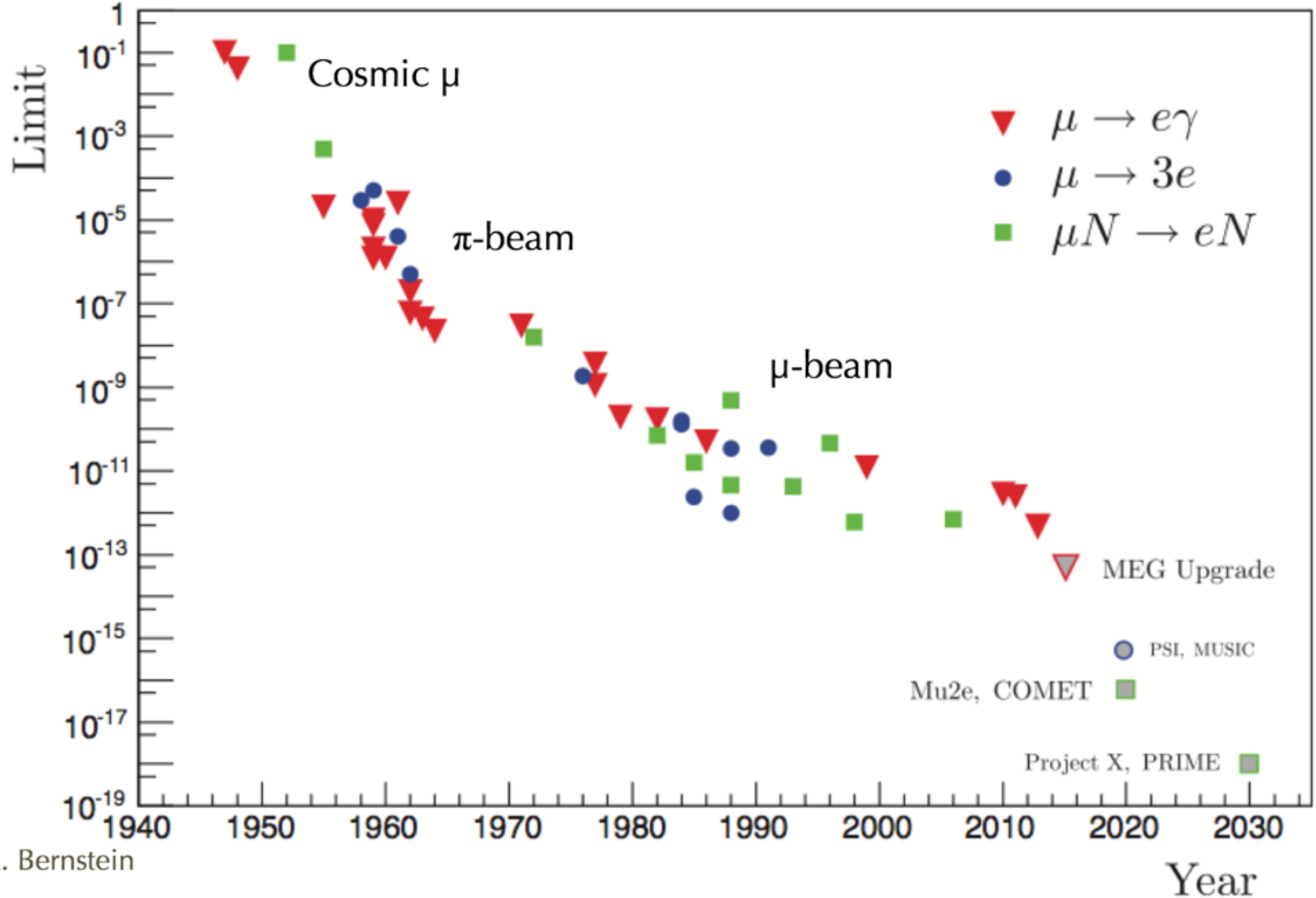


$< 1.0 \times 10^{-12}$

μ-capture:



$< 6.1 \times 10^{-13}$

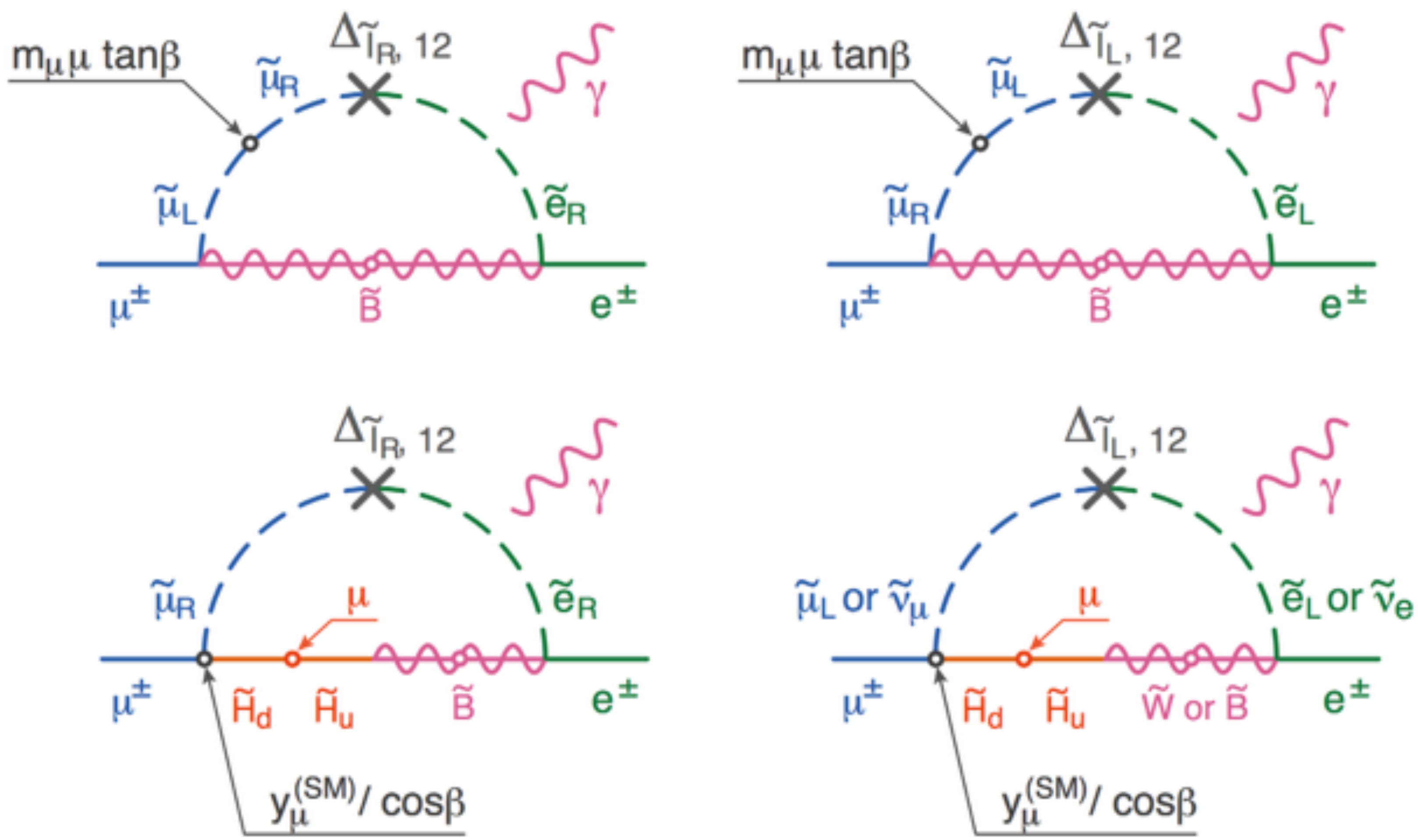


Limit
1
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-19}
1940
1950
1960
1970
1980
1990
2000
2010
2020
2030
Year
Cosmic μ
π-beam
μ-beam
$\mu \to e\gamma$
$\mu \to 3e$
$\mu N \to eN$
MEG Upgrade
PSI, MUSIC
Mu2e, COMET
Project X, PRIME

R. Bernstein

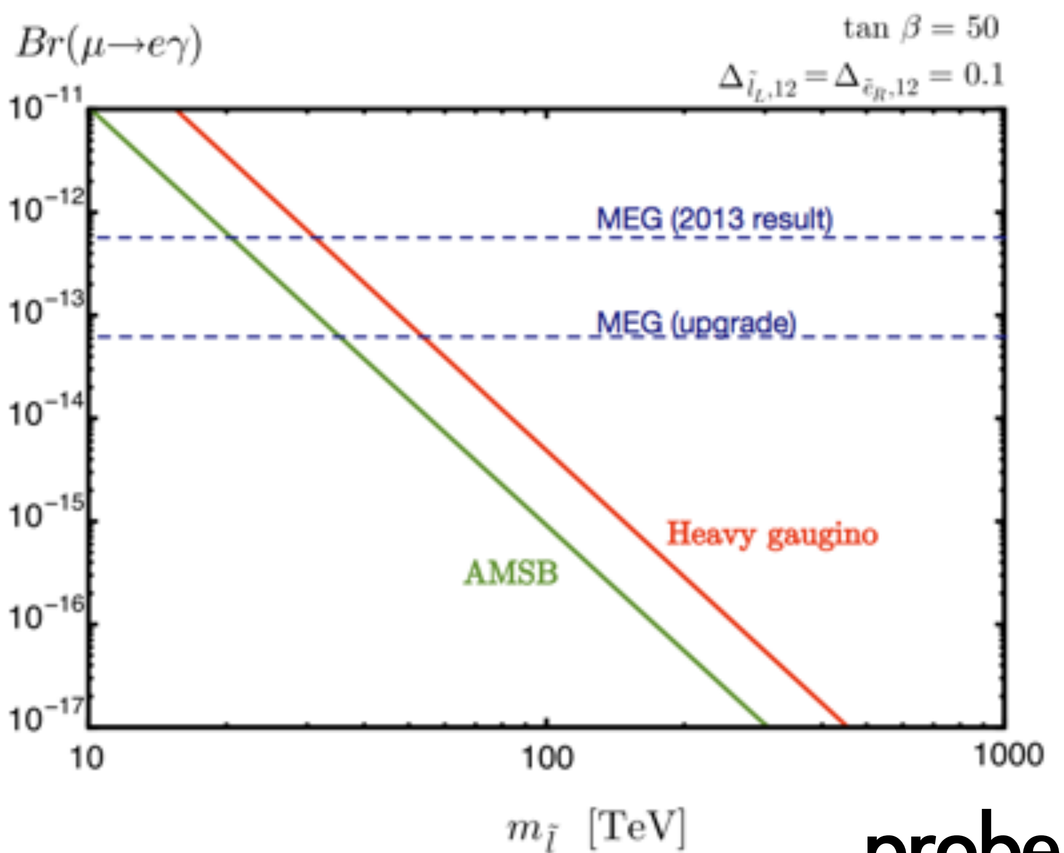
CLFV in the MSSM

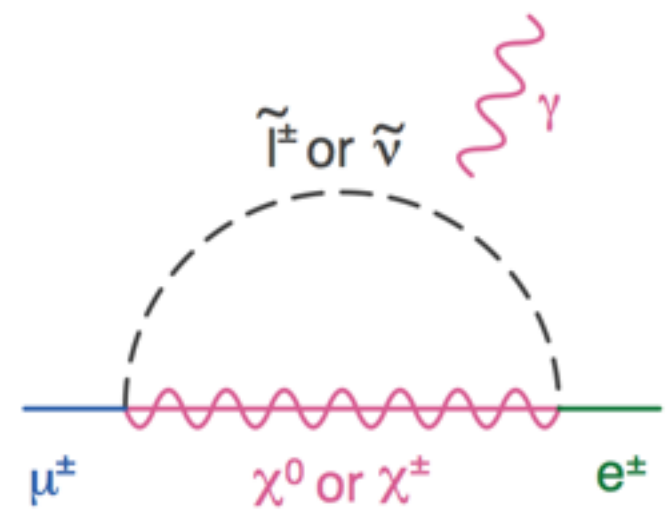
CLFV processes are proportional to $\tan\beta$



CLFV in the MSSM

$\mu \to e\gamma$ (with $\Delta^L_{e\mu} = \Delta^R_{e\mu} = 0.1,\ \tan\beta = 50,\ \cdots$)





probes MSSM far beyond LHC

- MEG (current bound): $Br < 5.7 \times 10^{-13}$
  [MEG experiment ('13)]
- MEG upgrade: $Br \lesssim 6 \times 10^{-14}$

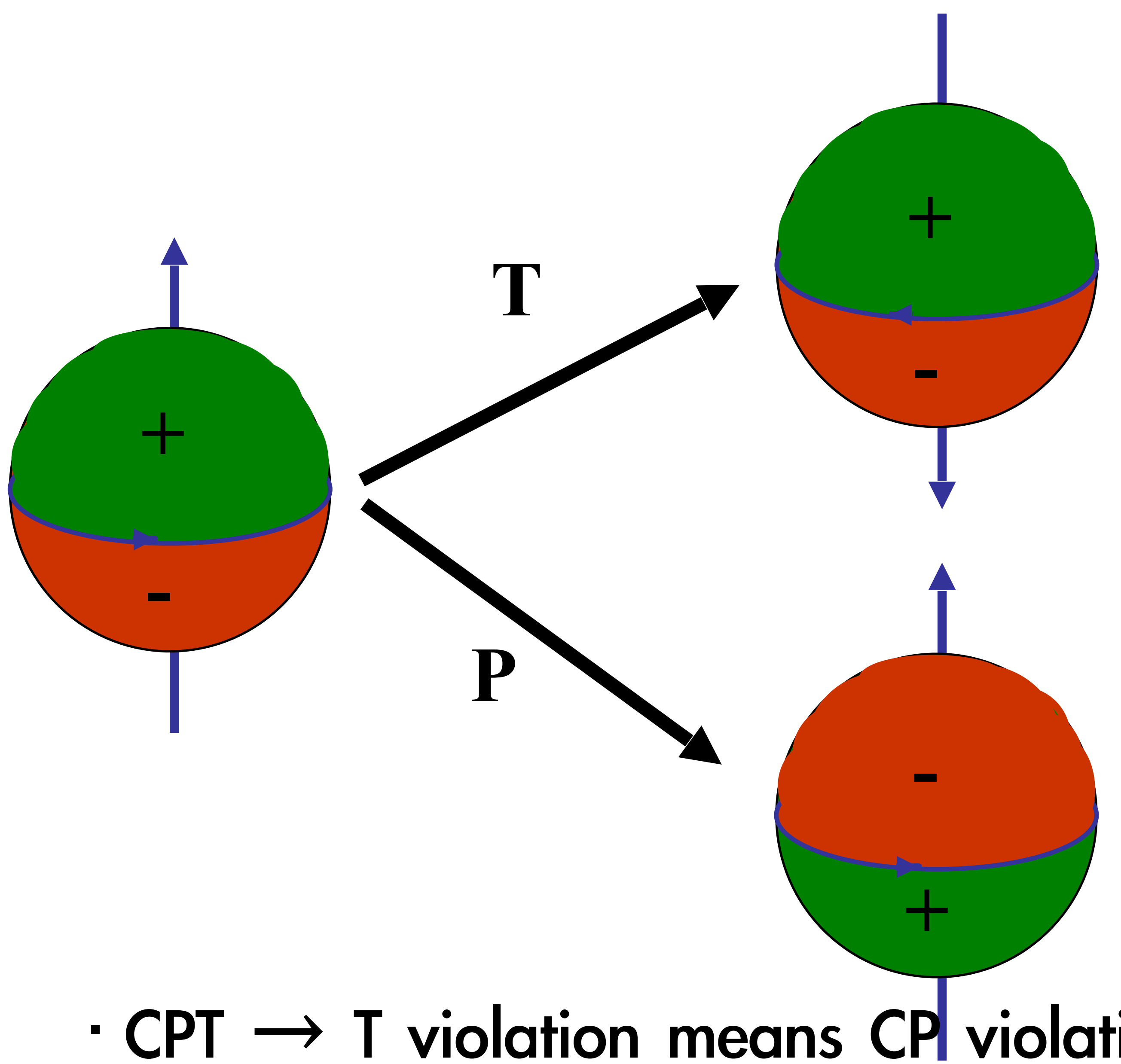
CLFV in the MSSM

x	$\left\|\left(\delta^l_{12}\right)_{LL}\right\|$	$\left\|\left(\delta^l_{12}\right)_{LR}\right\|$
0.3	4.1×10^{-3}	1.4×10^{-6}
1.0	7.7×10^{-3}	1.7×10^{-6}
5.0	3.2×10^{-2}	3.8×10^{-6}
x	$\left\|\left(\delta^l_{13}\right)_{LL}\right\|$	$\left\|\left(\delta^l_{13}\right)_{LR}\right\|$
0.3	15	8.9×10^{-2}
1.0	29	1.1×10^{-1}
5.0	1.2×10^{2}	2.4×10^{-1}
x	$\left\|\left(\delta^l_{23}\right)_{LL}\right\|$	$\left\|\left(\delta^l_{23}\right)_{LR}\right\|$
0.3	2.8	1.7×10^{-2}
1.0	5.3	2.0×10^{-2}
5.0	22	4.4×10^{-2}

Table 7: Limits on the $\left|\delta^d_{ij}\right|$ from $l_j \to l_i \gamma$ decays for an average slepton mass $m_{\tilde{l}} = 100\text{GeV}$ and for different values of $x = m^2_{\tilde{\gamma}}/m^2_{\tilde{l}}$. For different values of $m_{\tilde{l}}$, the limits can be obtained multiplying the ones in the table by $(m_{\tilde{l}}(\text{GeV})/100)^2$.

Electric Dipole Moment (EDM)

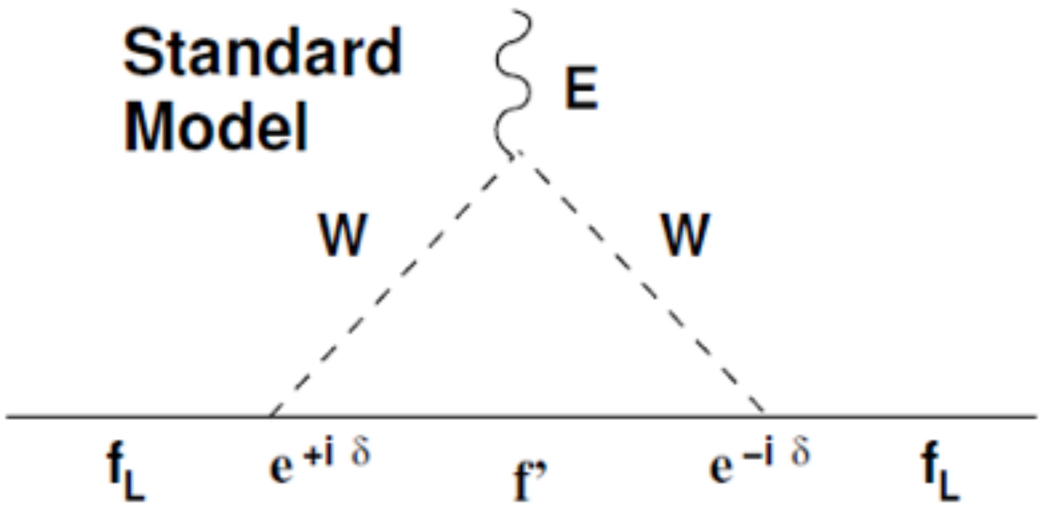
- A Permanent EDM Violates both T & P Symmetries:

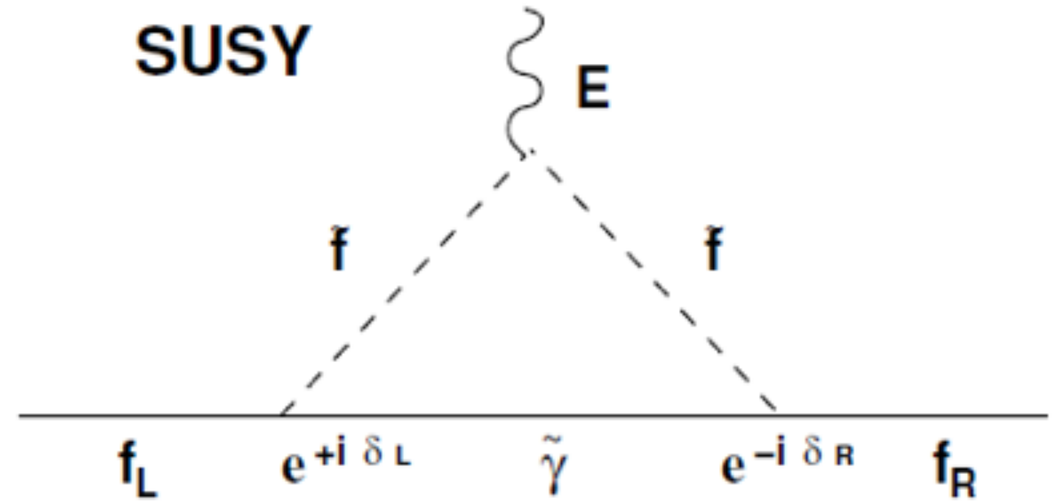


	P	T
$\vec{S}$	$\vec{S}$	$-\vec{S}$
$\vec{d}$	$-\vec{d}$	$\vec{d}$

- CPT $\longrightarrow$ T violation means CP violation

EDM in the SM/NP





EDM=0 at one-loop level (SM)

Non-zero EDM at one-loop level (MSSM)

$$\mathcal{L}_{\rm EDM} = -\frac{i}{2} d_f \overline{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu}$$

EDM in the SM/NP

Effective operators for EDM:

$$\mathcal{L}_{\text{eff}} = \sum_{i=1}^{3} C_i(Q) O_i(Q),$$

$$\begin{aligned} O_1 &= -\frac{i}{2}\bar{f}\sigma^{\mu\nu}\gamma_5 f F_{\mu\nu}, \\ O_2 &= -\frac{i}{2}\bar{f}\sigma^{\mu\nu}\gamma_5 T^a f G^a_{\mu\nu}, \\ O_3 &= -\frac{1}{6} f_{abc} G^a_{\mu\rho} G^{b\rho}{}_\nu G^c_{\lambda\sigma} \varepsilon^{\mu\nu\lambda\sigma} \end{aligned}$$

$$d_f = C_1^f(\Lambda_\chi) + \frac{e}{4\pi} C_2^f(\Lambda_\chi) + \frac{e\Lambda_\chi}{4\pi} C_3^f(\Lambda_\chi).$$

$$d_n = \frac{1}{3}(4d_d - d_u).$$

Experimental bound

In units of $e\,cm$, selected EDM limits are:

Cheng-Pang Liu (2007)

Particle	EDM limit	System	SM Prediction	New Physics
e	1.9×10^{-27}	Tl atom	10^{-38}	10^{-27}
μ	1.1×10^{-19}	rest frame $\vec{E}$	10^{-35}	10^{-22}
τ	3.1×10^{-16}	$e^+e^- \to \tau^+\tau^-\gamma$	10^{-34}	10^{-20}
p	6.5×10^{-23}	TlF molecule	10^{-31}	10^{-26}
n	2.9×10^{-26}	UCN	10^{-31}	10^{-26}
^{199}Hg	2.1×10^{-28}	atom cell	10^{-33}	10^{-28}

- Most precise measurements are taken in neutral systems.
- Results for charged particles (except μ) are inferred.

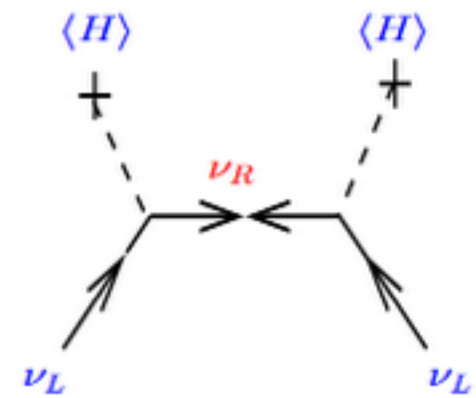
ACME (2014): $|d_e| < 8.7 \times 10^{-29}\,\mathrm{e\,cm}$

Flavor Physics and Neutrino Mass

- Conventional mechanism for small neutrino masses: seesaw mechanism

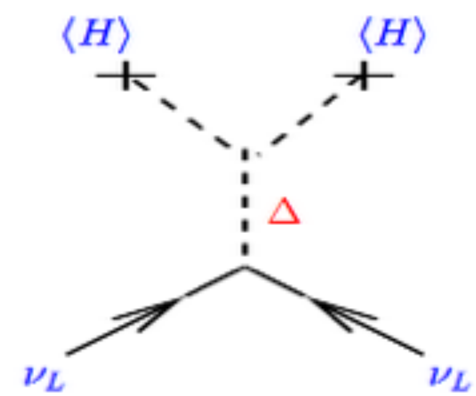
Seesaw type-I, right-handed neutrinos:

$$m_{1/2} \simeq \left(-\frac{Y_\nu^2 v^2}{M_M}, M_M\right)$$



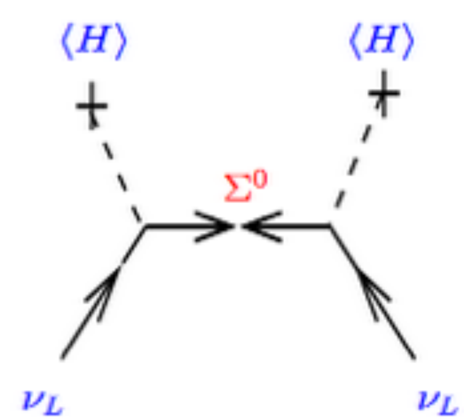
Seesaw type-II, scalar triplet:

$$m_\nu \simeq Y_T \langle \Delta_L^0 \rangle \simeq Y_T \frac{v^2}{m_\Delta}$$



Type-III: Replace ν_R by $\mathbf{\Sigma} = (\Sigma^+, \Sigma^0, \Sigma^-)$:

$$m_{1/2} \simeq \left(-\frac{Y_\Sigma^2 v^2}{M_\Sigma}, M_\Sigma\right)$$

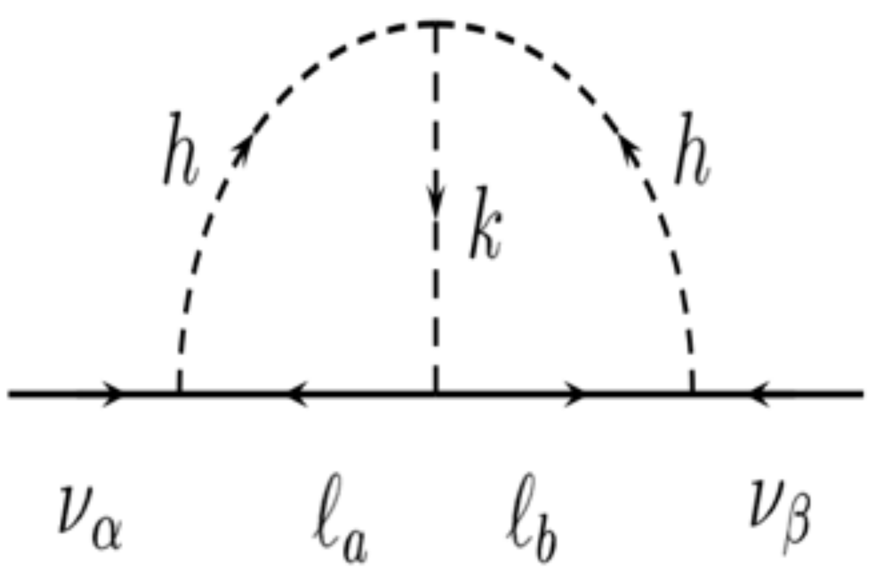


Flavor Physics and Neutrino Mass

- Alternative mechanism for small neutrino masses: radiative neutrino mass generation, eg. Zee-Babu model

Cheng & Li, 1980
Zee, 1985
Babu, 1988

$$\mathcal{L} = f(L^T L)h^+ + g(e_R^T e_R)k^{++} - \mu h^+ h^+ k^{--}$$



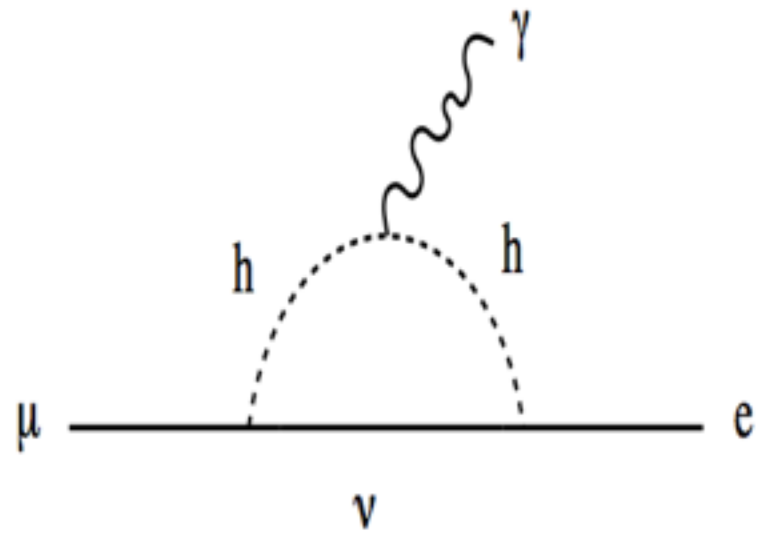
Neutrino mass is
2-loop suppressed!

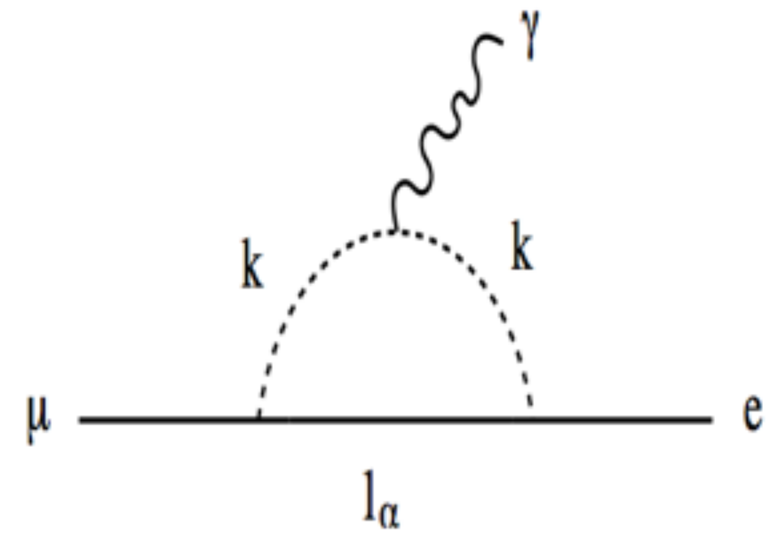
Babu & Macesanu, 2003
Aristizabal & Hirsch, 2006

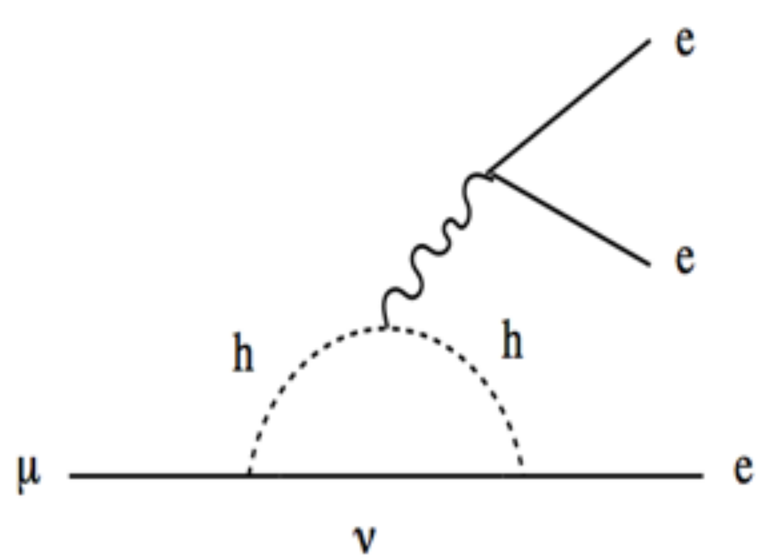
$$\mathcal{M}^\nu_{\alpha\beta} = \frac{8\mu}{(16\pi^2)^2 m_h^2} f_{\alpha a} m_a g_{xy} m_b f_{b\beta} \mathcal{I}\left(\frac{m_k^2}{m_h^2}\right),$$

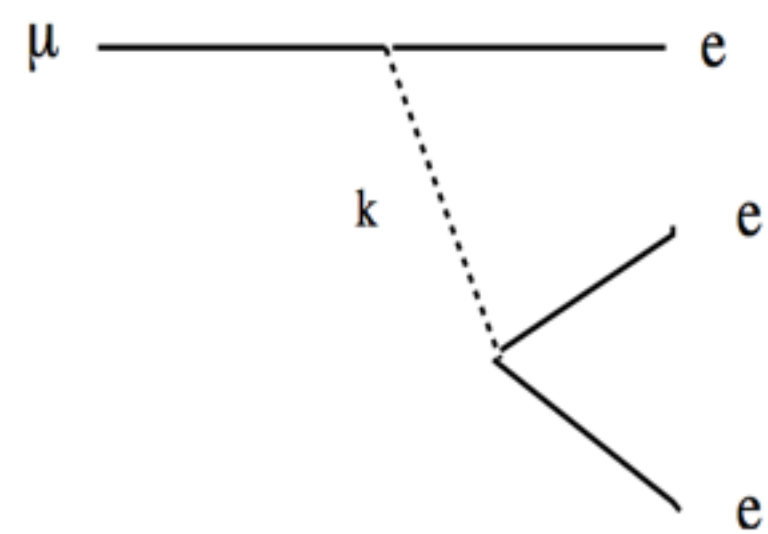
Large neutrino mixing angles require large CLFV

Flavor Physics and Neutrino Mass









Flavor Physics and Dark Matter

Ma, 0601225

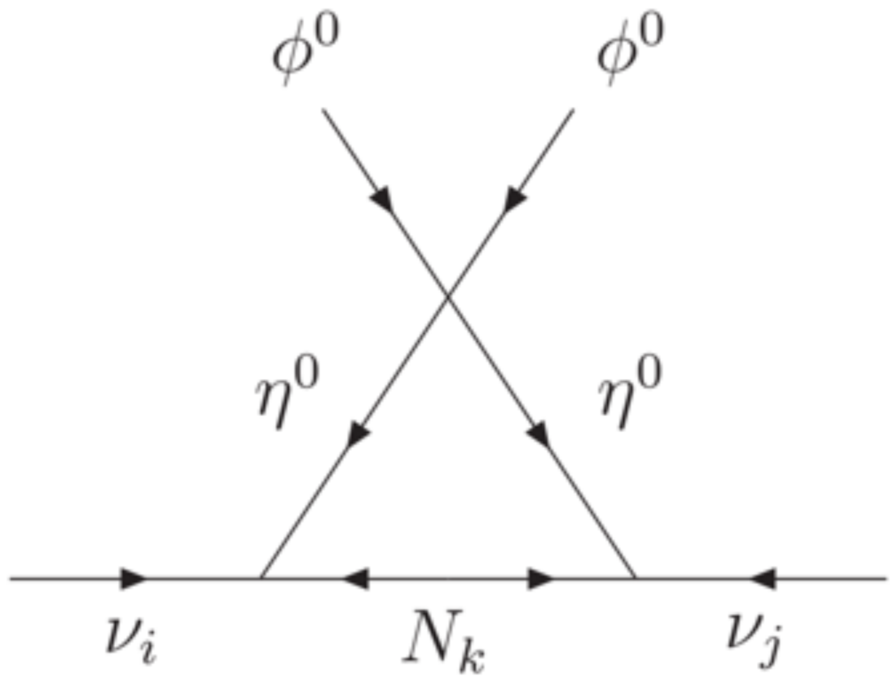
$$SU(2)_L \times U(1)_Y \times Z_2,$$

$$(\nu_i, l_i) \sim (2, -1/2; +), \quad l_i^c \sim (1, 1; +), \quad N_i \sim (1, 0; -), \tag{3}$$

$$(\phi^+, \phi^0) \sim (2, 1/2; +), \qquad (\eta^+, \eta^0) \sim (2, 1/2; -). \tag{4}$$

$$\mathcal{L}_Y = f_{ij}(\phi^- \nu_i + \bar{\phi}^0 l_i) l_j^c + h_{ij}(\nu_i \eta^0 - l_j \eta^+) N_j + \text{H.c.}$$

Flavor Physics and Dark Matter



Neutrino masses

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik} h_{jk} M_k}{16\pi^2} \left[\frac{m_R^2}{m_R^2 - M_k^2} \ln \frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2} \ln \frac{m_I^2}{M_k^2} \right],$$

Dark matter candidates: the lightest neutral state of Z2-odd particles

$$\eta^0, N_i$$