Flavor Physics, CP Violation and New Physics

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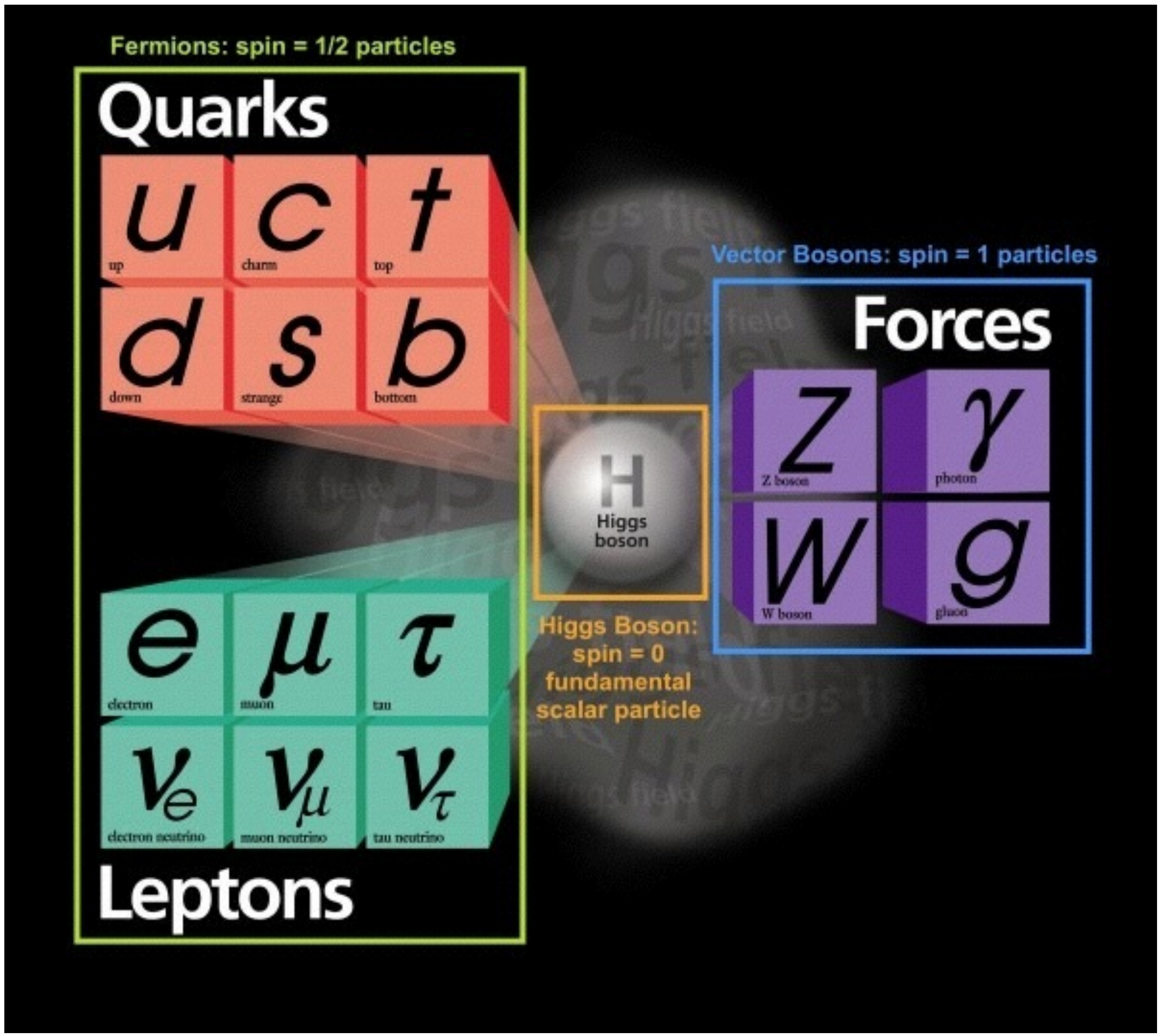
Outline

- Lecture 1
  - The Standard Model
  - New Physics beyond the SM
  - Minimal Supersymmetric Standard Model (MSSM)
- Lecture 2
  - Minimal Flavor Violation (MFV)
  - $(g-2)_\mu$, FCNC processes, EDM in the MSSM

References

- Peskin and Schroeder, "An Introduction to Quantum Field Theory"
- S. Martin, hep-ph/9709356
- S. Dawson, hep-ph/9712464
- A. Buras, hep-ph/9806471

The Standard Model(SM)



The SM: gauge group

- The SM interaction follows from SU(3)xSU(2)xU(1) gauge symmetry

$$V(x) = e^{i\alpha^a t^a}$$

$$\psi(x) \to V(x)\psi(x)$$

$$A^a_\mu(x)t^a \to V(x)\Big(A^a_\mu(x)t^a + \frac{i}{g}\partial_\mu\Big)V^\dagger(x)$$

- The particle contents

$$Q^i_L(3,2)_{+1/6}\,,\;\; U^i_R(3,1)_{+2/3}\,,\;\; D^i_R(3,1)_{-1/3}\,,\;\; L^i_L(1,2)_{-1/2}\,,\;\; E^i_R(1,1)_{-1}\,,$$

$$\phi(1,2)_{1/2}$$

The SM: Lagrangian

- The SM Lagrangian: gauge + Yukawa +Higgs

$$\mathcal{L}^{\rm SM}_{\rm gauge} = \sum_{i=1...3} \sum_{\psi=Q^i_L...E^i_R} \bar{\psi} i \not{D} \psi - \frac{1}{4} \sum_{a=1...8} G^a_{\mu\nu} G^a_{\mu\nu} - \frac{1}{4} \sum_{a=1...3} W^a_{\mu\nu} W^a_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B_{\mu\nu} \, .$$

$$Q^i_L(3,2)_{+1/6}\,,\ \ U^i_R(3,1)_{+2/3}\,,\ \ D^i_R(3,1)_{-1/3}\,,\ \ L^i_L(1,2)_{-1/2}\,,\ \ E^i_R(1,1)_{-1}\,,$$

$$-\mathcal{L}^{\rm SM}_{\rm Yukawa} = Y^{ij}_d \bar{Q}^i_L \phi D^j_R + Y^{ij}_u \bar{Q}^i_L \tilde{\phi} U^j_R + Y^{ij}_e \bar{L}^i_L \phi E^j_R + {\rm h.c.} \qquad (\tilde{\phi} = i\tau_2 \phi^\dagger)\,.$$

$$\mathcal{L}^{\rm SM}_{\rm Higgs} = D_\mu \phi^\dagger D^\mu \phi - \mu^2_\phi \phi^\dagger \phi - \lambda_\phi (\phi^\dagger \phi)^4 \qquad \phi(1,2)_{1/2}$$

SUSY? Neutrino Masses?

The SM: Lagrangian

- There are no mass terms:

$$\mathcal{L} \not\ni \frac{1}{2} m_A^2 A_\mu A^\mu, -m_\psi \overline{\psi}\psi, -\frac{1}{2} m_\phi^2 \phi^\dagger \phi$$

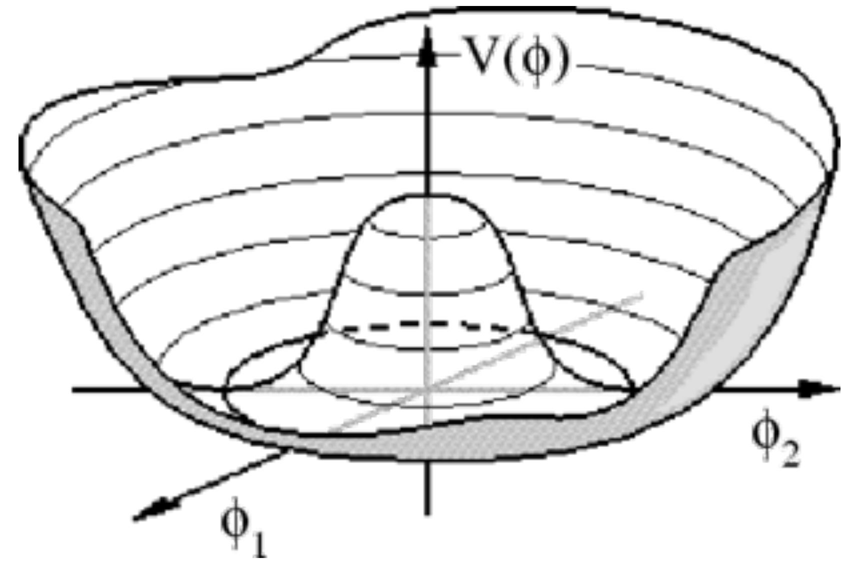
- All particles are "massless"

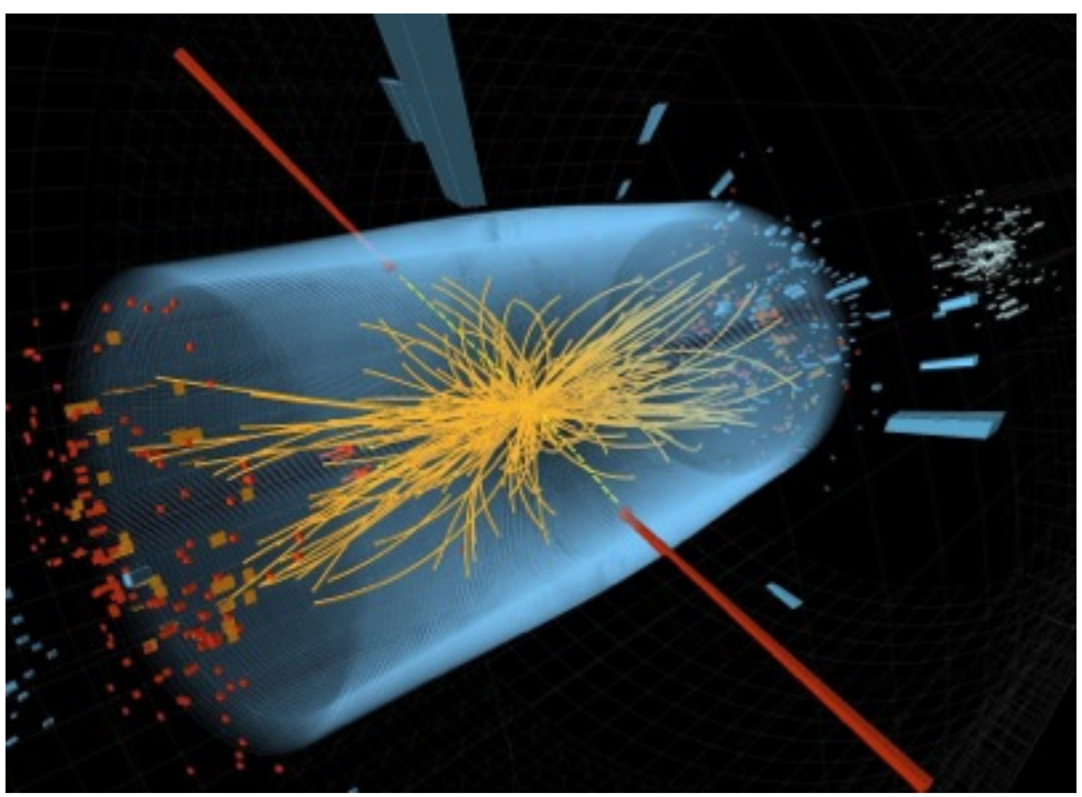
The SM: Higgs mechanism

- After the Higgs boson gets vev

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

the SM particles become massive.





The Standard Model(SM)

- No flavor-changing neutral current
- Flavor-changing charged current: CKM
- Arbitrary complex matrix can be diagonalized by bi-unitary transf.

$$u_L = V_L^u u'_L, \quad u_R = V_R^u u'_R, \mathrm{etc.}$$

Why no tree FCNC?

- Coupling is proportional to unit matrix in flavor space: flavor universal
    - eg: strong int., Z(γ)-mediated neutral coupling
- Coupling is flavor diagonal
    - eg: h-f-f coupling

(cf) Z' -mediated FCNC at tree-level?

Flavor Physics

- In the mass basis, we have FC charged current

$$\bar{u}_L \gamma^\mu d_L = \bar{u}'_L \gamma^\mu V_L^{u\dagger} V_L^d d'_L = \bar{u}'_L \gamma^\mu V_{\rm CKM} d'_L$$

- No tree-level FCNCs in the SM. First appears at loop-level. Sensitive to NP at $\Lambda \gg E_{\rm exp.}$

- Standard parametrization of CKM matrix

$$\hat{V}_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- Wolfenstein parametrization of CKM matrix

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \qquad \lambda \approx 0.22$$

- In the SM, CPV occurs at V_{13}, and therefore it is related to flavor violating processes.
- The SM flavor puzzle:
  - Why the off-diagonal components are small and the CKM matrix is hierarchical?
  - Why are the CKM and MNS matrix so different?

Improvement of Wolfenstein parameterization:

To improve the above equation in λ, adopt

$$s_{12} = \lambda, \quad s_{23} = A\lambda^2, \quad s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$$

in all orders of λ. Then we get

$$\begin{aligned}
V_{ud} &= 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 + \mathcal{O}(\lambda^6) \\
V_{us} &= \lambda + \mathcal{O}(\lambda^7) \\
V_{ub} &= A\lambda^3(\rho - i\eta) \\
V_{cd} &= -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] + \mathcal{O}(\lambda^7) \\
V_{cs} &= 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) + \mathcal{O}(\lambda^6) \\
V_{cb} &= A\lambda^2 + \mathcal{O}(\lambda^8) \\
V_{td} &= A\lambda^3[1 - (\rho + i\eta)(1 - \frac{1}{2}\lambda^2)] + \mathcal{O}(\lambda^7) \\
&= A\lambda^3[1 - \overline{\rho} - i\overline{\eta}] \\
V_{ts} &= -A\lambda^2 + \frac{1}{2}A(1 - 2\rho)\lambda^4 - i\eta A\lambda^4 + \mathcal{O}(\lambda^6) \\
V_{tb} &= 1 - \frac{1}{2}A^2\lambda^4 + \mathcal{O}(\lambda^6),
\end{aligned}$$

where $\overline{\rho} = \rho(1 - \frac{1}{2}\lambda^2), \overline{\eta} = \eta(1 - \frac{1}{2}\lambda^2)$.

Counting the # of parameters in the SM: revisit to C. Yu's lecture

- Gauge sector: 3-gauge couplings
- Higgs sector: 2
- Without flavor sector: approximate $[U(3)]^5$ global symmetry. We can use this symmetry to remove unphysical parameters in Yukawas
- quark sector: $[U(3)]^3 \to U(1)$: 2(9,9)-3(3,6)+(0,1)=(9,1)
- lepton sector: $[U(3)]^2 \to [U(1)]^3$:(9,9)-2(3,6)+3(0,1)=(3,0)
- total =18+1(QCD θ)=19

The SM: P & C

- P, C transformation (20.3 of Peskin)

- Parity

$$P\psi(x)P = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s \Big(\eta_a a^s_{-\mathbf{p}} u^s(p) e^{-ipx} + \eta_b^* b^{s\dagger}_{-\mathbf{p}} v^s(p) e^{ipx}\Big).$$

$$P\psi(t,\mathbf{x})P = \eta_a \gamma^0 \psi(t,-\mathbf{x}).$$

- Charge conjugation

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$C a^s_{\mathbf{p}} C = b^s_{\mathbf{p}}; \qquad C b^s_{\mathbf{p}} C = a^s_{\mathbf{p}}.$$

$$C\psi(x)C = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s \Big(-i\gamma^2 b^s_{\mathbf{p}} \big(v^s(p)\big)^* e^{-ipx} - i\gamma^2 a^{s\dagger}_{\mathbf{p}} \big(u^s(p)\big)^* e^{ipx}\Big)$$

$$= -i\gamma^2\psi^*(x) = -i\gamma^2(\psi^\dagger)^T = -i\big(\bar\psi\gamma^0\gamma^2\big)^T. \qquad (3.145)$$

The charge conjugation matrix, one of which representations is $C = i\gamma^0\gamma^2$, satisfies

(i) $C^\dagger = C^{-1}$.

(ii) $C^\dagger = C^T = -C$.

(iii) For $\Gamma = \{1, \gamma_\mu, \sigma_{\mu\nu}, \gamma_\mu\gamma_5, \gamma_5\}$,

$$\Gamma^C \equiv C\Gamma^T C^\dagger = \epsilon\Gamma. \quad \epsilon = \begin{cases} +1 & \text{for} \quad 1, \gamma_\mu\gamma_5, \gamma_5 \\ -1 & \text{for} \quad \gamma_\mu, \sigma_{\mu\nu} \end{cases}$$

Some properties of charge conjugation of fermion fields:

(i) $\psi^c = C\overline{\psi}^T \Leftrightarrow \psi^* = \gamma^0 C^\dagger \psi^c$.

(ii) $\overline{\psi^c} = -\psi^T C^\dagger \Leftrightarrow \psi^T = -\overline{\psi^c} C$.

(Proof)

$$\overline{\psi^c} = \overline{(C\overline{\psi}^T)} = \overline{(C\gamma^0\psi^*)} = \psi^T\gamma^0 C^\dagger \gamma^0 = -\psi^T C^\dagger.$$

In the last step, we used $C^\dagger\gamma^0 C = -\gamma^0$.

(iii) $(\psi^c)^c = \psi$.

(iv) $(\psi_L)^c = (\psi^c)_R$

(v) $\overline{\psi_1}\Gamma\psi_2 = \epsilon\overline{\psi_2^c}\Gamma\psi_1^c$.

CP transformation

term	$\overline{\psi}_i\psi_j$	$i\overline{\psi}_i\gamma^5\psi_j$	$\overline{\psi}_i\gamma^\mu\psi_j$	$\overline{\psi}_i\gamma^\mu\gamma^5\psi_j$
CP−transformed term	$\overline{\psi}_j\psi_i$	$-i\overline{\psi}_j\gamma^5\psi_i$	$-(-1)^\mu\overline{\psi}_j\gamma^\mu\psi_i$	$-(-1)^\mu\overline{\psi}_j\gamma^\mu\gamma^5\psi_i$

term	H	A	$W^{\pm\mu}$	∂_μ
CP−transformed term	H	$-A$	$-(-1)^\mu W^{\mp\mu}$	$(-1)^\mu\partial_\mu$

The SM violates C, P maximally

$$\mathcal{L}_m = -\lambda_d^{ij}\overline{Q}_L^i \cdot \phi\, d_R^j - \lambda_u^{ij}\epsilon^{ab}\overline{Q}_{La}^i\phi_b^\dagger u_R^j + \text{h.c.},$$

transformed into h.c. after CP

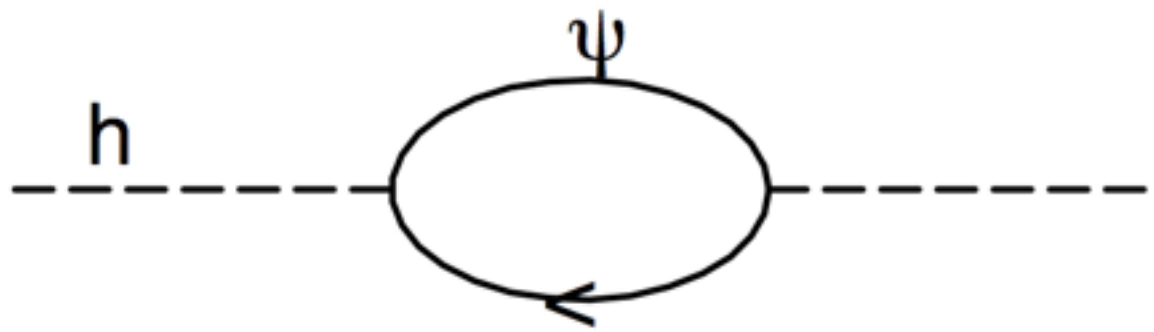
Or effectively, $\lambda_d^{ij} \to (\lambda_d^{ij})^*, \qquad \lambda_u^{ij} \to (\lambda_u^{ij})^*.$

New Physics beyond the SM

- Higgs mass is not protected by any symmetry ⇒ Hierarchy problem.
- SM has 19 unknown parameters whose value are to be set experimentally.
- No cold dark matter candidate.
- Neutrinos are massless in SM.
- Does not explain fermion mass hierarchy.
- It can not explain baryogenesis and leptogenesis.
- It does not give the gauge coupling unification at some high scale.
- Finally, it does not include gravity.

Gauge hierarchy problem

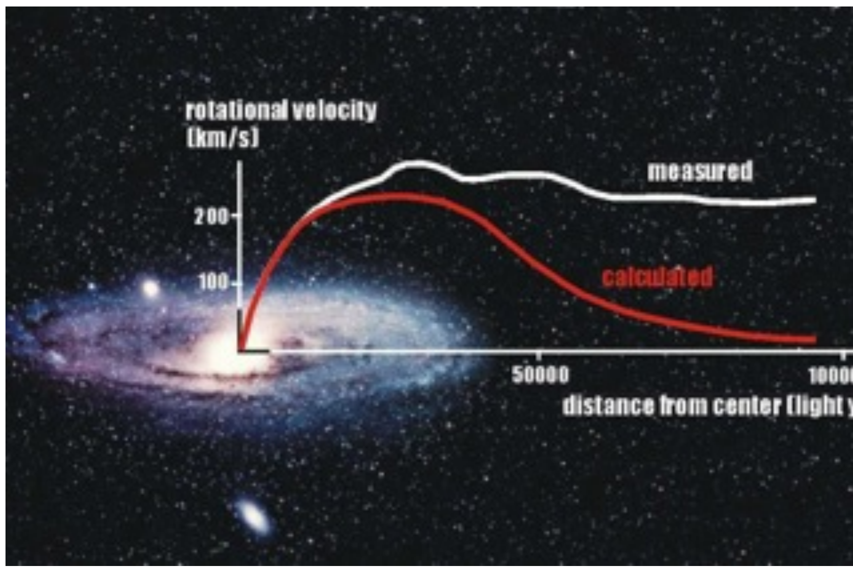
$$\mathcal{L}_\phi = \overline{\psi}(i\gamma^\mu \partial_\mu)\psi + |\partial_\mu \phi|^2 - m_S^2 |\phi|^2 - \left(\frac{\lambda_F}{2}\overline{\psi}\psi\phi + \mathrm{h.c.}\right)$$

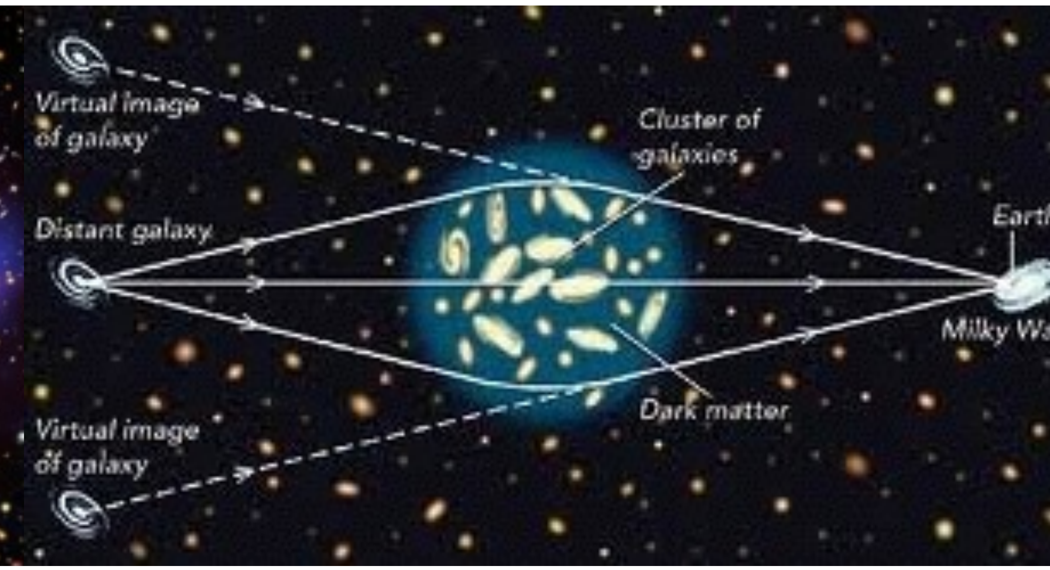


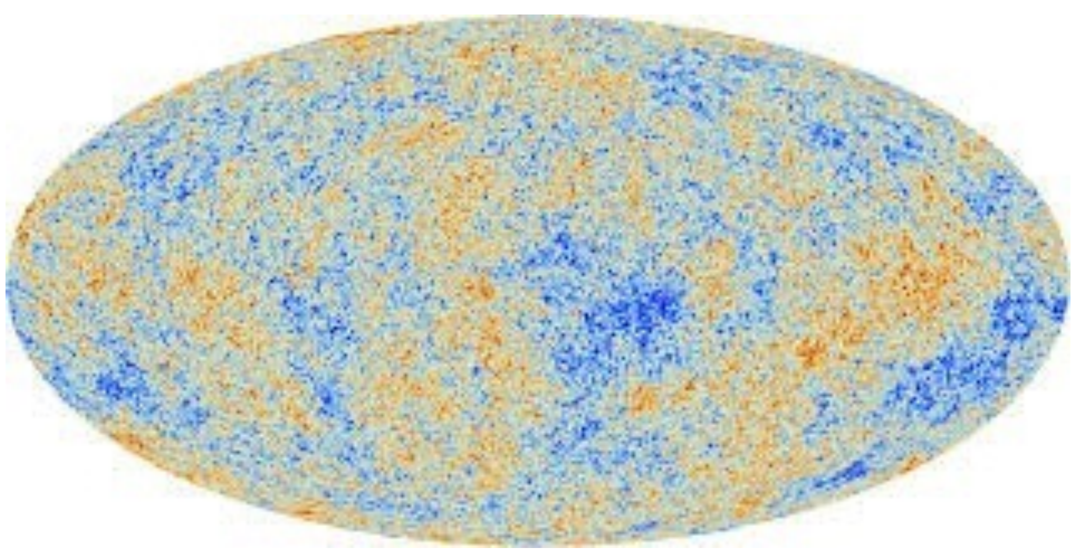
$$(\delta M_h^2)_a = -\frac{\lambda_F^2}{8\pi^2}\left[\Lambda^2 + (m_S^2 - 6m_F^2)\log\left(\frac{\Lambda}{m_F}\right) + (2m_F^2 - \frac{m_S^2}{2})\left(1 + I_1\left(\frac{m_S^2}{m_F^2}\right)\right)\right] + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$$

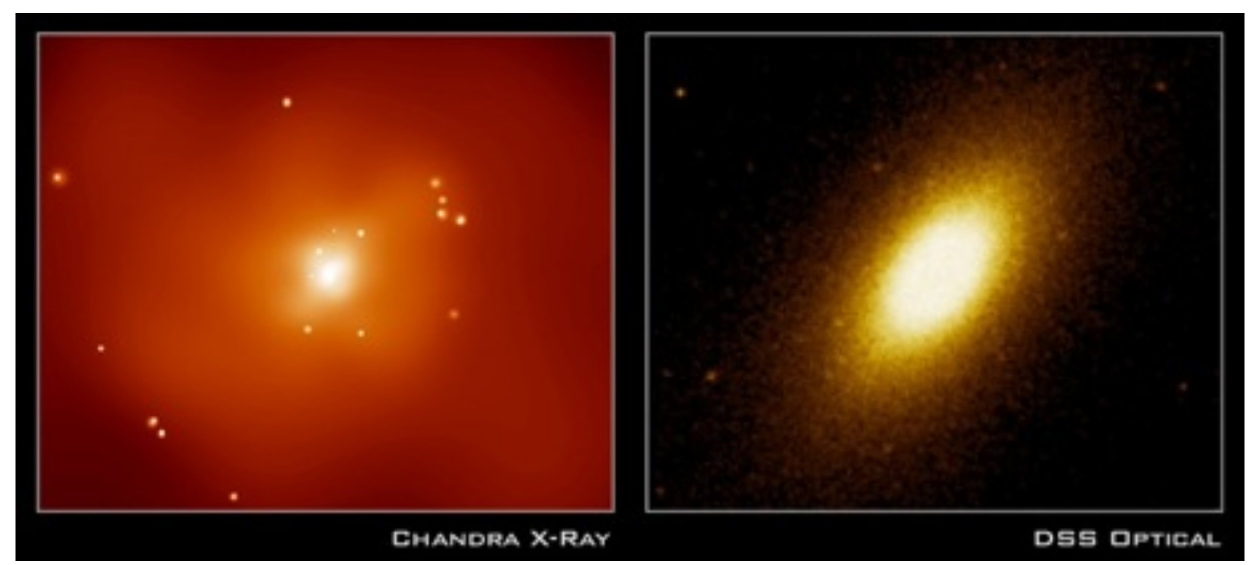
Correction to scalar mass is quadratic divergent. Fine tuning is required to explain the observed Higgs mass. →Unnatural.

Dark Matter


rotational velocity
(km/s)
200
100
measured
calculated
50000
distance from center

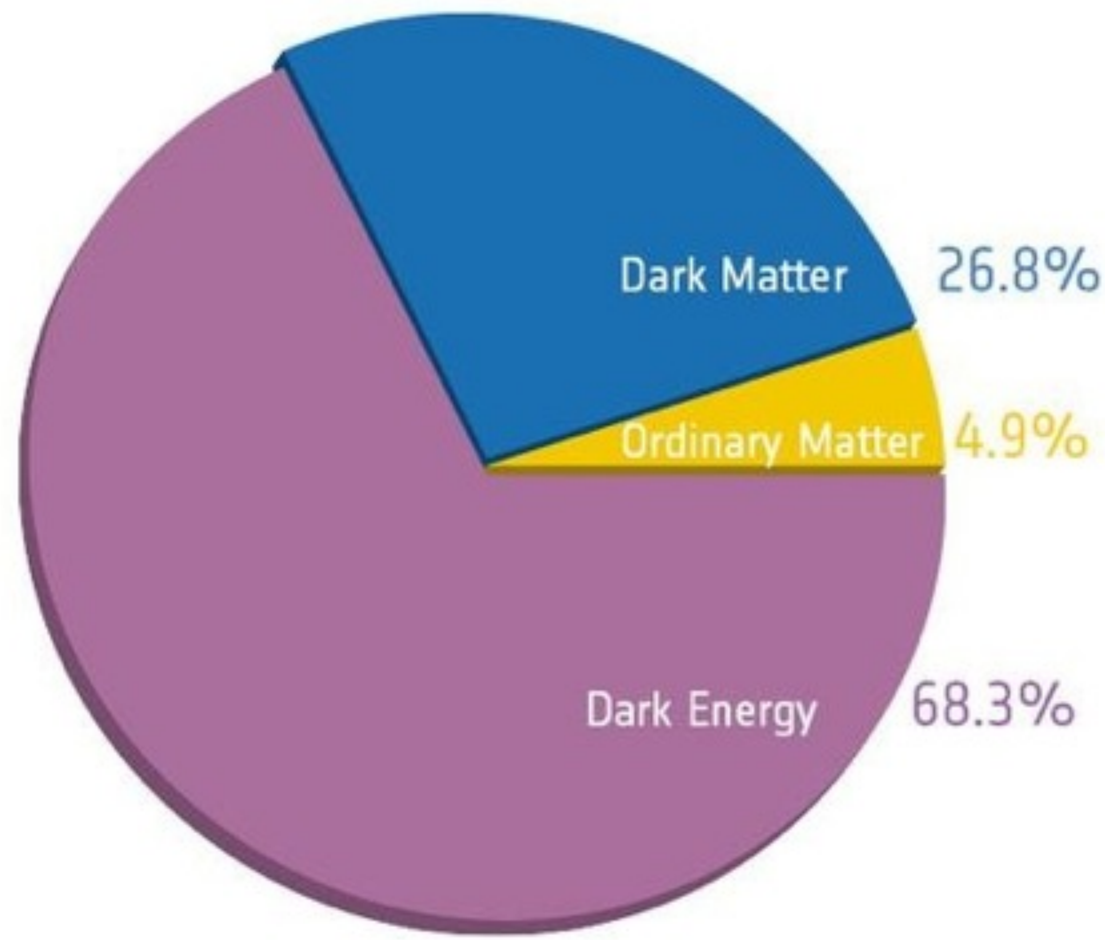

Virtual image
of galaxy
Cluster of
galaxies
Distant galaxy
Earth
Milky Way
Dark matter
Virtual image
of galaxy




CHANDRA X-RAY
DSS OPTICAL

Dark Matter

- Dark matter(s) should be stable, i.e., the lifetime should be much longer than the age of universe.
- Should be electrically neutral.
- No SM particle can be dominant component of DM



Neutrino oscillation

For massive neutrinos, one can introduce in analogy to the quark mixing a mixing matrix describing the relation between mass and flavor states:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 3} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Pontecorvo-Maki-Nakagawa-Sakata matrix

Massive neutrinos develop differently in time.

$$|\nu_i(t)\rangle = |\nu_i(0)\rangle e^{-iE_i t} = |\nu_i(0)\rangle e^{-i(p_i + \frac{m_i^2}{2p_i})}$$

for masses $m_i << E_i$:

$$E_i = \sqrt{p^2 + m_{i^2}} = p_i + \frac{m_{i^2}}{2p_i}$$

→ there will be a mixing of the flavor states with time.

$$|\nu(t)\rangle_\alpha = \sum_i U_{\alpha i} e^{-iE_i t} |\nu_i(0)\rangle = \sum_{i,\beta} U_{\alpha i} U^*_{\beta i} e^{-iE_i t} |\nu_\beta\rangle$$

Two-Flavor mixing (for simplicity)

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Definite momentum p; same for all mass eigenstate components

$$E_i = \sqrt{p^2 + m_i^2} = p + \frac{m_i^2}{2p}$$

$$E_2 - E_1 = \frac{m_1^2 - m_2^2}{2p} \approx \frac{\Delta m^2}{2E}$$

(assuming p_i is the same)

$t = L/\beta$ w/ $\beta \approx 1$:

$$(E_2 - E_1)\, t = \frac{\Delta m^2}{2E} L$$

Time development for an initially pure $|\nu_\alpha\rangle$ beam:

$$|\nu_\alpha(t)\rangle = \cos\theta\, e^{-iE_1 t} |\nu_1\rangle + \sin\theta\, e^{-iE_2 t} |\nu_2\rangle$$

$$= \left[\cos^2\theta\, e^{-iE_1 t} + \sin^2\theta\, e^{-iE_2 t}\right] \cdot |\nu_\alpha\rangle$$

$$+ \left[\cos\theta \sin\theta \left(e^{-iE_1 t} - e^{-iE_2 t} \right)\right] \cdot |\nu_\beta\rangle$$

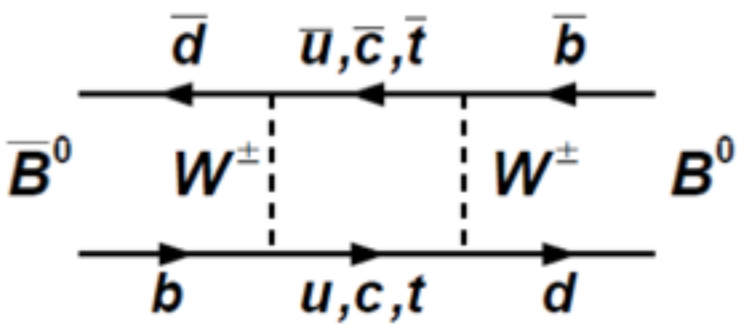
Mixing probability:

$$P(\nu_\alpha \to \nu_\beta, t) = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = 2(\cos\theta \sin\theta)^2 \left[1 - \cos^2 \frac{E_2 - E_1}{2} t \right]$$

$$P(\nu_\alpha \to \nu_\beta, t) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{4E} L \right) = \sin^2 2\theta \sin^2 \left(\frac{1.27 \cdot \Delta m^2 [eV]}{4E[GeV]} L[km] \right)$$

Mass difference is required for neutrino oscillation.

$B^0 - \bar{B}^0$ Mixing :reminder



$$H = M - \frac{i}{2}\Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

"dispersive" part "absorptive" part

$$|B_L\rangle = p|B^0\rangle + q|\overline{B}^0\rangle$$

$$|B_H\rangle = p|B^0\rangle - q|\overline{B}^0\rangle$$

$$|p|^2 + |q|^2 = 1$$

$$\frac{q}{p} = \frac{1-\tilde{\epsilon}}{1+\tilde{\epsilon}} = \sqrt{\frac{H_{21}}{H_{12}}} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}$$

$$\begin{aligned} \overline{m} &\equiv (m_H + m_L)/2 \\ \overline{\Gamma} &\equiv (\Gamma_H + \Gamma_L)/2 \\ \Delta m &\equiv m_H - m_L \quad > 0 \\ \Delta\Gamma &\equiv \Gamma_H - \Gamma_L \end{aligned}$$

$$|B_H(t)\rangle = \left(p \cdot |B^0\rangle - q \cdot |\overline{B}^0\rangle\right) \cdot e^{-im_H t} \cdot e^{-\Gamma_H t/2}$$

$$|B_L(t)\rangle = \left(p \cdot |B^0\rangle + q \cdot |\overline{B}^0\rangle\right) \cdot e^{-im_L t} \cdot e^{-\Gamma_L t/2}$$

$$|B^0_{t=0}(t)\rangle = \frac{1}{2p}\left(|B_H\rangle e^{-im_H t} e^{-\Gamma_H t/2} + |B_L\rangle e^{-im_L t} e^{-\Gamma_L t/2}\right)$$

$$|\overline{B}^0_{t=0}(t)\rangle = \frac{1}{2q}\left(|B_L\rangle e^{-im_L t} e^{-\Gamma_L t/2} - |B_H\rangle e^{-im_H t} e^{-\Gamma_H t/2}\right)$$

$$|B^0_{t=0}(t)\rangle = g_+(t) \cdot |B^0\rangle + \frac{q}{p} \cdot g_-(t) \cdot |\overline{B}^0\rangle$$

$$|\overline{B}^0_{t=0}(t)\rangle = g_+(t) \cdot |\overline{B}^0\rangle + \frac{p}{q} \cdot g_-(t) \cdot |B^0\rangle$$

with $g_\pm(t) = \frac{1}{2} e^{-i\overline{m}t} e^{-\frac{\overline{\Gamma} t}{2}} \left(e^{i\frac{\Delta m\, t}{2}} e^{+\frac{\Delta\Gamma\, t}{4}} \pm e^{-i\frac{\Delta m\, t}{2}} e^{-\frac{\Delta\Gamma\, t}{4}} \right)$

Neutrino oscillation parameters

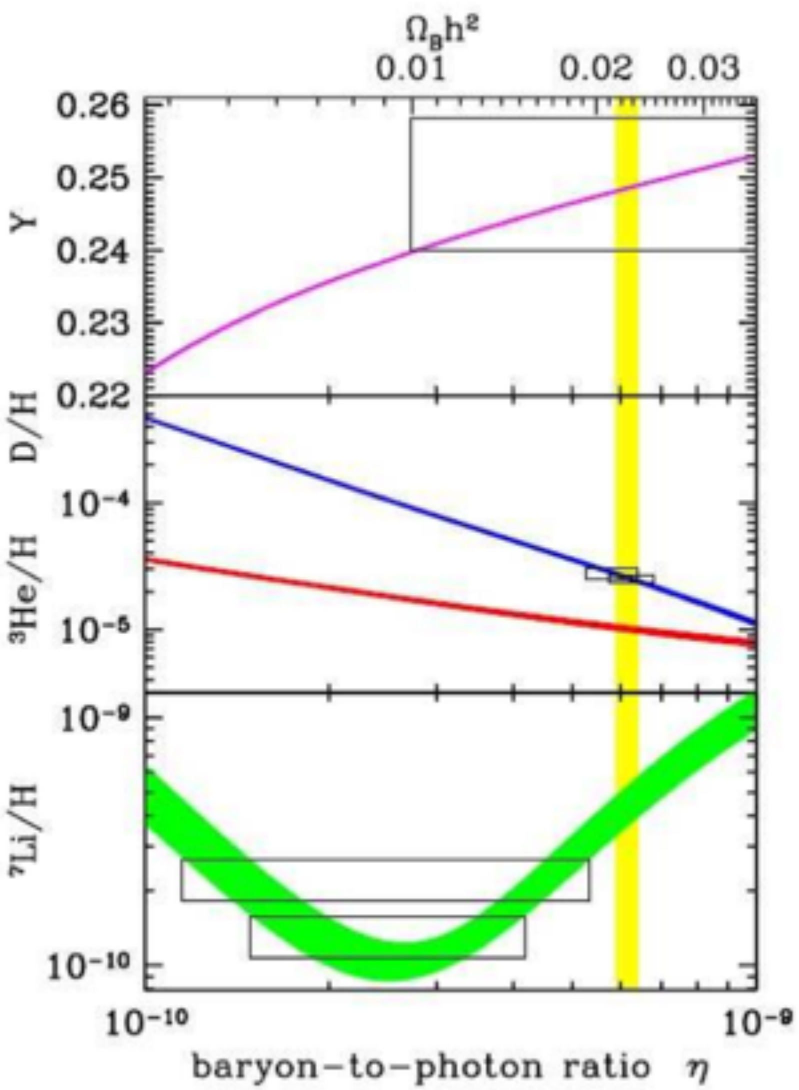
Parameter	best-fit ($\pm 1\sigma$)	3σ
Δm^2_{21} [10^{-5} eV ^{2}]	$7.54^{+0.26}_{-0.22}$	$6.99 - 8.18$
$\|\Delta m^2\|$ [10^{-3} eV ^{2}]	$2.43^{+0.06}_{-0.10}$ ($2.42^{+0.07}_{-0.11}$)	$2.19(2.17) - 2.62(2.61)$
$\sin^2\theta_{12}$	$0.307^{+0.018}_{-0.016}$	$0.259 - 0.359$
$\sin^2\theta_{23}$	$0.386^{+0.024}_{-0.021}$ ($0.392^{+0.039}_{-0.022}$)	$0.331(0.335) - 0.637(0.663)$
$\sin^2\theta_{13}$ [173]	0.0241 ± 0.0025 ($0.0244^{+0.0023}_{-0.0025}$)	$0.0169(0.0171) - 0.0313(0.0315)$

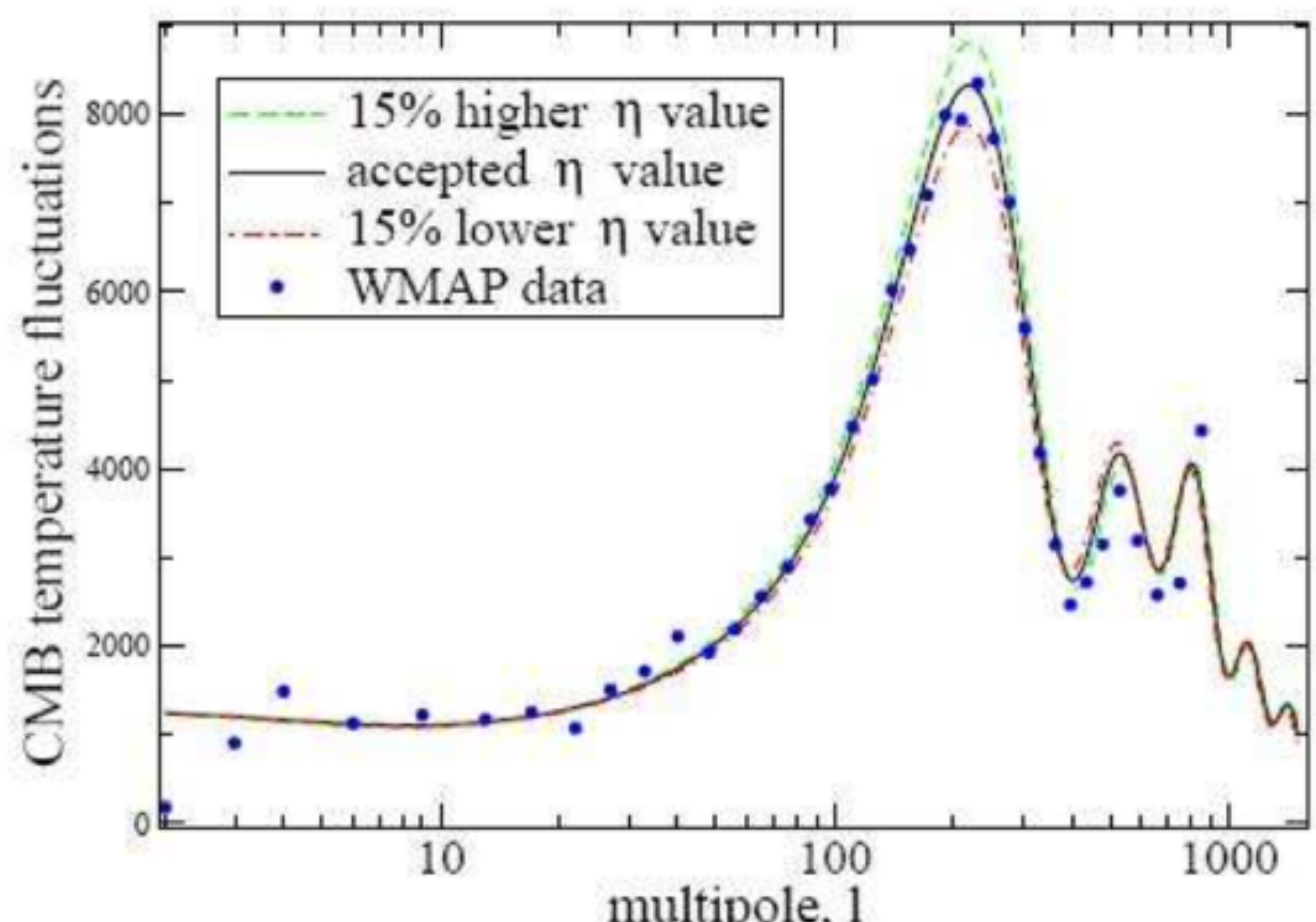
Neutrinos are massive as opposed to the SM.

Baryogenesis

$$n_B = n_b - n_{\bar{b}} \qquad n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3 \qquad \eta = \frac{n_B}{n_\gamma}$$

$$\eta = 10^{-10} \times \begin{cases} 6.28 \pm 0.35 \\ 5.92 \pm 0.56 \end{cases} \qquad \eta = (6.14 \pm 0.25) \times 10^{-10}$$





Baryogenesis

- To explain matter-antimatter asymmetry of the universe (baryogenesis), we need 3 Sakharov conditions
  - B-violation
  - C and CP violation
  - Out of thermal equilibrium condition
- The conservation of baryon number should be violated: obvious
- If C or CP conserved, we get $n_b = n_{\bar{b}}$

$$
\begin{array}{ccc}
X \to \overline{\nu}_R d & \overset{C}{\leftrightarrow} & \overline{X} \to \nu_R \bar{d} \\
P \updownarrow & & P \updownarrow \\
X \to \overline{\nu}_L d & \overset{C}{\leftrightarrow} & \overline{X} \to \nu_L \bar{d}
\end{array}
$$

- $(CPT)H(CPT)^{-1} = H, \quad (CPT)B(CPT)^{-1} = -B.$

In thermal equilibrium, $\langle B \rangle = \mathrm{Tr}\left[e^{-\beta H} B\right]$

$$
\begin{aligned}
\langle B \rangle &= \mathrm{Tr}\left[(CPT) e^{-\beta H} B (CPT)^{-1}\right] \\
&= \mathrm{Tr}\left[e^{-\beta H} (CPT) B (CPT)^{-1}\right] = -\langle B \rangle.
\end{aligned}
$$

Baryogenesis

- The SM satisfies all the three conditions
  - B: violated by the anomaly -> very small, but can be large at finite T (sphaleron)
  - C: maximally violated, CP: violated by a complex phase in the CKM matrix of quark mixing
  - out of thermal equilibrium: electroweak phase transition

•But the CP violation in the SM is too small

• The strong first order phase transition needed to explain the current baryon asymmetry requires Higgs mass < 40 GeV

- We need new physics beyond the SM with new sources of CPV

We can estimate the magnitude of the baryon asymmetry of the Universe caused by KM CP violation

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \sim \frac{J \times P_u \times P_d}{M^{12}}$$

$$J = \cos(\theta_{12})\cos(\theta_{23})\cos^2(\theta_{13})\sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin(\delta)$$
$$P_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$
$$P_d = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

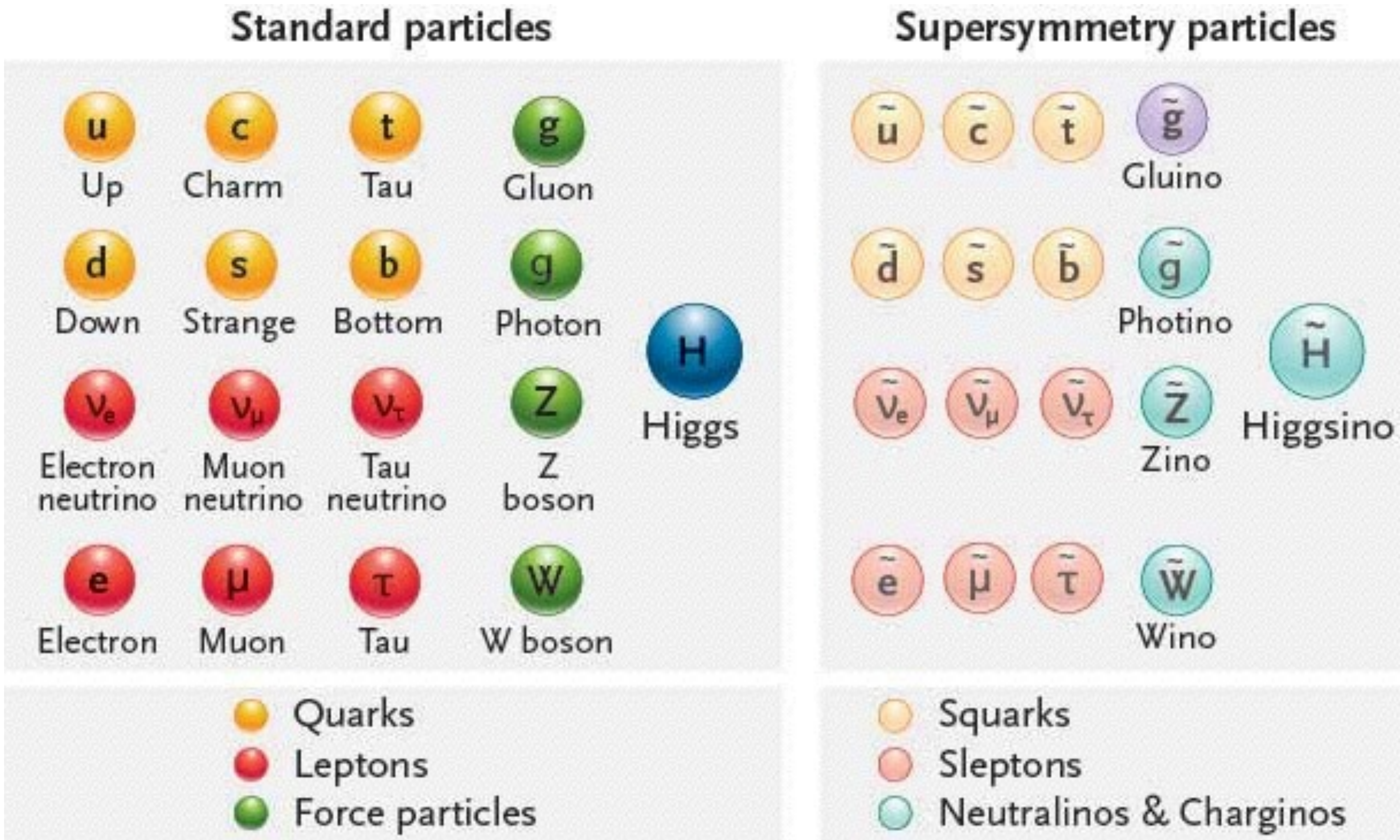
- The Jarlskog parameter J is a parametrization invariant measure of CP violation in the quark sector: J ~ O(10^{-5})
- The mass scale M can be taken to be the electroweak scale O(100 GeV)
- This gives an asymmetry O(10^{-17}):

much much below the observed value of O(10^{-10})

New Physics in Flavor

- No signal of new physics in B-decays
- Why bother?
  - Can probe NP scale much higher than LHC
  - New physics in other sector may also affect quark sector: B-physics can give strong constraint
  - Precision measurements at LHC and Belle 2
  - There can be interplay between flavor physics and other sector, e.g., dark matter

Supersymmetry

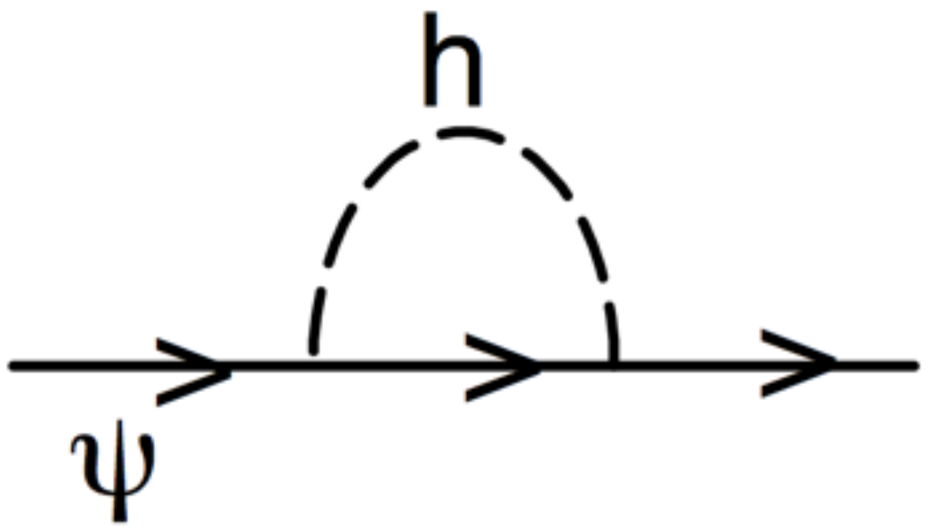


Supersymmetry

- Solves gauge hierarchy problem
- Gauge coupling unification
- Provides dark matter candidate
- Radiative electroweak symmetry breaking

Quantum correction to m_F, m_V

$$\mathcal{L}_\phi = \overline{\psi}(i\gamma^\mu \partial_\mu)\psi + \mid \partial_\mu \phi \mid^2 - m_S^2 \mid \phi \mid^2 - \left(\frac{\lambda_F}{2}\overline{\psi}\psi\phi + \text{h.c.}\right)$$



$$\delta m_F = -\frac{3\lambda_F^2 m_F}{64\pi^2}\log\left(\frac{\Lambda^2}{m_F^2}\right) + \ldots.$$

In the $m_F \rightarrow 0$, the Lagrangian in invariant under the chiral transf.

$$\psi_L \rightarrow e^{i\theta_L}\psi_L$$
$$\psi_R \rightarrow e^{i\theta_R}\psi_R,$$

The Yukawa int. breaks this sym. and the correction should be proportional to m_F. Fermion masses are natural.

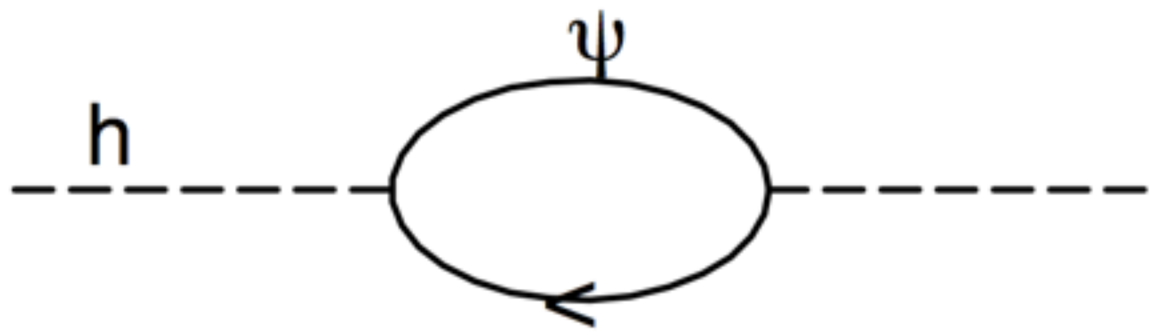
Quantum correction to m_F, m_V

- Similarly, the gauge boson masses are protected by gauge symmetry.

- Gauge boson masses are natural.

Gauge hierarchy problem

$$\mathcal{L}_\phi = \overline{\psi}(i\gamma^\mu\partial_\mu)\psi + \mid \partial_\mu\phi \mid^2 - m_S^2 \mid \phi \mid^2 - \left(\frac{\lambda_F}{2}\overline{\psi}\psi\phi + \text{h.c.}\right)$$

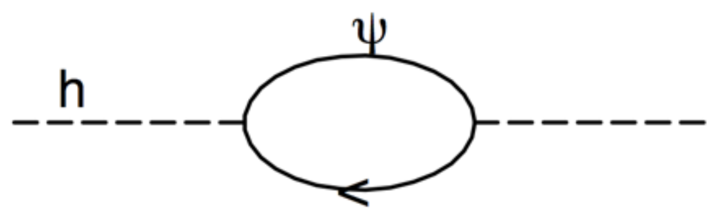


$$\begin{aligned}(\delta M_h^2)_a \quad = \quad & -\frac{\lambda_F^2}{8\pi^2}\Big[\Lambda^2 + (m_S^2 - 6m_F^2)\log\left(\frac{\Lambda}{m_F}\right) \\ & +(2m_F^2 - \frac{m_S^2}{2})\left(1 + I_1\left(\frac{m_S^2}{m_F^2}\right)\right)\Big] + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)\end{aligned}$$

Correction to scalar mass is quadratic divergent. Fine tuning is required to explain the observed Higgs mass. →Unnatural.

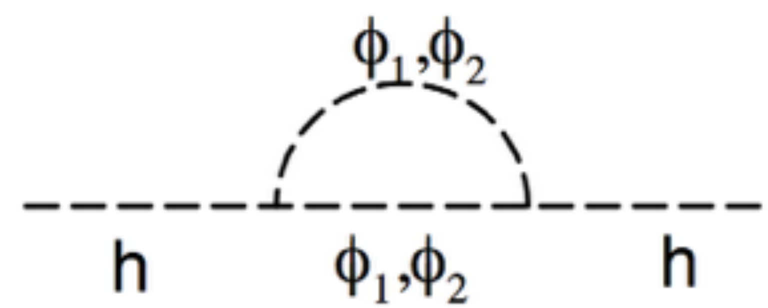
Supersymmetry

- Solves gauge hierarchy problem



$$(\delta M_h^2)_{tot} = \frac{\lambda_F^2}{4\pi^2}\left\{m_{s_1}^2 \log\left(\frac{\Lambda}{m_{s_1}}\right) - m_F^2 \log\left(\frac{\Lambda}{m_F}\right)\right\} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$$

$$\begin{aligned}\mathcal{L} \; &= \; |\,\partial_\mu\phi_1\,|^2 + |\,\partial_\mu\phi_2\,|^2 - m_{s_1}^2\,|\,\phi_1\,|^2 - m_{s_2}^2\,|\,\phi_2\,|^2 \\ &\quad + \lambda_S\,|\,\phi\,|^2\Big(\,|\,\phi_1\,|^2 + |\,\phi_1\,|^2\Big) + \mathcal{L}_\phi \quad .\end{aligned}$$



$$\begin{aligned}(\delta M_h^2)_b \;&=\; -\lambda_S \int \frac{d^4k}{(2\pi)^4}\left[\frac{i}{k^2 - m_{s_1}^2} + \frac{i}{k^2 - m_{s_2}^2}\right] \\ &=\; \frac{\lambda_S}{16\pi^2}\left\{2\Lambda^2 - 2m_{s_1}^2 \log\left(\frac{\Lambda}{m_{s_1}}\right) - 2m_{s_2}^2 \log\left(\frac{\Lambda}{m_{s_2}}\right)\right\} \\ &\quad + \mathcal{O}\left(\frac{1}{\Lambda^2}\right).\end{aligned}$$

- Fermion and boson contributions have different sign.

Supersymmetry

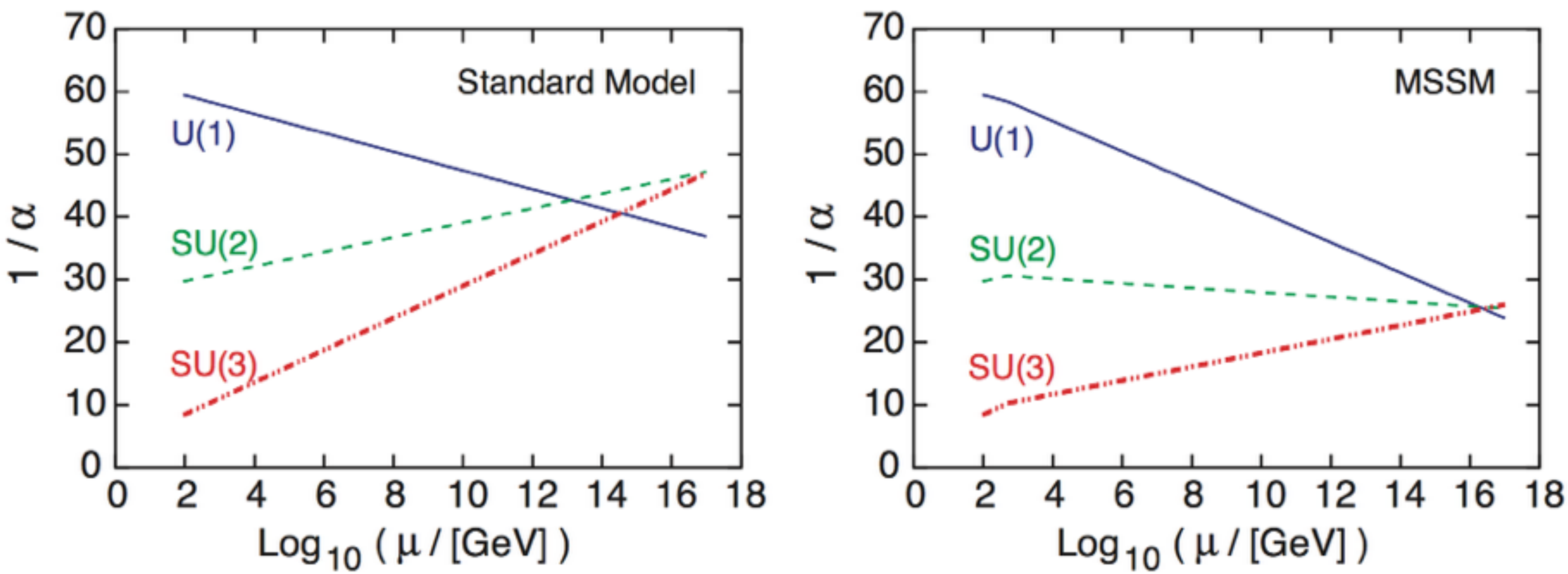
- If $\lambda_S = \lambda_F^2$, (true in SUSY) the quadratic divergences cancel each other.
- $$(\delta M_h^2)_{tot} = \frac{\lambda_F^2}{4\pi^2}\left\{m_{s_1}^2 \log\left(\frac{\Lambda}{m_{s_1}}\right) - m_F^2 \log\left(\frac{\Lambda}{m_F}\right)\right\} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$$
- For small fermion and scalar mass difference, $\delta m^2 \equiv m_F^2 - m_{s_1}^2$,

$$(\delta M_h^2)_{tot} = \frac{\lambda_F^2}{4\pi^2}\delta^2$$

- The scalar masses become natural for small mass difference.

Supersymmetry

- Suggests gauge coupling unification: supports GUT



MSSM

Table 1.1: Particle Contents of MSSM: Weak Eigenstates

Spin-0	Spin-1/2	Spin-1	Color	I_3	Y	B	L
$\widetilde{Q}_L = \begin{pmatrix} \widetilde{u}_L \\ \widetilde{d}_L \end{pmatrix}$	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$		$\mathbf{3}$	$+\frac{1}{2}$	$+\frac{1}{6}$	$+\frac{1}{3}$	0
$\widetilde{u}_R^*$	$(u^c)_L$		$\bar{\mathbf{3}}$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0
$\widetilde{d}_R^*$	$(d^c)_L$		$\bar{\mathbf{3}}$	0	$+\frac{1}{3}$	$-\frac{1}{3}$	0
$\widetilde{L}_L = \begin{pmatrix} \widetilde{\nu} \\ \widetilde{e}_L \end{pmatrix}$	$L_L = \begin{pmatrix} \nu \\ e_L \end{pmatrix}$		$\mathbf{1}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	+1
$\widetilde{e}_R^*$	$(e^c)_L$		$\mathbf{1}$	0	+1	0	−1
	$\widetilde{g}$	g	$\mathbf{8}$	0	0	0	0
	$\begin{pmatrix} \widetilde{W}^+ \\ \widetilde{W}^0 \\ \widetilde{W}^- \end{pmatrix}$	$\begin{pmatrix} W^+ \\ W^0 \\ W^- \end{pmatrix}$	$\mathbf{1}$	$\begin{matrix} +1 \\ 0 \\ -1 \end{matrix}$	0	0	0
	$\widetilde{B}$	B	$\mathbf{1}$	0	0	0	0
$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}$	$\widetilde{H}_{1L} = \begin{pmatrix} \widetilde{H}_{1L}^0 \\ \widetilde{H}_{1L}^- \end{pmatrix}$		$\mathbf{1}$	$\begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix}$	$-\frac{1}{2}$	0	0
$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$	$\widetilde{H}_{2L} = \begin{pmatrix} \widetilde{H}_{2L}^+ \\ \widetilde{H}_{2L}^0 \end{pmatrix}$		$\mathbf{1}$	$\begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix}$	$+\frac{1}{2}$	0	0

Higgs sector: Type II two Higgs doublet model.

Supersymmetric Lagrangian

- Write superpotential with d<4

$$W = \sum_i k_i \phi_i + \frac{1}{2}\sum_{ij} m_{ij}\phi_i\phi_j + \frac{1}{6}\sum_{ijk} g_{ijk}\phi_i\phi_j\phi_k,$$

holomorphic function:
no ϕ *

- SUSY Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\rm SUSY} &= -\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} + \frac{1}{2}\lambda^a i\not{\mathcal{D}}\overline{\lambda}^b + (\mathcal{D}^\mu\phi)^\dagger(\mathcal{D}^\mu\phi) + \overline{\psi}i\not{\mathcal{D}}\psi \\ & -\sum_i \left|\frac{\partial W}{\partial\phi_i}\right|^2 - \frac{1}{2}\left(\sum_{jk}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + h.c\right) \\ & +i\sqrt{2}g(\phi^*\lambda^a T^a\psi + h.c) - \frac{1}{2}\sum_a D^a D^a, \end{aligned} \qquad (1.133)$$

where the D-term $D^a = -g\sum_{ij}\phi_i^* T^a_{ij}\phi_j$, and the covariant derivative $\mathcal{D}_\mu = \partial + igA^aT^a$. Use also the representation $(T^c_G)_{ab} = -if^{cab}$ for the adjoint representation.

Superpartners: $(\phi, \psi), (\lambda, A)$

→ Majorana fermion (mass? charge?)

MSSM superpotential

- $$W = u^c \mathbf{Y}_u Q H_u + d^c \mathbf{Y}_d Q H_d + e^c \mathbf{Y}_e L H_d + \mu H_u H_d,$$

$$eg)\ QH_u = \epsilon^{ij} Q^i H_u^j \to \epsilon^{ij} U^{ik} U^{jl} Q^k H_u^l = \det(U) \epsilon^{kl} Q^k H_u^l$$

- No SUSY particle has been found→SUSY must be broken

- SUSY breaking soft terms

$$\begin{aligned} V_{\rm soft} &= \frac{1}{2}(M_3 \widetilde{g}^G \widetilde{g}^G + M_2 \widetilde{W}^a \widetilde{W}^a + M_1 \widetilde{B}\widetilde{B}) \\ &+ m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + \widetilde{q}_L^\dagger M_{\widetilde{Q}}^2 \widetilde{q}_L + \widetilde{u}_R^\dagger M_{\widetilde{u}}^2 \widetilde{u}_R + \widetilde{d}_R^\dagger M_{\widetilde{d}}^2 \widetilde{d}_R + \widetilde{l}_L^\dagger M_{\widetilde{L}}^2 \widetilde{l}_L + \widetilde{e}_R^\dagger M_{\widetilde{e}}^2 \widetilde{e}_R \\ &+ \widetilde{u}_R^\dagger A_U \widetilde{Q} H_u - \widetilde{d}_R^\dagger A_D \widetilde{Q} H_d - \widetilde{e}_R^\dagger A_E \widetilde{Q} H_d + H.c \\ &+ B\mu H_u H_d + H.c, \end{aligned}$$

where $\widetilde{q}_L^\dagger M_{\widetilde{Q}}^2 \widetilde{q}_L = \widetilde{q}_{Li}^{\alpha\dagger} \left(M_{\widetilde{Q}}^2\right)_{ij} \widetilde{q}_{L\alpha j}$.

When mass parameters→0, SUSY is recovered. Scalar masses become natural.

MSSM mass (squark)

F-term contribution

$$W = -\mu(H_d^0 H_u^0 - H_d^- H_u^+) + Y_u^{ij}\tilde{u}_{Ri}^*(\tilde{u}_{Lj}H_u^0 - \tilde{d}_{Lj}H_u^*) + Y_d^{ij}\tilde{d}_{Ri}^*(\tilde{u}_{Lj}H_d^- - \tilde{d}_{Lj}H_d^0) + Y_e^{ij}\tilde{e}_{Ri}^*(\tilde{\nu}_{Lj}H_d^- - \tilde{e}_{Lj}H_d^0)$$

$$\frac{\partial W}{\partial \tilde{u}_L^a} = H_2^0 Y_U^{ab}\tilde{u}_R^{b*}$$

$$\frac{\partial W}{\partial \tilde{u}_R^{b*}} = H_2^0 \tilde{u}_L^a Y_U^{ab}$$

$$\frac{\partial W}{\partial H_2^0} = -\mu H_1^0 + \tilde{u}_L^a Y_U^{ab}\tilde{u}_R^{b*},$$

$$V_F = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 = |H_2^0|^2 \tilde{u}_R^{0\dagger} Y_U^T Y_U^* \tilde{u}_R^0 + |H_2^0|^2 \tilde{u}_L^{0\dagger} Y_U^* Y_U^T \tilde{u}_L^0 - \mu^* H_1^{0*} \tilde{u}_R^{0\dagger} Y_U^T \tilde{u}_L^0 - \mu H_1^0 \tilde{u}_L^{0\dagger} Y_U^* \tilde{u}_R^0,$$

$$\tilde{u}_L^0 = V_L^{U\dagger}\tilde{u}_L$$

$$\tilde{u}_R^0 = V_R^{U\dagger}\tilde{u}_R$$

(Super-CKM basis)

$$V_F = \tilde{u}_R^\dagger m_U^2 \tilde{u}_R + \tilde{u}_L^\dagger m_U^2 \tilde{u}_L - \mu \cot\beta\, \tilde{u}_L^\dagger m_U \tilde{u}_R - \mu^* \cot\beta\, \tilde{u}_R^\dagger m_U \tilde{u}_L,$$

MSSM mass (squark)

D-term contribution

$$U(1) \;:\; D' = g'(y_Q|\widetilde{Q}|^2 - y_u|\widetilde{u}_R|^2 - y_d|\widetilde{d}_R|^2 + y_L|\widetilde{L}|^2 - y_e|\widetilde{e}_R|^2 + y_{H_u}|H_u|^2 + y_{H_d}|H_d|^2)$$

$$SU(2) \;:\; D^a = \frac{1}{2}(\widetilde{Q}_i^* \tau_{ij}^a \widetilde{Q}_j + \widetilde{L}_i^* \tau_{ij}^a \widetilde{L}_j + H_{u_i}^* \tau_{ij}^a H_{u_j} + H_{d_i}^* \tau_{ij}^a H_{d_j})$$

$$\begin{aligned}
V_D &= \frac{1}{2}\sum_a (D^a)^2 \\
&= \frac{g'^2}{4}(v_u^2 - v_d^2)(y_Q|\widetilde{Q}|^2 - y_u|\widetilde{u}_R|^2 - y_d|\widetilde{d}_R|^2 + y_L|\widetilde{L}|^2 - y_e|\widetilde{e}_R|^2) \\
&\quad -\frac{g^2}{8}(v_u^2 + v_d^2)(|\widetilde{Q}|^2 + |\widetilde{L}|^2) \\
&\quad +\frac{g^2}{4}(v_u^2|\widetilde{d}_L|^2 + v_d^2|\widetilde{u}_L|^2 + v_u^2|\widetilde{e}_L|^2 + v_d^2|\widetilde{\nu}_L|^2) \\
&= \frac{g^2}{4cw^2}(v_d^2 - v_u^2)\Big[(\frac{1}{2} - Q_u sw^2)|\widetilde{u}_L|^2 + Q_u sw^2|\widetilde{u}_R|^2 + (-\frac{1}{2} - Q_u sw^2)|\widetilde{d}_L|^2 + Q_u sw^2|\widetilde{d}_R|^2 \\
&\quad +(\frac{1}{2} - Q_\nu sw^2)|\widetilde{\nu}_L|^2 + (-\frac{1}{2} - Q_u sw^2)|\widetilde{e}_L|^2 + Q_u sw^2|\widetilde{e}_R|^2\Big]
\end{aligned}$$

MSSM mass (squark)

- In terms of $(\widetilde{u}_L, \widetilde{u}_R)$ (SuperCKM basis)

$$M^2_{\tilde{u}} = \begin{pmatrix} V_L^U M^2_{\tilde{Q}} V_L^{U\dagger} + m_U^2 + m_Z^2 \cos 2\beta \; g_V^{u_L} & -\mu m_U \cot\beta + V_L^U A_U^* V_R^{U\dagger} v_2/\sqrt{2} \\ -\mu^* m_U \cot\beta + V_R^U A_U^T V_L^{U\dagger} v_2/\sqrt{2} & V_R^U m^2_{\tilde{U}} V_R^{U\dagger} + m_U^2 + m_Z^2 \cos 2\beta \; g_V^{(u^c)_L} \end{pmatrix}$$

- 6x6 mass matrix: LL, LR (RL), RR 3x3 submatrices

- In general, sizes of off-diagonal terms in LL, LR, RL matrices are of the same order with the diagonal terms. When squarks couple to gluino, these lead to SUSY flavor problem.

- Many complex phases (SUSY CP problem)

MSSM mass (squark, sneutrino, slepton)

Similarly,

$$
\begin{aligned}
M^2_{\tilde{d}} &= \begin{pmatrix} V_L^D M^2_{\tilde{Q}} V_L^{D\dagger} + m_D^2 + m_Z^2 \cos 2\beta \, g_V^{d_L} & -\mu m_D \tan\beta + V_L^D A_D^* V_R^{D\dagger} v_1/\sqrt{2} \\ -\mu^* m_D \tan\beta + V_R^D A_D^T V_L^{D\dagger} v_1/\sqrt{2} & V_R^U m^2_{\tilde{D}} V_R^{U\dagger} + m_D^2 + m_Z^2 \cos 2\beta \, g_V^{(d^c)_L} \end{pmatrix} \\
M^2_{\tilde{\nu}} &= V_L^E M^2_{\tilde{L}} V_L^{E\dagger} + m_Z^2 \cos 2\beta \, g_V^{\nu_L} \\
M^2_{\tilde{e}} &= \begin{pmatrix} V_L^E M^2_{\tilde{L}} V_L^{E\dagger} + m_E^2 + m_Z^2 \cos 2\beta \, g_V^{e_L} & -\mu m_E \tan\beta + V_L^E A_E^* V_R^{E\dagger} v_1/\sqrt{2} \\ -\mu^* m_E \tan\beta + V_R^E A_E^T V_L^{E\dagger} v_1/\sqrt{2} & V_R^E m^2_{\tilde{e}} V_R^{E\dagger} + m_E^2 + m_Z^2 \cos 2\beta \, g_V^{(e^c)_L} \end{pmatrix} .
\end{aligned}
$$

with $g_V^f = (T_3^f)_L - \sin^2\theta_w Q^f$.

MSSM mass (chargino)

$$
\begin{aligned}
\mathcal{L}_\chi &= \frac{\mu}{2}(\widetilde{H}^0_{1L}\widetilde{H}^0_{2L} + \widetilde{H}^0_{2L}\widetilde{H}^0_{1L} - \widetilde{H}^-_{1L}\widetilde{H}^+_{2L} - \widetilde{H}^+_{2L}\widetilde{H}^-_{1L}) + h.c \quad (\leftarrow \mathrm{F-term}) \\
&+ \frac{1}{2}M_1\lambda_B\lambda_B + \frac{1}{2}M_2\lambda^a\lambda^a + h.c \quad (\leftarrow \mathrm{soft-term}) \\
&+ \frac{ig}{\sqrt{2}}(H^*_{1i}\lambda^a\tau^a_{ij}\widetilde{H}_{1Lj} + H^*_{2i}\lambda^a\tau^a_{ij}\widetilde{H}_{2Lj}) + h.c \quad (\leftarrow i\sqrt{2}g(\phi^*\lambda\psi - \overline{\lambda\psi}\phi)) \\
&+ \frac{ig'}{\sqrt{2}}(-H^*_{1i}\lambda_B\widetilde{H}_{1Li} + H^*_{2i}\lambda_B\widetilde{H}_{2Li}) + h.c \quad (\leftarrow i\sqrt{2}g(\phi^*\lambda\psi - \overline{\lambda\psi}\phi))
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{L}_{\chi^\pm} &= \mu\widetilde{H}^-_{1L}\widetilde{H}^+_{2L} - M_2\lambda^-\lambda^+ - \frac{ig}{\sqrt{2}}v_1\widetilde{H}^-_{1L}\lambda^+ - \frac{ig}{\sqrt{2}}v_2\lambda^-\widetilde{H}^+_{2L} + h.c \\
&= \begin{pmatrix} -i\lambda^- & \widetilde{H}^-_{1L} \end{pmatrix} \begin{pmatrix} M_2 & \sqrt{2}m_W\sin\beta \\ \sqrt{2}m_W\cos\beta & \mu \end{pmatrix} \begin{pmatrix} -i\lambda^+ \\ \widetilde{H}^+_{2L} \end{pmatrix} + h.c
\end{aligned}
$$

Gaugino-Higgsino mixing after EW sym. br.

MSSM mass (neutralino)

$$-\mathcal{L}_0 = \frac{1}{2}\left(-i\lambda_B \quad -i\lambda^3 \quad \widetilde{H}^0_{1L} \quad \widetilde{H}^0_{2L} \right) \mathcal{M}_N \begin{pmatrix} -i\lambda_B \\ -i\lambda^3 \\ \widetilde{H}^0_{1L} \\ \widetilde{H}^0_{2L} \end{pmatrix} + h.c\,.$$

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -m_Z s_w c_\beta & m_Z s_w s_\beta \\ 0 & M_2 & m_Z c_w c_\beta & -m_Z c_w s_\beta \\ -m_Z s_w c_\beta & m_Z c_w c_\beta & 0 & -\mu \\ m_Z s_w s_\beta & -m_Z c_w s_\beta & -\mu & 0 \end{pmatrix}$$

MSSM interaction

See Dreiner, et.al, 0812.1594 for 2 comp. spinor

$$
\begin{aligned}
& i\sqrt{2} g \phi^* \lambda \psi + h.c \\
\to \quad & i\sqrt{2} g_s \widetilde{Q}^{Ij*}_{Li} \lambda^a T^a_{jk} Q^{Ik}_{Li} + i\sqrt{2} g_s \widetilde{u}^{Ij}_R \lambda^a (-T^{a*}_{jk})(u^c)^{Ik}_L + i\sqrt{2} g_s \widetilde{d}^{Ij}_R \lambda^a (-T^{a*}_{jk})(d^c)^{Ik}_L + h.c \\
\overset{4-comp}{=} \quad & -\sqrt{2} g_s \widetilde{Q}^\dagger \overline{\widetilde{g}^a} T^a P_L Q + \sqrt{2} g_s \widetilde{u}_R \overline{u} P_L T^a \widetilde{g}^a + \sqrt{2} g_s \widetilde{d}_R \overline{d} P_L T^a \widetilde{g}^a + h.c. \\
\overset{4-comp}{=} \quad & -\sqrt{2} g_s \widetilde{Q}^\dagger \overline{\widetilde{g}^a} T^a P_L Q + \sqrt{2} g_s \widetilde{u}^\dagger_R \overline{\widetilde{g}^a} T^a P_R u + \sqrt{2} g_s \widetilde{d}^\dagger_R \overline{\widetilde{g}^a} T^a P_R d + h.c. \\
\overset{SCKM}{=} \quad & -\sqrt{2} g_s e^{-i\phi_3/2} \widetilde{Q}^\dagger \overline{\widetilde{g}^a} T^a P_L Q + \sqrt{2} g_s e^{i\phi_3/2} \widetilde{u}^\dagger_R \overline{\widetilde{g}^a} T^a P_R u + \sqrt{2} g_s e^{i\phi_3/2} \widetilde{d}^\dagger_R \overline{\widetilde{g}^a} T^a P_R d + h.c. \\
= \quad & -\sqrt{2} g_s \sum_{q=u,d} \left(\widetilde{q}^\dagger_L \overline{\widetilde{g}^a} T^a P_L q - \widetilde{q}^\dagger_R \overline{\widetilde{g}^a} T^a P_R q \right) + h.c,
\end{aligned}
$$

$$
\begin{aligned}
P_L q^0 &= V^{Q\dagger}_L P_L q \\
P_R q^0 &= V^{Q\dagger}_R P_R q \\
\widetilde{q}^0_L &= V^{Q\dagger}_L \widetilde{q}_L = V^{Q\dagger}_L \Gamma^{Q\dagger}_L \widetilde{q} \\
\widetilde{q}^0_R &= V^{Q\dagger}_R \widetilde{d}_R = V^{Q\dagger}_R \Gamma^{Q\dagger}_R \widetilde{q},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\rm int} = & -\sqrt{2} g_s \widetilde{u}^\dagger \overline{\widetilde{g}^a} T^a (e^{-i\phi_3/2} \Gamma^u_L P_L - e^{i\phi_3/2} \Gamma^u_R P_R) u \\
& -\sqrt{2} g_s \widetilde{d}^\dagger \overline{\widetilde{g}^a} T^a (e^{-i\phi_3/2} \Gamma^d_L P_L - e^{i\phi_3/2} \Gamma^d_R P_R) d.
\end{aligned}
$$

Mixing in the squark generates gluino mediated FCNC.

MSSM interaction

- chargino-quark-squark

In the flavor eigenstates,

$$\begin{aligned}\mathcal{L} &= -g\widetilde{u}_L^\dagger \overline{\widetilde{W}^C} V_{\mathrm{CKM}} P_L d \\ &+ g\widetilde{u}_L^\dagger \overline{\widetilde{H}^C} V_{\mathrm{CKM}} \frac{m_D}{\sqrt{2} m_W \cos\beta} P_R d \\ &+ g\widetilde{u}_R^\dagger \overline{\widetilde{H}^C} \frac{m_U}{\sqrt{2} m_W \sin\beta} V_{\mathrm{CKM}} P_L d \\ &+ h.c.\end{aligned}$$

In the mass eigenstates,

$$\begin{aligned}\mathcal{L}_{\mathrm{int}}(\widetilde{\chi}^{\pm}\widetilde{f}f) &= g\widetilde{u}^\dagger \overline{\widetilde{\chi}_I^C}(X_I^{d_L} P_L + X_I^{d_R} P_R) d \\ &+ g\widetilde{d}^\dagger \overline{\widetilde{\chi}_I}(X_I^{u_L} P_L + X_I^{u_R} P_R) u\end{aligned}$$

$$\begin{aligned}X_I^{d_L} &= \left(-V_{I1}^* \Gamma_L^u + V_{I2}^* \Gamma_R^u \frac{\mathbf{m}_U}{\sqrt{2} m_W \sin\beta}\right) V_{\mathrm{CKM}} \\ &= \left(-V_{I1}^* \Gamma_L^u + V_{I2}^* \Gamma_R^u \hat{\hat{Y}}_u\right) V_{\mathrm{CKM}} \\ X_I^{d_R} &= U_{I2} \Gamma_L^u V_{\mathrm{CKM}} \frac{\mathbf{m}_D}{\sqrt{2} m_W \cos\beta} \\ &= U_{I2} \Gamma_L^u V_{\mathrm{CKM}} \hat{\hat{Y}}_d \\ X_I^{u_L} &= \left(-U_{I1}^* \Gamma_L^d + U_{I2}^* \Gamma_R^d \frac{\mathbf{m}_D}{\sqrt{2} m_W \cos\beta}\right) V_{\mathrm{CKM}}^\dagger \\ &= \left(-U_{I1}^* \Gamma_L^d + U_{I2}^* \Gamma_R^d \hat{\hat{Y}}_d\right) V_{\mathrm{CKM}}^\dagger \\ X_I^{u_R} &= V_{I2} \Gamma_L^d V_{\mathrm{CKM}}^\dagger \frac{\mathbf{m}_U}{\sqrt{2} m_W \sin\beta} \\ &= V_{I2} \Gamma_L^d V_{\mathrm{CKM}}^\dagger \hat{\hat{Y}}_u\end{aligned}$$

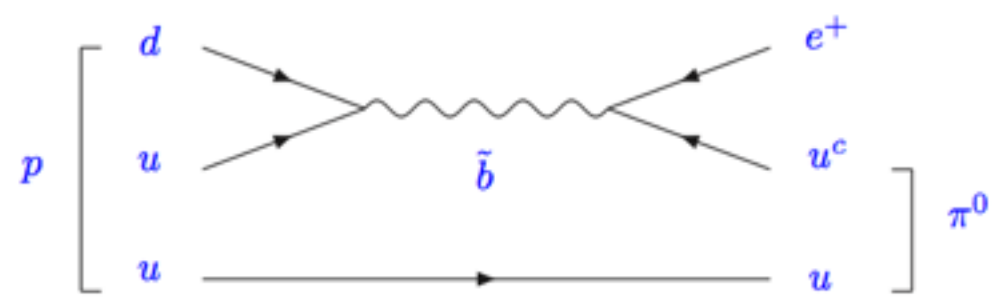
R-parity conservation

The SM accidentally preserves baryon number and therefore the proton is stable.
(Actually $U(1)_{B+L}$ is anomalous and L number is violated if the neutrinos are Majorana fermions, but such effects are important only at high temperature) $\longrightarrow$ BARYOGENESIS ?

In a supersymmetric model we can have additional renormalizable couplings that do violate explicitely

baryon and lepton number. $$W = \lambda LLE^c + \lambda' LQD^c + \lambda'' U^c D^c D^c + \mu_i L_i H_2$$

$\Rightarrow$ Dimension 4 proton decay operators



$\propto \frac{1}{m_{\tilde{b}}^2}$

To avoid fast proton decay, impose a discrete symmetry called R-parity, which forbids these terms:

$$q_L, u_R, d_R, l_L, e_R, g, \gamma, Z, W^\pm, H_u, H_d \quad \rightarrow \quad +q_L, u_R, d_R, l_L, e_R, g, \gamma, Z, W^\pm, H_u, H_d$$
$$\tilde{q}_L, \tilde{u}_R, \tilde{d}_R, \tilde{l}_L, \tilde{e}_R, \tilde{g}, \tilde{\gamma}, \tilde{Z}, \tilde{W}^\pm, \tilde{H}_u, \tilde{H}_d \quad \rightarrow \quad -\tilde{q}_L, \tilde{u}_R, \tilde{d}_R, \tilde{l}_L, \tilde{e}_R, \tilde{g}, \tilde{\gamma}, \tilde{Z}, \tilde{W}^\pm, \tilde{H}_u, \tilde{H}_d$$

So couplings only involve pairs of superpartners and therefore the supersymmetric particles can only be produced or destroyed in pairs ! $\Rightarrow$ The Lightest Susy Particle is stable !

MSSM interaction

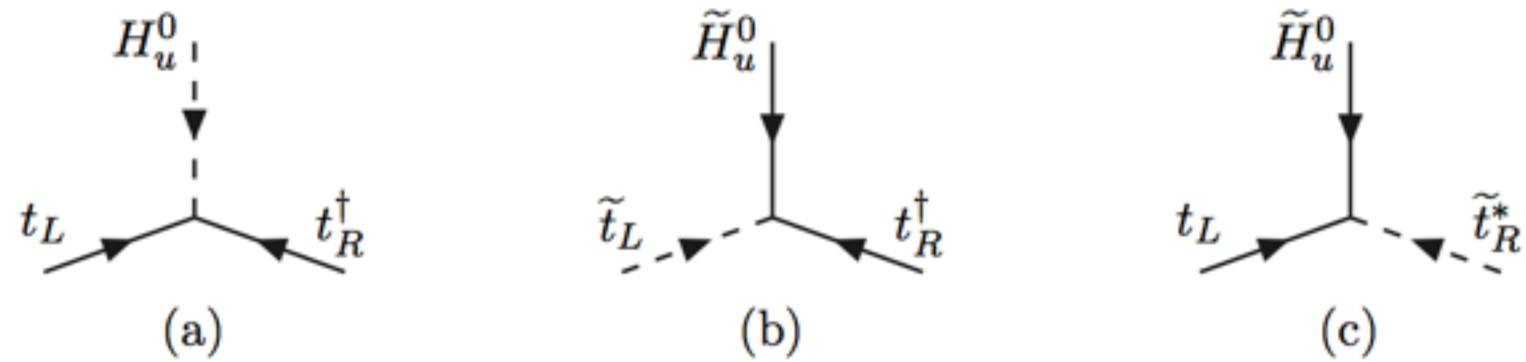


Figure 5.1: The top-quark Yukawa coupling (a) and its "supersymmetrizations" (b), (c), all of strength y_t.

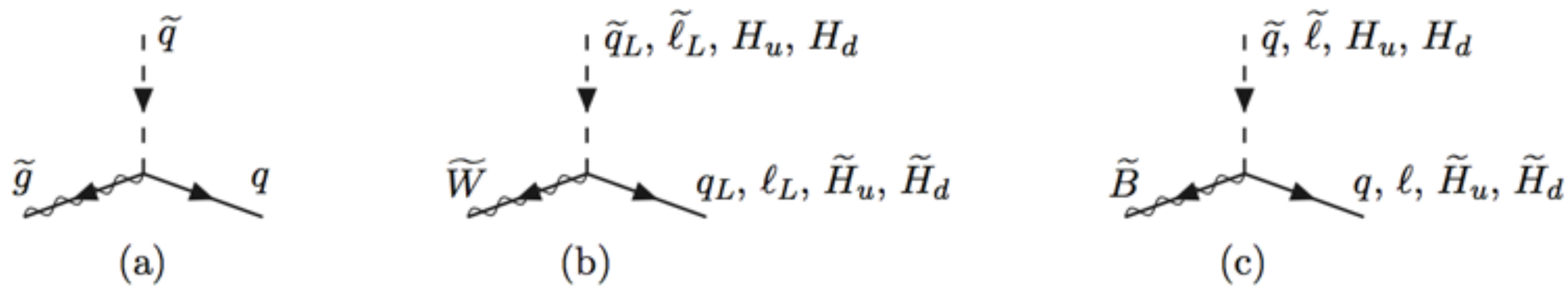


Figure 5.3: Couplings of the gluino, wino, and bino to MSSM (scalar, fermion) pairs.

MSSM interaction

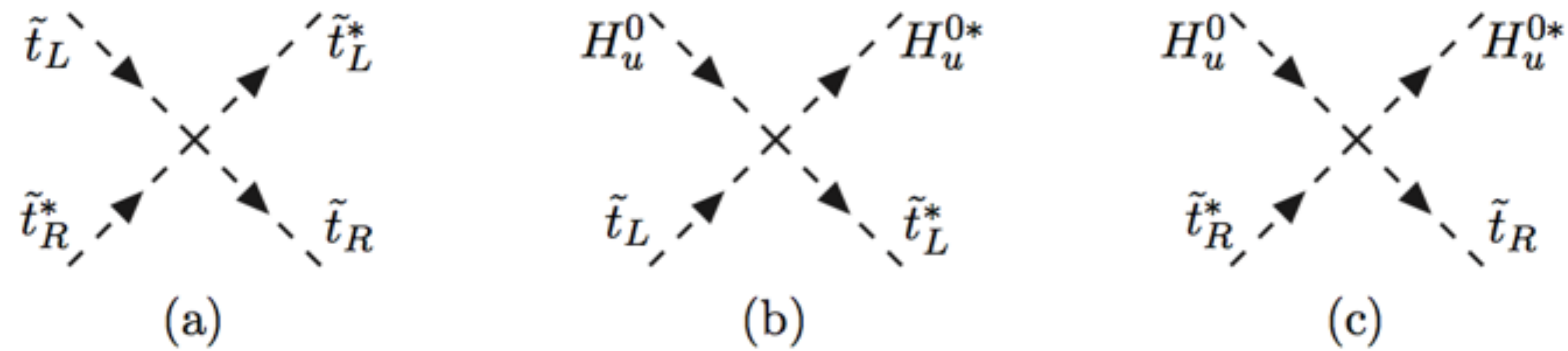


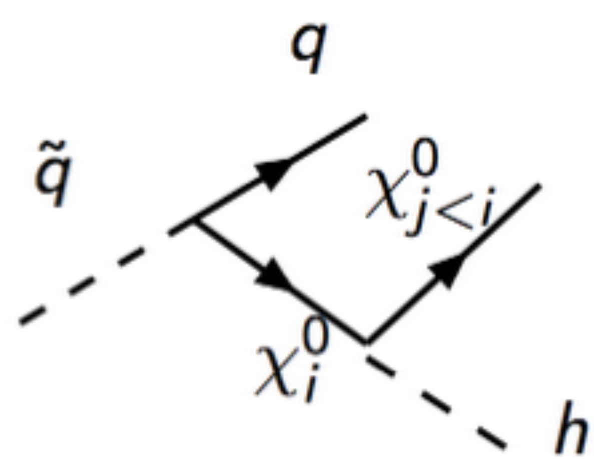
Figure 5.2: Some of the (scalar)^4 interactions with strength proportional to y_t^2.

MSSM searches at collider

- R-parity, for each particle is defined as

$$P_R = (-1)^{3(\mathrm{B-L})+2\mathrm{s}} \tag{1}$$

- Here R-parity is conserved.
  $\Rightarrow$ super-particles appear in pairs in each vertices involving super-particles.
- LSP(Lightest supersymmetric particle) can not decay.



- $\not{E}_T$ candidate at the colliders.

MSSM RGE

Coupling constants are not "constants".
They run as energy changes.

Gauge couplings: $t = \frac{1}{16\pi^2} \log Q$

$$\frac{d}{dt} g_a = b_a g_a^3,$$

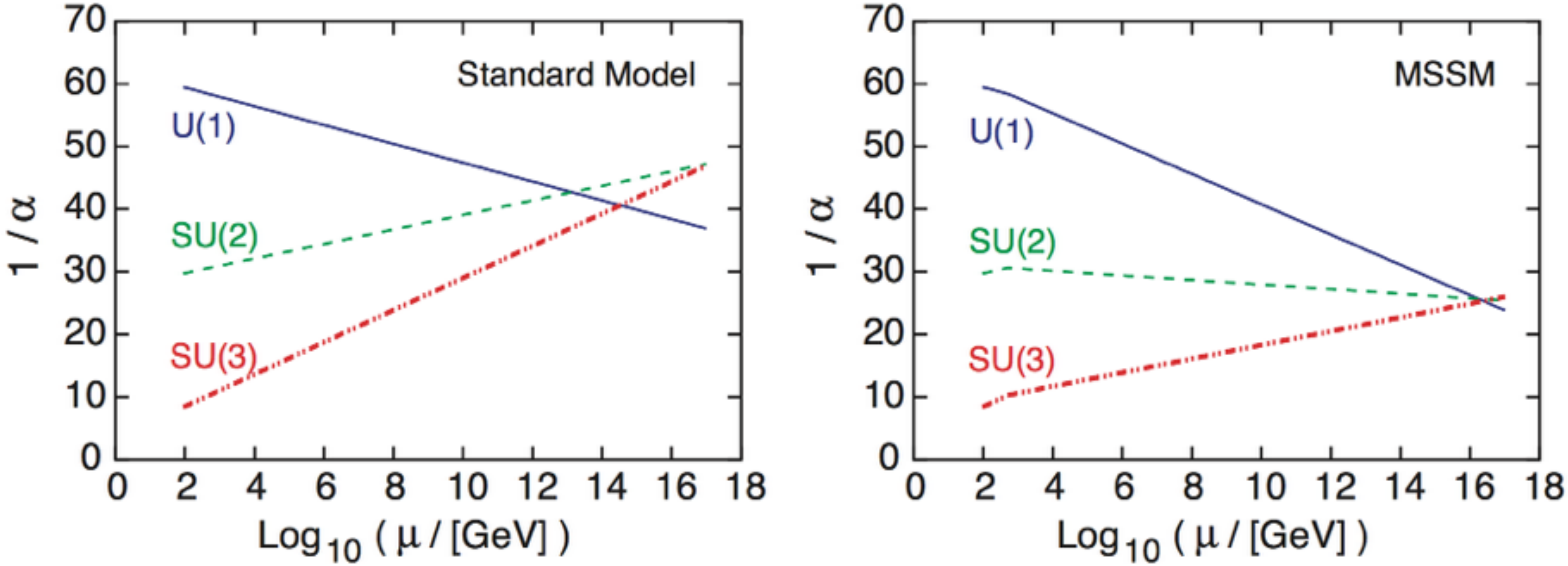
where $b_a = \left(\frac{33}{5}, 1, -3\right)$.

$$b_a = -\frac{11}{3} C_2(G) + \frac{2}{3} C_2(\lambda) + \frac{2}{3} \sum_\chi C(\chi) + \frac{1}{3} \sum_\phi C(\phi),$$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
C_2(Adjoint)	3	2	0
C_2(Fundamental)	$\frac{4}{3}$	$\frac{3}{4}$	Y^2
C(Fundamental)	$\frac{1}{2}$	$\frac{1}{2}$	Y^2

Supersymmetry

- Gauge coupling unification



RGEs for gaugino mass parameters, mu-term, and Yukawa couplings.

Gaugino masses:

$$\frac{d}{dt}M_a = 2b_a g_a^2 M_a.$$

Superpotential parameters:

$$\begin{aligned}
\frac{d}{dt}\mu &= \mu\Big\{\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 3g_2^2 - \frac{3}{5}g_1^2\Big\},\\
\frac{d}{dt}\mathbf{Y}_u &= \mathbf{Y}_u\Big\{3\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + 3\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_d^\dagger\mathbf{Y}_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Big\},\\
\frac{d}{dt}\mathbf{Y}_d &= \mathbf{Y}_d\Big\{\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) + 3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_u^\dagger\mathbf{Y}_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\Big\},\\
\frac{d}{dt}\mathbf{Y}_e &= \mathbf{Y}_e\Big\{\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) + 3\mathbf{Y}_e^\dagger\mathbf{Y}_e - 3g_2^2 - \frac{9}{5}g_1^2\Big\}.
\end{aligned}$$

Soft-breaking trilinear scalar couplings:

$$\begin{aligned}
\frac{d}{dt}\mathbf{A}_u &= \mathbf{A}_u\Big\{3\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + 5\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_d^\dagger\mathbf{Y}_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Big\}, \\
&\quad +\mathbf{Y}_u\Big\{6\mathrm{Tr}(\mathbf{A}_u\mathbf{Y}_u^\dagger) + 4\mathbf{Y}_u^\dagger\mathbf{A}_u + 2\mathbf{Y}_d^\dagger\mathbf{A}_d + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{26}{15}g_1^2M_1\Big\}, \\
\frac{d}{dt}\mathbf{A}_d &= \mathbf{A}_d\Big\{\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) + 5\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_u^\dagger\mathbf{Y}_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\Big\} \\
&\quad +\mathbf{Y}_d\Big\{\mathrm{Tr}(6\mathbf{A}_d\mathbf{Y}_d^\dagger + 2\mathbf{A}_e\mathbf{Y}_e^\dagger) + 4\mathbf{Y}_d^\dagger\mathbf{A}_d + 2\mathbf{Y}_u^\dagger\mathbf{A}_u + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{14}{15}g_1^2M_1\Big\}, \\
\frac{d}{dt}\mathbf{A}_e &= \mathbf{A}_e\Big\{\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) + 5\mathbf{Y}_e^\dagger\mathbf{Y}_e - 3g_2^2 - \frac{9}{5}g_1^2\Big\} \\
&\quad +\mathbf{Y}_e\Big\{\mathrm{Tr}(6\mathbf{A}_d\mathbf{Y}_d^\dagger + 2\mathbf{A}_e\mathbf{Y}_e^\dagger) + 4\mathbf{Y}_e^\dagger\mathbf{A}_e + 6g_2^2M_2 + \frac{18}{5}g_1^2M_1\Big\}.
\end{aligned}$$

Scalar (mass)^2 of the type b^{ij}:

$$\begin{aligned}
\frac{d}{dt}B &= B\Big\{\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 3g_2^2 - \frac{3}{5}g_1^2\Big\} \\
&\quad +\mu\Big\{\mathrm{Tr}(6\mathbf{A}_u\mathbf{Y}_u^\dagger + 6\mathbf{A}_d\mathbf{Y}_d^\dagger + 2\mathbf{A}_e\mathbf{Y}_e^\dagger) + 6g_2^2M_2 + \frac{6}{5}g_1^2M_1\Big\}.
\end{aligned}$$

Scalar (mass)^2 terms of the $(m^2)_i^j$ type: it is convenient to define the quantities

$$\mathcal{S} \;=\; m_{H_u}^2 - m_{H_d}^2 + \mathrm{Tr}[\mathbf{m}_Q^2 - 2\mathbf{m}_u^2 + \mathbf{m}_d^2 - \mathbf{m}_L^2 + \mathbf{m}_e^2].$$

Then

$$\begin{aligned}
\frac{d}{dt} m_{H_u}^2 &= 6\mathrm{Tr}[(m_{H_u}^2 + \mathbf{m}_Q^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + \mathbf{A}_u^\dagger \mathbf{A}_u] \\
&\quad -6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 + \frac{3}{5}g_1^2\mathcal{S}, \\
\frac{d}{dt} m_{H_d}^2 &= \mathrm{Tr}\Big[6(m_{H_d}^2 + \mathbf{m}_Q^2)\mathbf{Y}_d^\dagger \mathbf{Y}_d + 6\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + 2(m_{H_d}^2 + \mathbf{m}_L^2)\mathbf{Y}_e^\dagger \mathbf{Y}_e + 2\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e \\
&\qquad +6\mathbf{A}_d^\dagger \mathbf{A}_d + 2\mathbf{A}_e^\dagger \mathbf{A}_e\Big] - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2\mathcal{S}, \\
\frac{d}{dt}\mathbf{m}_Q^2 &= (\mathbf{m}_Q^2 + 2m_{H_u}^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u + (\mathbf{m}_Q^2 + 2m_{H_d}^2)\mathbf{Y}_d^\dagger \mathbf{Y}_d + [\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d]\mathbf{m}_Q^2 + 2\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u \\
&\quad +2\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + 2\mathbf{A}_u^\dagger \mathbf{A}_u + 2\mathbf{A}_d^\dagger \mathbf{A}_d \\
&\quad -\frac{32}{3}g_3^2|M_3|^2 - 6g_2^2|M_2|^2 - \frac{2}{15}g_1^2|M_1|^2 + \frac{1}{5}g_1^2\mathcal{S}, \\
\frac{d}{dt}\mathbf{m}_u^2 &= (2\mathbf{m}_u^2 + 4m_{H_u}^2)\mathbf{Y}_u \mathbf{Y}_u^\dagger + 4\mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_u^\dagger + 2\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{m}_u^2 + 4\mathbf{A}_u \mathbf{A}_u^\dagger \\
&\quad -\frac{32}{3}g_3^2|M_3|^2 - \frac{32}{15}g_1^2|M_1|^2 - \frac{4}{5}g_1^2\mathcal{S}, \\
\frac{d}{dt}\mathbf{m}_d^2 &= (2\mathbf{m}_d^2 + 4m_{H_d}^2)\mathbf{Y}_d \mathbf{Y}_d^\dagger + 4\mathbf{Y}_d \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger + 2\mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{m}_d^2 + 4\mathbf{A}_d \mathbf{A}_d^\dagger \\
&\quad -\frac{32}{3}g_3^2|M_3|^2 - \frac{8}{15}g_1^2|M_1|^2 + \frac{2}{5}g_1^2\mathcal{S}, \\
\frac{d}{dt}\mathbf{m}_L^2 &= (\mathbf{m}_L^2 + 2m_{H_d}^2)\mathbf{Y}_e^\dagger \mathbf{Y}_e + 2\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e + \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{m}_L^2 + 2\mathbf{A}_e^\dagger \mathbf{A}_e \\
&\quad -6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2\mathcal{S}, \\
\frac{d}{dt}\mathbf{m}_e^2 &= (2\mathbf{m}_e^2 + 4m_{H_d}^2)\mathbf{Y}_e \mathbf{Y}_e^\dagger + 4\mathbf{Y}_e \mathbf{m}_L^2 \mathbf{Y}_e^\dagger + 2\mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{m}_e^2 + 4\mathbf{A}_e \mathbf{A}_e^\dagger \\
&\quad -\frac{24}{5}g_1^2|M_1|^2 + \frac{6}{5}g_1^2\mathcal{S}, \\
\frac{d}{dt}\mathcal{S} &= \frac{66}{5}g_1^2\mathcal{S}.
\end{aligned}$$

Counting # of MSSM parameters

How many free physical parameters (including phases) in the MSSM Lagrangian?

- Gauge sector

Parameter	(Re,Im)	Symm	Phys. (Re,Im)
Gauge couplings	$(3,0)$		$(3,0)$
Gaugino masses	$(3,3)$	$U(1)_R$	$(3,2)$

- Higgs sector

Parameter	(Re,Im)	Symm	Phys. (Re,Im)
$m^2_{H_u}, m^2_{H_d}$	$(2,0)$		$(2,0)$
μ, B	$2\times(1,1)$	$U(1)_{PQ}$	$(2,1)$

- Flavor sector

Parameter	(Re,Im)	Symm	Phys. (Re,Im)
3 Yukawa, 3 trilinear	$6\times(9,9)$	$[U(3)_f]^5$	$(39,26)^*$
5 scalar mass matrices	$5\times(6,3)$		$(30,15)$

$^* = 6\times(9,9) - 5\times(3,6) + (0,2)(B,L) = (39,26)$

- Real parameters $= 6 + 4 + 69 = 79$

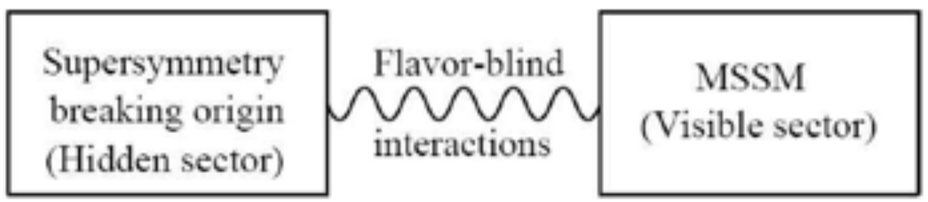
  Phases $= 2 + 1 + 41 + 1(\theta_{QCD}) = 45$

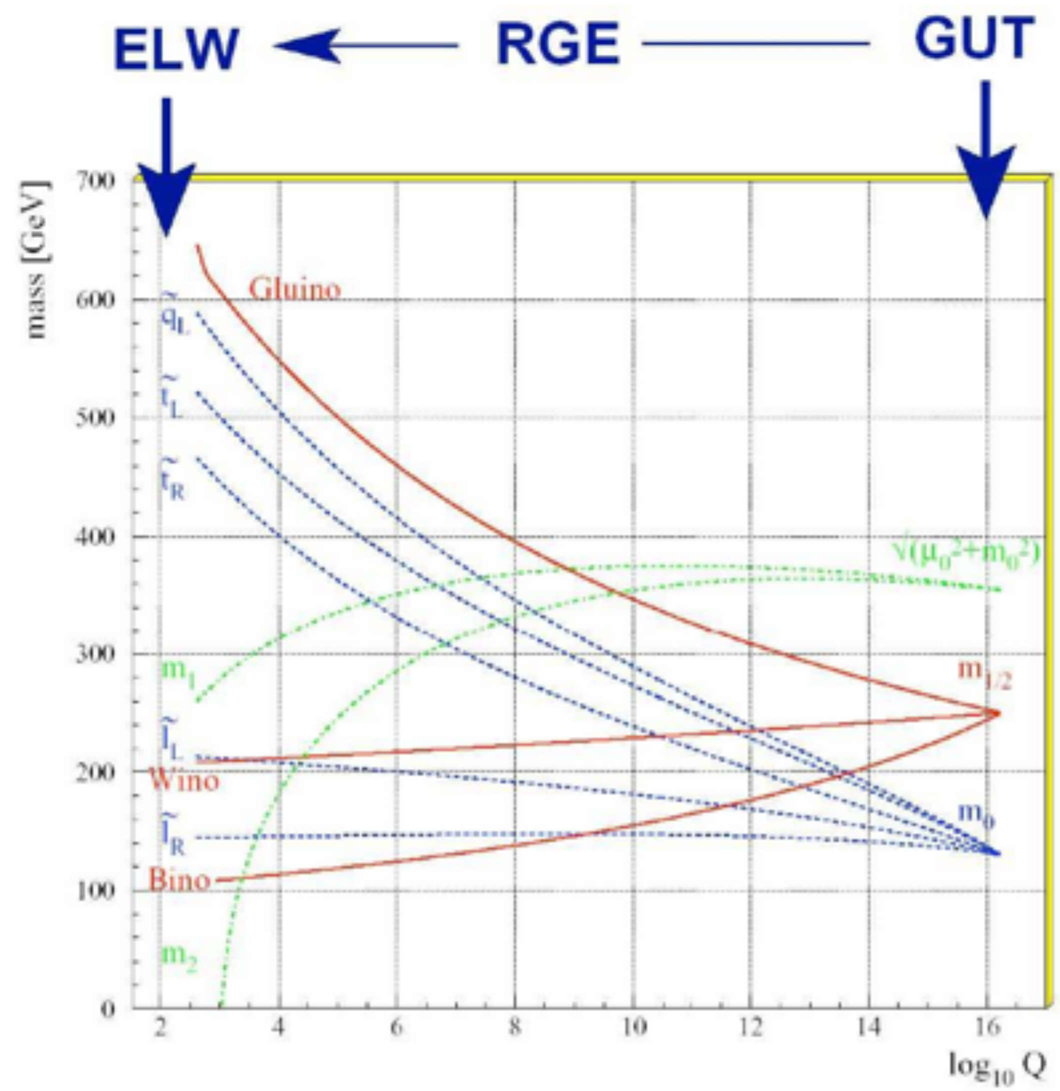
  $\Rightarrow$ MSSM-124

MSSM parameters

- mSUGRA

SUSY is broken by gravity





Assume universal masses at GUT scale:

m_0 – common mass of scalars (squarks, sleptons, Higgs bosons)

$m_{1/2}$ – common mass of gauginos and higgsinos

A_0 – common trilinear coupling

$\tan\beta$ – ratio of Higgs vacuum expectation values

$\mathrm{sign}\,\mu = \pm 1$ – sign of μ SUSY conserving Higgsino mass parameter

Radiative EW sym. br.

- Why is $\mu^2 < 0$? $\mathcal{L}_{\rm Higgs}^{\rm SM} = D_\mu \phi^\dagger D^\mu \phi - \mu_\phi^2 \phi^\dagger \phi - \lambda_\phi (\phi^\dagger \phi)^4$

- SUSY provides an answer. $M_h^2 = m_0^2 + \mu^2$

increases

$$\frac{d}{d\log(Q)} \begin{pmatrix} M_h^2 \\ M_{\tilde{t}_R}^2 \\ M_{\tilde{Q}_L^3}^2 \end{pmatrix} = -\frac{8\alpha_s}{3\pi} M_3^2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \frac{\lambda_T^2}{8\pi^2} \left(M_{\tilde{Q}_L^3}^2 + M_{\tilde{t}_R}^2 + M_h^2 + A_T^2 \right) \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

decreases

- Gaugino interactions increase the masses. Yukawa interactions decrease the masses.

- Higgs mass becomes negative at EW scale!!

MSSM parameters

Off-diagonal components are generated by RGE even starting with flavor diagonal universal b.c. at high energy scale as in the case of mSUGRA.

$$
\begin{aligned}
\frac{d}{dt}\mathbf{A}_u \;&=\; \mathbf{A}_u\left\{3\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger)+5\mathbf{Y}_u^\dagger\mathbf{Y}_u+\mathbf{Y}_d^\dagger\mathbf{Y}_d-\frac{16}{3}g_3^2-3g_2^2-\frac{13}{15}g_1^2\right\},\\
&+\mathbf{Y}_u\left\{6\mathrm{Tr}(\mathbf{A}_u\mathbf{Y}_u^\dagger)+4\mathbf{Y}_u^\dagger\mathbf{A}_u+2\mathbf{Y}_d^\dagger\mathbf{A}_d+\frac{32}{3}g_3^2M_3+6g_2^2M_2+\frac{26}{15}g_1^2M_1\right\},
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\mathbf{m}_Q^2 \;&=\; (\mathbf{m}_Q^2+2m_{H_u}^2)\mathbf{Y}_u^\dagger\mathbf{Y}_u+(\mathbf{m}_Q^2+2m_{H_d}^2)\mathbf{Y}_d^\dagger\mathbf{Y}_d+[\mathbf{Y}_u^\dagger\mathbf{Y}_u+\mathbf{Y}_d^\dagger\mathbf{Y}_d]\mathbf{m}_Q^2+2\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u\\
&+2\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d+2\mathbf{A}_u^\dagger\mathbf{A}_u+2\mathbf{A}_d^\dagger\mathbf{A}_d\\
&-\frac{32}{3}g_3^2|M_3|^2-6g_2^2|M_2|^2-\frac{2}{15}g_1^2|M_1|^2+\frac{1}{5}g_1^2\mathcal{S},
\end{aligned}
$$

gluino-mediated FCNC

- In the mass basis, the flavor change and CPV occurs at the vertex: SUSY flavor/CP problem

- Approximately, we can think gluino vertex preserves flavor, but the flavor change and CPV occurs at the propagator (mass insertion approximation)