Problem: B^0-$\bar{B}^0$ mixing

In the Standard Model, the B^0-$\bar{B}^0$ mixing diagram occurs at one-loop level as a box diagram.

1. Draw the two box diagrams. Note that there are additional diagrams in the Feynman gauge, where W propagators are replaced by the charged Goldstone components.

2. Show that these diagrams are manifestly finite by power counting. Then we can calculate the diagrams in 4 dimensions.

3. All the extra momenta can be set to be zero because they are small compared to M_W and heavy quark masses in the loop. In the Feynman gauge, show that the amplitude for the B^0-$\bar{B}^0$ transition is given by

$$iT(\bar{b}d \to b\bar{d}) = 2\left(\frac{g}{\sqrt{s}}\right)^4 \sum_{i,j=u,c,t} (V_{ib}V_{id}^*)\left(V_{jb}V_{jd}^*\right) \int \frac{d^4k}{(2\pi)^4} \left(\frac{-i}{k^2 - M_W^2}\right)^2 \times \left(\bar{d}_L \gamma^\mu \frac{/k + m_i}{k^2 - m_i^2} \gamma^\nu b_L\right)^2 \left(\bar{d}_L \gamma_\nu \frac{/k + m_j}{k^2 - m_j^2} \gamma_\mu b_L\right)^2, \tag{1}$$

for the W exchange boxes. Find the amplitudes for the other box diagrams.

4. Calculate the following integrals and find out the explicit form of $A(x_i, x_j)$ and $B(x_i, x_j)$.

$$I^{\mu\nu}(i,j) \equiv \int \frac{d^4k k^\mu k^\nu}{(k^2 - M_W^2)^2 (k^2 - m_i^2)(k^2 - m_j^2)} = -\frac{i\pi^2}{4M_W^2} A(x_i, x_j) g^{\mu\nu}, \tag{2}$$

$$I(i,j) \equiv \int \frac{d^4k}{(k^2 - M_W^2)^2 (k^2 - m_i^2)(k^2 - m_j^2)} = -\frac{i\pi^2}{M_W^4} B(x_i, x_j), \tag{3}$$

where

$$x_i = \frac{m_i^2}{M_W^2}. \tag{4}$$

5. Using the identity

$$\gamma^\mu \gamma^\alpha \gamma^\nu = g^{\mu\alpha}\gamma^\nu + g^{\nu\alpha}\gamma^\mu - g^{\mu\nu}\gamma^\alpha - i\epsilon^{\mu\alpha\nu\beta}\gamma_5\gamma_\beta, \tag{5}$$

show the following relation:

$$[\gamma^\mu\gamma^\alpha\gamma^\nu(1-\gamma_5)/2]\cdots[\gamma_\nu\gamma_\alpha\gamma_\mu(1-\gamma_5)/2] = 4[\gamma^\alpha(1-\gamma_5)/2]\cdots[\gamma_\alpha(1-\gamma_5)/2]. \tag{6}$$

6. Find out the transition amplitude

$$T(\bar{b}d \to b\bar{d}) = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi \sin^2\theta_W}(\bar{d}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu s_L)\sum_{i,j=u,c,t}(V_{ib}V_{id}^*)\left(V_{jb}V_{jd}^*\right)J(x_i,x_j) \tag{7}$$

with the explicit form of $J(x_i, x_j)$.

7. Show that the effective Lagrangian is given by

$$\mathcal{L}_{\rm eff}^{\Delta B=2} = -\frac{G_F}{\sqrt{s}}\frac{\alpha}{16\pi}\left(\frac{1}{M_W\sin\theta_W}\right)^2\lambda\mathcal{O}_{JJ}, \tag{8}$$

where

$$\mathcal{O}_{JJ} = [\bar{d}\gamma^\mu(1-\gamma_5)s][\bar{d}\gamma_\mu(1-\gamma_5)s], \tag{9}$$

$$\lambda = M_W^2\sum_{i,j=u,c,t}(V_{ib}V_{id}^*)\left(V_{jb}V_{jd}^*\right)A(x_i,x_j). \tag{10}$$