Flavour Physics 2: CP violation

Chaehyun Yu

KOREA INSTITUTE FOR ADVANCED STUDY

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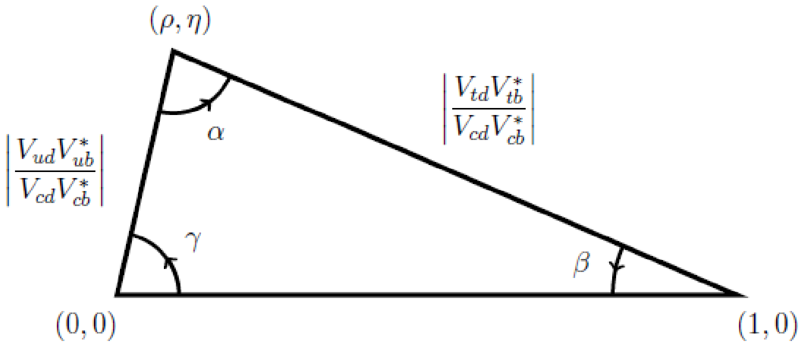
Jarlskog invariant



$$\text{(area)}=\left(\frac{1}{2}\text{(base)}\times\text{(height)}\right)\times\text{(scale factor)}=\frac{1}{2}\mathrm{Im}\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)\times\left|V_{cd}V_{cb}^*\right|^2$$

$$=\frac{1}{2}\mathrm{Im}\left(-V_{ud}V_{ub}^*V_{cd}^*V_{cb}\right)=\frac{1}{2}\mathrm{Im}\left(c_{12}c_{13}s_{13}e^{i\delta}\left(s_{12}c_{23}+c_{12}s_{23}s_{13}e^{-i\delta}\right)s_{23}c_{13}\right)$$

$$=\frac{1}{2}c_{12}c_{23}c_{13}^2s_{12}s_{23}s_{13}\sin\delta$$

- basis-independent quantity that identifies CP violation = Jarlskog invariant

$$\mathrm{Im}\left[V_{ij}V_{kl}V_{il}^*V_{kj}^*\right]=J\sum_{n,m}\epsilon_{ikn}\epsilon_{jlm}\quad\text{(no sum in } i,j,k,l\text{)}$$

$$J=c_{12}c_{23}c_{13}^2s_{12}s_{23}s_{13}\sin\delta\sim\lambda^6A^2\eta\quad\text{(in the SM)}$$

A brief history of CPV

- 1964 – 2000
  - $|\varepsilon| = (2.284 \pm 0.014) \times 10^{-3}$; $\mathcal{R}e(\varepsilon'/\varepsilon) = (1.67 \pm 0.26) \times 10^{-3}$
- 2000 – 2011
  - $S_{\psi K_S} = +0.67 \pm 0.02$
  - $S_{\phi K_S} = +0.56 \pm 0.18$, $S_{\eta' K_S} = +0.59 \pm 0.07$, $S_{\pi^0 K_S} = +0.57 \pm 0.17$, $S_{f_0 K_S} = +0.62 \pm 0.12$
  - $S_{K^+K^-K_S} = -0.82 \pm 0.07$, $S_{K_S K_S K_S} = +0.74 \pm 0.17$
  - $S_{\pi^+\pi^-} = -0.65 \pm 0.07$, $C_{\pi^+\pi^-} = -0.38 \pm 0.06$
  - $S_{\psi\pi^0} = -0.93 \pm 0.15$, $S_{DD} = -0.89 \pm 0.26$, $S_{D^*D^*} = -0.77 \pm 0.14$
  - $\mathcal{A}_{K^\mp\rho^0} = +0.37 \pm 0.11$, $\mathcal{A}_{\eta K^\mp} = -0.37 \pm 0.09$, $\mathcal{A}_{f_2 K^\mp} = -0.68 \pm 0.20$
  - $\mathcal{A}_{K^\mp\pi^\pm} = -0.098 \pm 0.012$, $\mathcal{A}_{\eta K^{*0}} = +0.19 \pm 0.05$
  - ...

CP violation in K decay

- Two kinds of neutral K mesons and make isospin doublet with charged K

$$K^0 = d\bar{s}\,(S=+1) \qquad \bar{K}^0 = s\bar{d}\,(S=-1)$$

$$K^+ = u\bar{s} \qquad K^- = s\bar{u}$$

- CP properties of neutral K mesons are

$$\mathrm{CP}\left|K^0\right\rangle = e^{i\xi_{\mathrm{CP}}}\left|\bar{K}^0\right\rangle = \eta_{\mathrm{CP}}\left|\bar{K}^0\right\rangle$$

$$\mathrm{CP}\left|\bar{K}^0\right\rangle = e^{-i\xi_{\mathrm{CP}}}\left|K^0\right\rangle = \eta_{\mathrm{CP}}^*\left|K^0\right\rangle$$

$\xi_{\mathrm{CP}} = 0$ by convention

- If CP is conserved in weak interactions, CP is a good quantum number and CP eigenstates can be defined by

$$\left|K_1\right\rangle = \left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right)/\sqrt{2}$$

$$\left|K_2\right\rangle = \left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)/\sqrt{2}$$

$$\mathrm{CP}\left|K_1\right\rangle = \left|K_1\right\rangle$$

$$\mathrm{CP}\left|K_2\right\rangle = -\left|K_2\right\rangle$$

- They decay to 2π or 3π, whose CP is +1 and -1, respectively.

$$K_1 \to 2\pi,\ K_2 \to 3\pi$$

CP($\pi\pi$)

- $\pi^+\pi^-$ $J^P(\pi^\pm) = 0^-$

  $P\pi = -\pi$ and $P(2\pi) = (-1)^l$ where l is the reletive angular momentum of 2π

  C operation exchange $\pi^+ \leftrightarrow \pi^-$

  $\Rightarrow$ $CP(\pi^+\pi^-) = (-1)^{2l+2} = +1$

- $\pi^0\pi^0$ $J^{PC}(\pi^0) = 0^{-+}$

  $CP\pi^0 = -\pi^0$

  Bose statistics forces l of $2\pi^0$ to be even.

  $\Rightarrow$ $CP(\pi^0\pi^0) = (-1)^{l+2} = +1$

$m_K \sim 497$ MeV
$m_\pi \sim 140$ MeV

- K_1 decays into $\pi\pi$, but K_2 does not.
  - The lifetime of K_1 is short (or its decay width is large) because of mass difference between K_1 and $\pi\pi$.

CP($\pi\pi\pi$)

- $\pi^+\pi^-\pi^0$

$$\mathrm{CP}(\pi^+\pi^-\pi^0) = (-1)^3(-1)^{2l}(-1)^L = (-1)^{2l+L+3}$$

l : the orbital angular momentum of $\pi^+\pi^-$

L : the orbital angular momentum of π^0 and $(\pi^+\pi^-)$

$l = L$ because the spin of K is 0.

$$\mathrm{CP}(\pi^+\pi^-\pi^0) = (-1)^{3l+3}$$

- The dominant component is CP=-1.

- $\pi^0\pi^0\pi^0$

$$\mathrm{CP}(\pi^0\pi^0\pi^0) = (-1)^3(-1)^l(-1)^L = (-1)^{l+L+3}$$

Bose statistics forces l of $2\pi^0$ to be even.

$$\Rightarrow \mathrm{CP}(\pi^0\pi^0\pi^0) = (-1)^{3+2l} = -1$$

$m_K \sim 497$ MeV

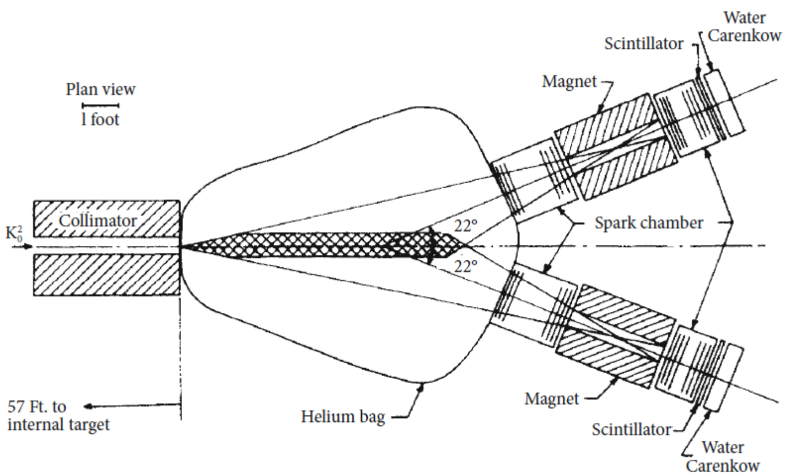
$m_\pi \sim 140$ MeV

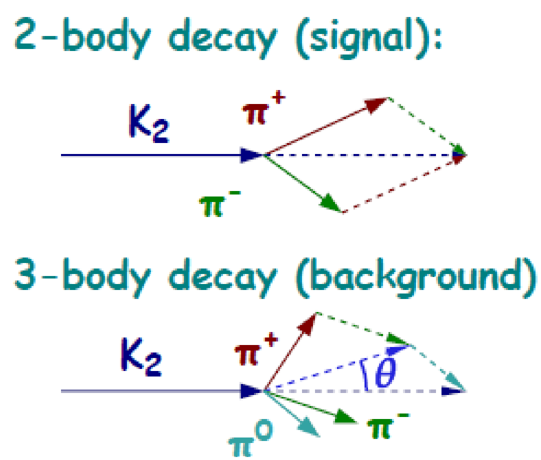
- K_2 decays into $\pi\pi\pi$.

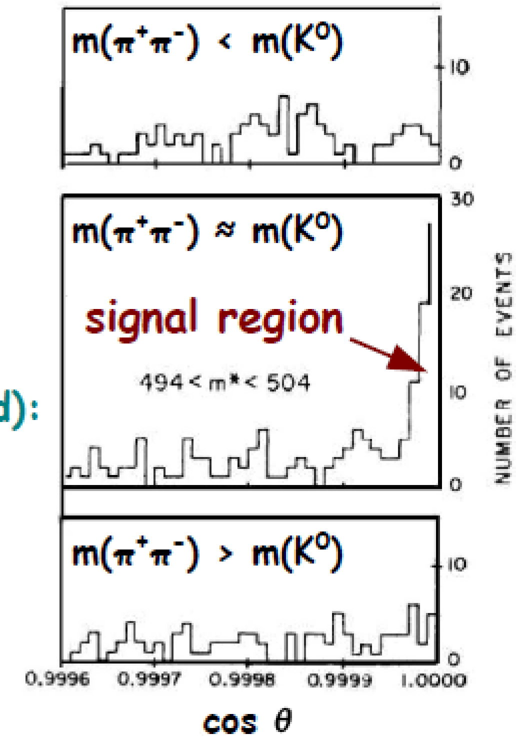
- The lifetime of K_2 is larger than K_1 because of phase space suppression in $\pi\pi\pi$ decay.

Discovery of CP violation

- Observation of $K_2 \to \pi^+\pi^-$ (Christension,Cronin,Fitch,Turlay,1964)
  - produce K^0 (mix of K_1 and K_2) and let them propagate in vacuum tube long enough for K_1 component to decay away → pure K_2 beam
  - search for CP-forbidden decay, $K_2 \to \pi^+\pi^-$







  - find excess of 56 events ? ⇒ BR $(K_2 \to \pi^+\pi^-) \approx 2 \times 10^{-3}$

Kaon CP-violation observables

- Observed neutral Kaons are not eigenstates of CP.

$$|K_S\rangle = \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}(|K_1\rangle + \tilde{\epsilon}|K_2\rangle) \longrightarrow \pi\pi$$
$$|K_L\rangle = \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}(|K_2\rangle + \tilde{\epsilon}|K_1\rangle) \longrightarrow \pi\pi\pi$$

Br	$\pi^0\pi^0$	$\pi^+\pi^-$	$\pi^+\pi^-\pi^0$	$\pi^0\pi^0\pi^0$
K_S	$30.69(5)\,\%$	$69.20(5)\,\%$	$3.5(1.1)\times10^{-7}$	$<1.2\times10^{-7}$
K_L	$8.64(6)\times10^{-4}$	$1.967(10)\times10^{-3}$	$12.54(5)\,\%$	$19.52(12)\,\%$

Discussion. Br($K_S\rightarrow2\pi$)~0.998, but Br($K_L\rightarrow3\pi$)~0.32. In the K_L decays, the semileptonic decay modes are dominant. Why is the branching ratio of the semileptonic decay of K_S quite small?

Kaon CP-violation observables

$$\eta_{+-} \equiv \frac{\mathcal{A}\left(K_L \to \pi^+\pi^-\right)}{\mathcal{A}\left(K_S \to \pi^+\pi^-\right)} = \varepsilon + \varepsilon'$$

$$\eta_{00} \equiv \frac{\mathcal{A}\left(K_L \to \pi^0\pi^0\right)}{\mathcal{A}\left(K_S \to \pi^0\pi^0\right)} = \varepsilon - 2\,\varepsilon'$$

$$\varepsilon \simeq \tilde{\epsilon} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0} = \tilde{\epsilon}_{\mathrm{WY}}$$

Indirect CP violation

$$\varepsilon' \simeq \frac{i}{\sqrt{2}}\, e^{i(\delta_2-\delta_0)}\, \frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]$$

Direct CP violation

$$\left|\pi^+\pi^-\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\pi^+,\pi^-\right\rangle + \left|\pi^-,\pi^+\right\rangle\right) = \frac{1}{\sqrt{3}}\left|2,0\right\rangle + \frac{\sqrt{2}}{\sqrt{3}}\left|0,0\right\rangle$$

$$\left|\pi^0\pi^0\right\rangle = \frac{\sqrt{2}}{\sqrt{3}}\left|2,0\right\rangle - \frac{1}{\sqrt{3}}\left|0,0\right\rangle$$

Indirect CP violation

$$\varepsilon = f\left(\hat{B}_K, V_{\mathrm{CKM}}, m_c, m_t, \ldots\right) \xrightarrow{\text{Theory}} |\varepsilon| = 1.90(26) \times 10^{-3}$$

$$|\varepsilon|_{\exp} = 2.228(11) \times 10^{-3} \qquad \arg\left(\varepsilon\right)_{\exp} = 44(7)^\circ$$

$$\frac{\Gamma\left(K_L \to \pi^-\ell^+\nu_\ell\right) - \Gamma\left(K_L \to \pi^+\ell^-\overline{\nu}_\ell\right)}{\Gamma\left(K_L \to \pi^-\ell^+\nu_\ell\right) + \Gamma\left(K_L \to \pi^+\ell^-\overline{\nu}_\ell\right)} = \frac{2\,\mathrm{Re}\,\varepsilon}{1+|\varepsilon|^2}$$

Kaon CP-violation observables

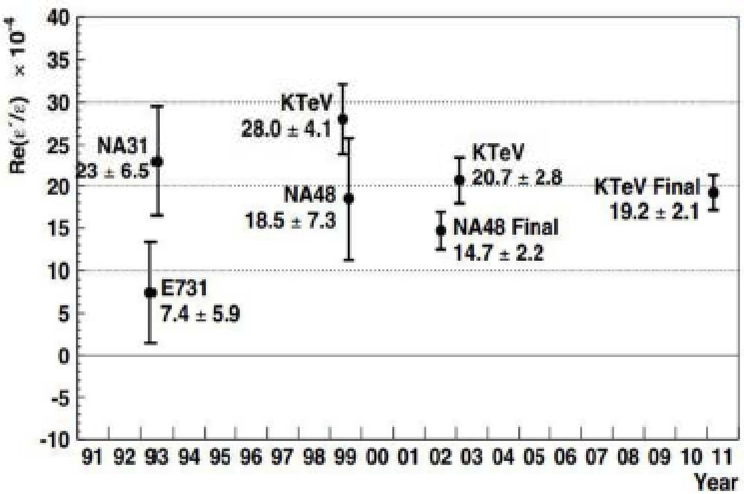
Direct CP violation

$$\varepsilon' \simeq \frac{i}{\sqrt{2}} e^{i(\delta_2-\delta_0)} \frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right)$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{exp}} = 16.8\,(1.4) \times 10^{-4}$$

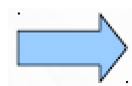
$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{theo\ SM}} = 19\,(11) \times 10^{-4}$$



$$\arg(\varepsilon') = \chi_2 - \chi_0 + \frac{\pi}{2} = 42.5\,(9)^\circ$$

Importance of B physics

- Large mass m_b
  - Variety of final states to decay to
  - determination of several CKM elements
  - allows us to use expansion in $1/m_b$ to estimate non-perturbative effects systematically

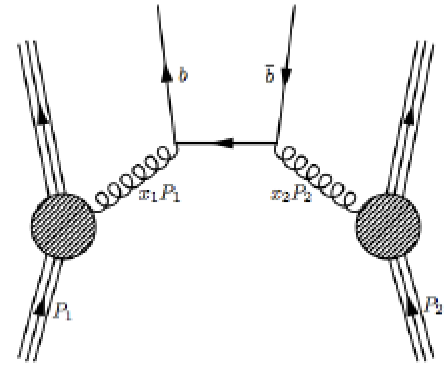
- CPV phase in V_{ub}  CPV effects

- Rare decays of B mesons due to loop suppression
  - sensitive to New Physics

- K and D physics have relatively large theoretical uncertainties

$b\bar{b}$ production mechanism

Hadron colliders: *e.g.* Tevatron, LHC



gluon-gluon fusion is the leading mechanism at LHCb

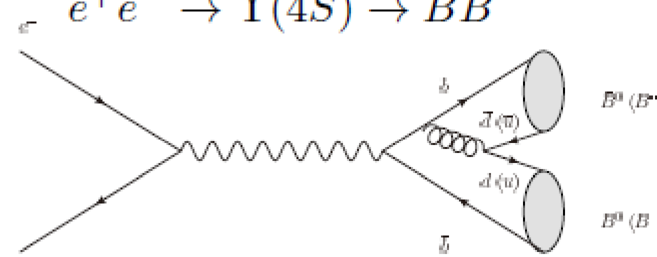
- $b\bar{b}$ from QCD mediated process
- incoherent production of *b* hadrons
- not defined hadron energy

Tevatron $\sigma(b\bar{b}) \sim 10\mu\text{b}$ at $p\bar{p}$ collisions, $E_{CM} = 1.96$ TeV

LHCb $\sigma(b\bar{b}) \sim 150\mu\text{b}$ at pp collisions, $E_{CM} = 14$ TeV

Electron colliders: *e.g.* *B* factories

$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$$



- coherent production of $B\bar{B}$ at E_{CM}=10.58 GeV
- well defined *B* meson energy

$\sigma(B\bar{B}) \sim 1.1\text{nb}$ at e^+e^- collisions, $E_{CM} = 10.58$ GeV

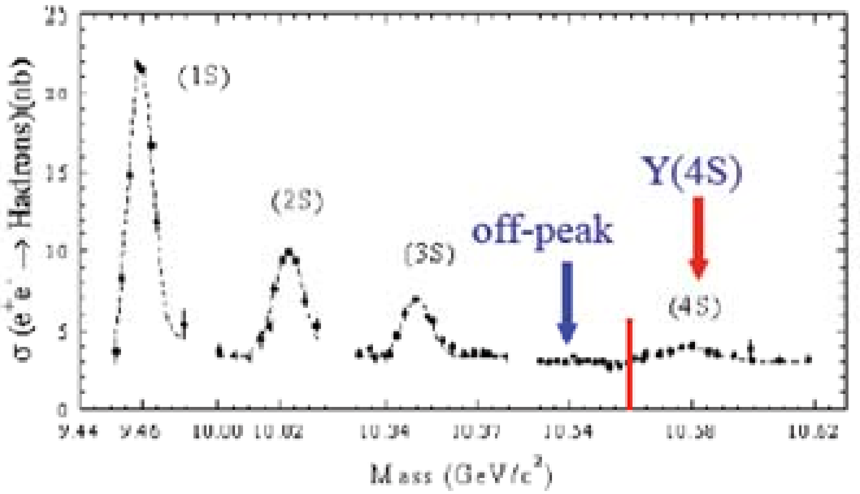
B meson production at B factories

- Collide electrons and positrons at $\sqrt{s}$=10.58 GeV/c²

resonance at q=m $\sim \frac{1}{q^2 - m^2 + im\Gamma}$

$e^+e^- \to$	Cross-section (nb)
$b\bar{b}$	1.05
$c\bar{c}$	1.30
$s\bar{s}$	0.35
$d\bar{d}$	0.35
$u\bar{u}$	1.39
$\tau^+\tau^-$	0.92
$\mu^+\mu^-$	1.16
e^+e^-	~ 40

many types of interaction occur.



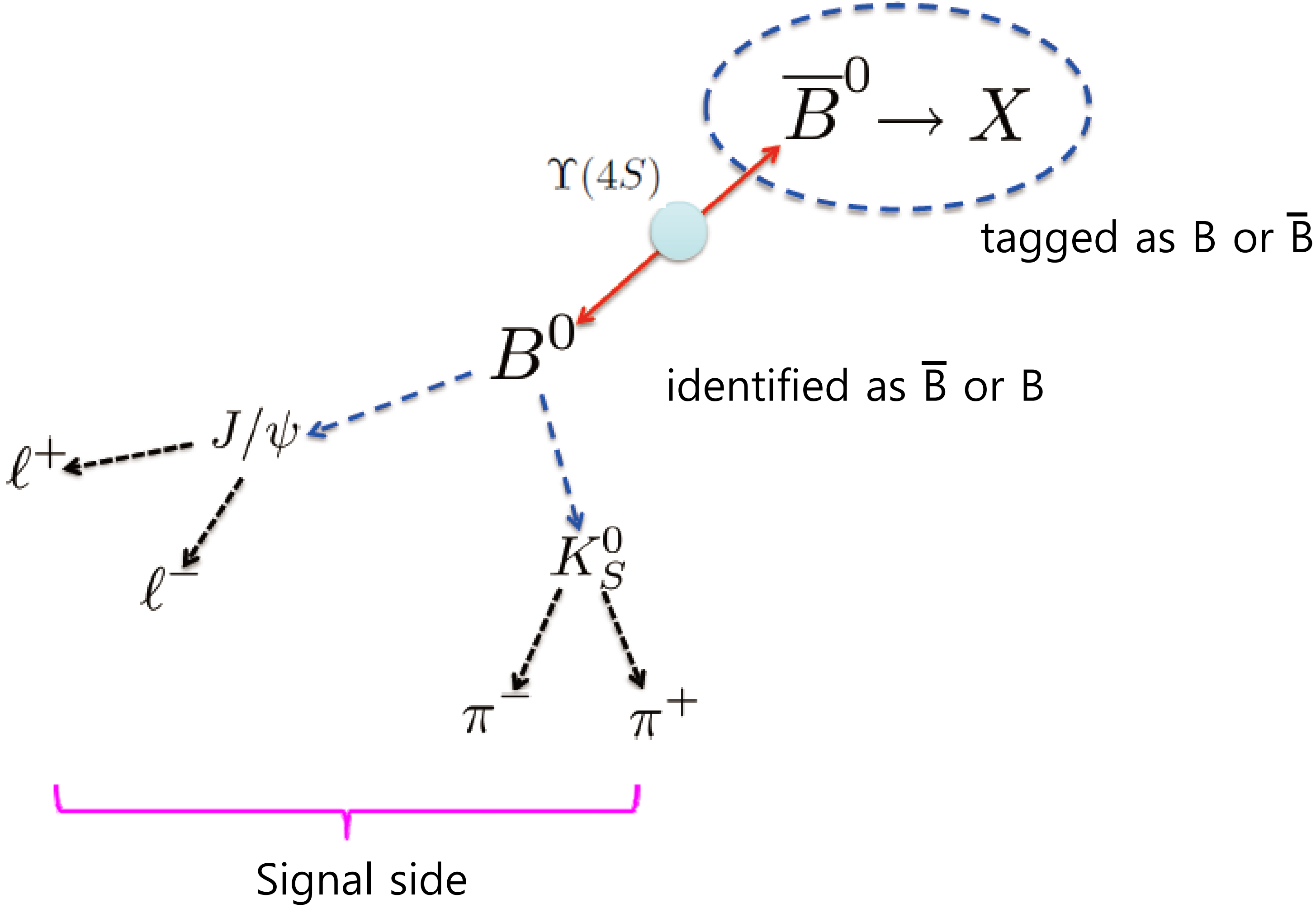
- We are interested in $e^+e^- \to \Upsilon(4S) \to B\bar{B}$ for B physics

$$B(\Upsilon(4S) \to B\bar{B}) \sim 100\% \qquad \frac{\mathcal{B}(\Upsilon(4S) \to B^0\bar{B}^0)}{\mathcal{B}(\Upsilon(4S) \to B^+B^-)} \simeq 1$$

$\sigma_{b\bar{b}} \approx$ 1 nb $\Rightarrow$ with 1 fb^{-1} produce 10^6 $B\bar{B}$ pairs

N.B. $\Upsilon(5S) \to B_s\bar{B}_s$ is possible, but it is not the main target of B factories

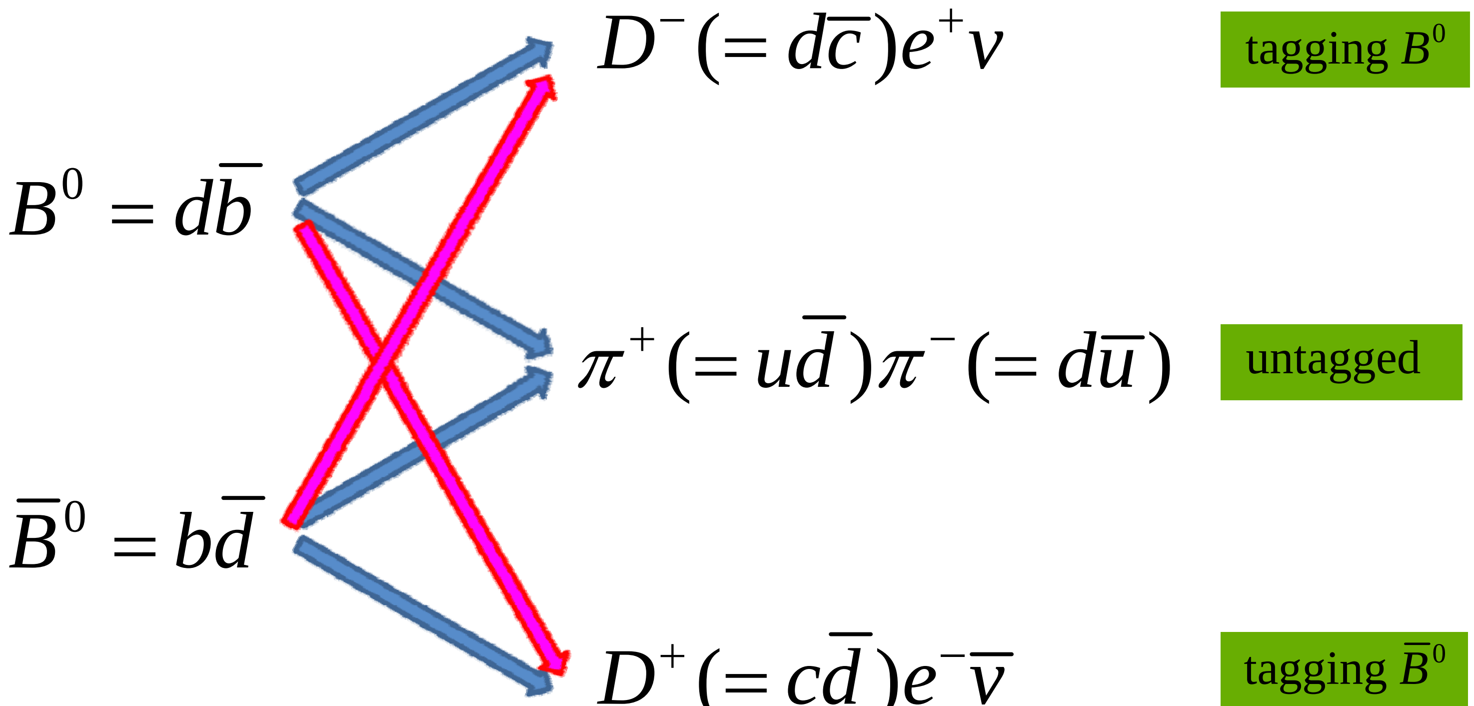
How to identify B or $\overline{B}$



Tagging B or $\overline{B}$

- Charged B mesons can easily be tagged by measuring the charge of its decay products.

- Neutral B mesons



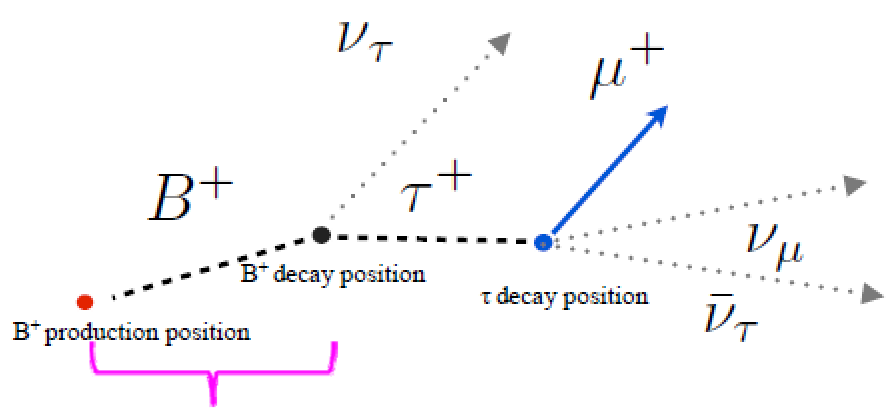
Travel distance of B meson

For a relativistic particle, the travel distance:

$$d = (\beta c\ \tau)\gamma \approx (300\ \mu m)(\frac{\tau}{10^{-12}\ s})\ \gamma$$

The lifetime of B meson $\tau_B \sim 10^{-12} s$

$$B^+ \to \tau^+ \nu_\tau$$
$$\searrow \mu^+ \nu_\mu \bar{\nu}_\tau$$



$\sim$ O(100) μm

b-hadrons:

$\tau \approx 1.5$ ps, $c\tau \approx 450$ µm
at p = 20 GeV → dist ≈ 1.8 mm
$m_b \approx 4.2$ GeV

c-hadrons:

D^+: ≈ 312 µm, D^0: ≈ 123 µm
$m_c \approx 1.9$ GeV

Why need asymmetric B factories?

- Many observables require measurement of time-dependent asymmetries
- But, B mesons are produced almost at rest in the $\Upsilon(4S)$ rest frame

  $\Rightarrow$ difficult to resolve the vertex of B decays

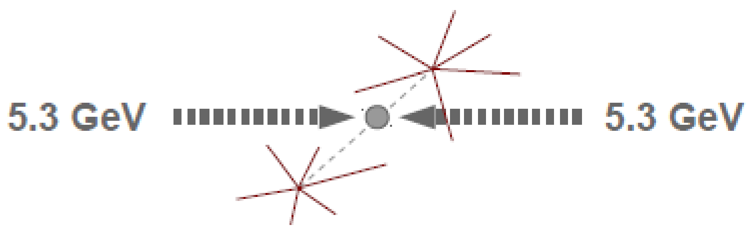
- $\Upsilon(4S)$ decays produce $B\bar{B}$ pairs in a coherent quantum state

  $J_{\Upsilon(4s)} = 1$, $J_B = 0 \Rightarrow L_{B\bar{B}} = 1 \Rightarrow$ **wave function anti-symmetric**

- Bose-Einstein statistics implies flavour wave-function must be anti-symmetric

  $\Rightarrow$ $B\bar{B}$ must oscillate in phase until one of them decays

- $\Upsilon(4S)$ is produced at rest at a symmetric collider



back-to-back
cannot construct production vertex

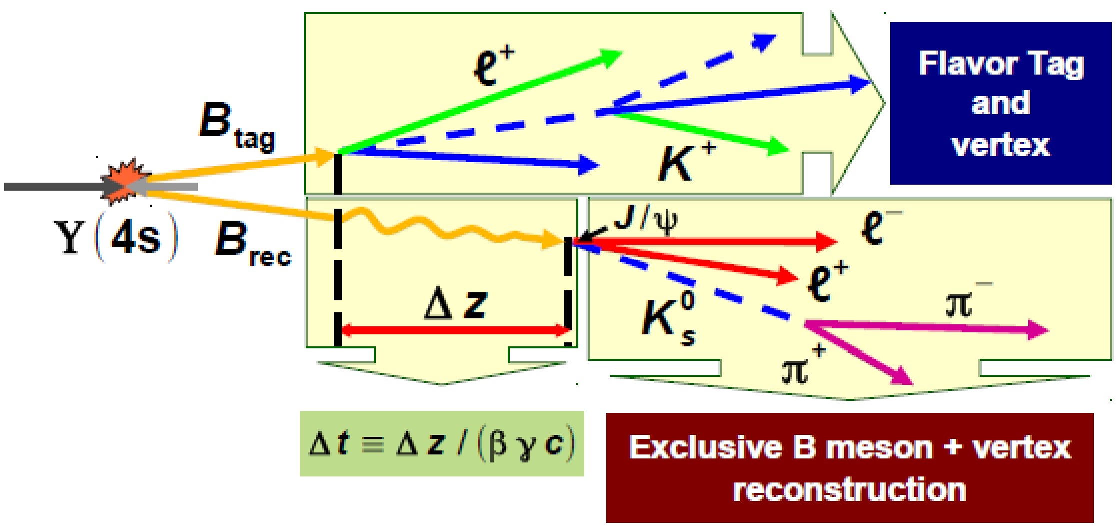
Asymmetric B factories

- PEP-II: 9 GeV e⁻ + 3.1 GeV e⁺

$\beta\gamma = 0.56$
$\langle \Delta z \rangle \approx 260\ \mu m$

- KEKB: 8 GeV e⁻ + 3.5 GeV e⁺

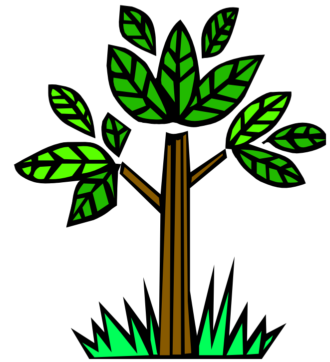
$\beta\gamma = 0.425$
$\langle \Delta z \rangle \approx 200\ \mu m$



Home discussion

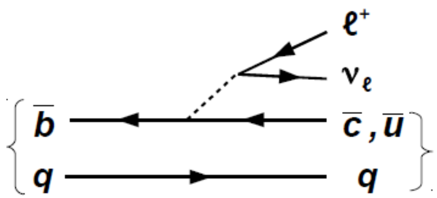
This is a counter example of the famous EPR paradox. Discuss the EPR paradox.

Classification of B decays



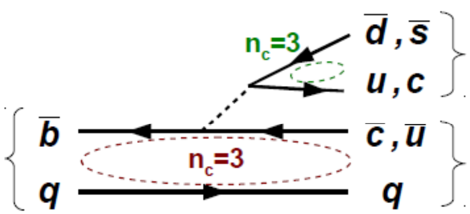
Tree decays

- semileptonic



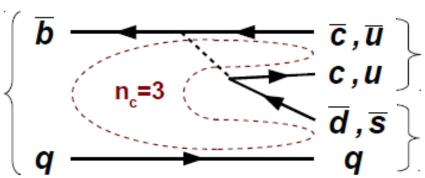
$Br \sim 11\%$ per each lepton(e, μ, τ)

- color-allowed tree



$Br \sim$ up to a few %

- color-suppressed tree



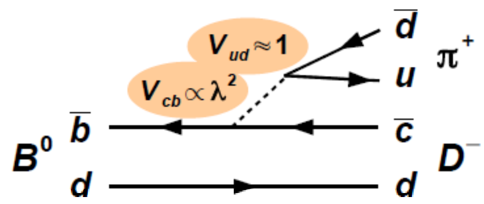
$Br \sim 1/10$ of color-allowed tree

nonleptonic

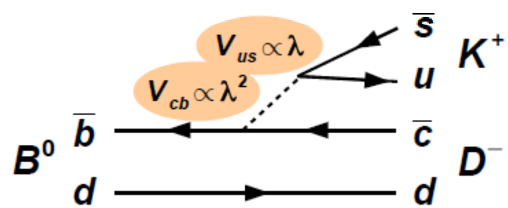
Classification of B decays

Tree decays by orders of λ

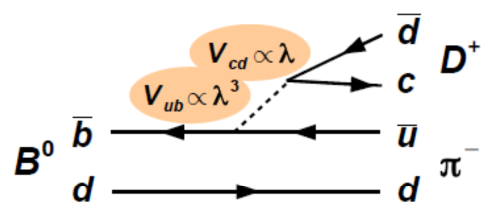
- Cabibbo-favored (λ^2)



- Cabibbo-suppressed (λ^3)



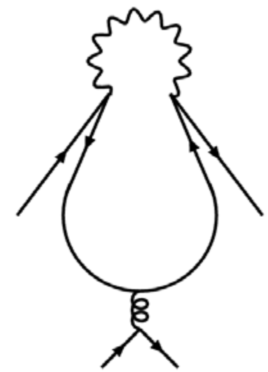
- doubly Cabibbo-suppressed (λ^4)

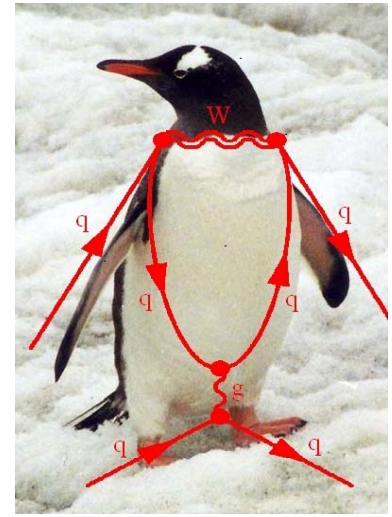


Classification of B decays

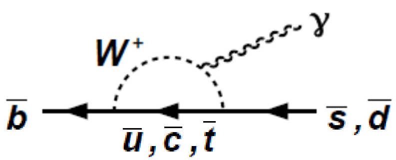
Penguin decays

~FCNC, loop suppressed, sensitive to new physics

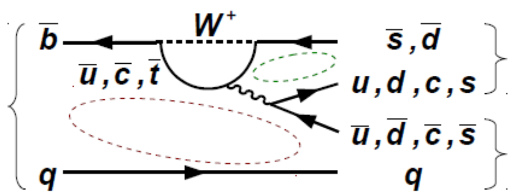




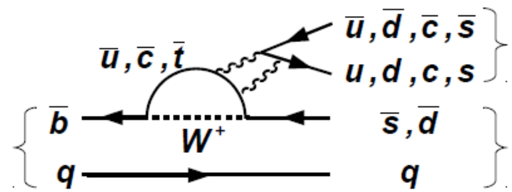
- radiative



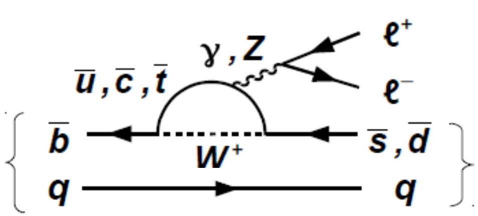
- (internal) gluonic penguin



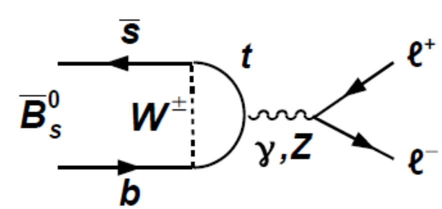
- external gluonic penguin



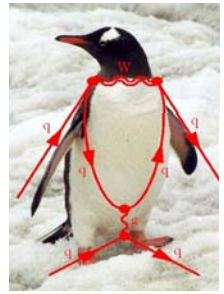
- electroweak penguin



- leptonic



Penguin diagram



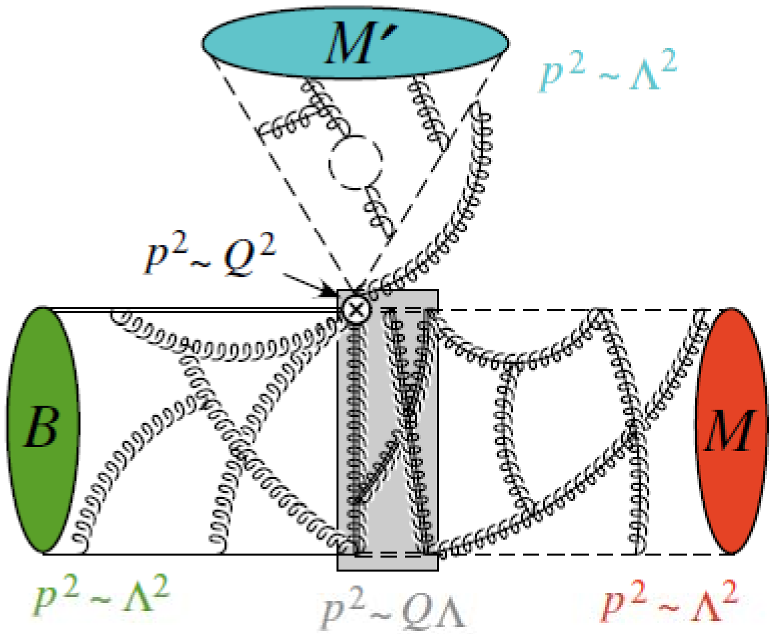
Origin of the name [edit]

John Ellis was the first to refer to a certain class of Feynman diagrams as **penguin diagrams**, due in part to their shape, and in part to a legendary bar-room bet with Melissa Franklin. According to John Ellis:[2]

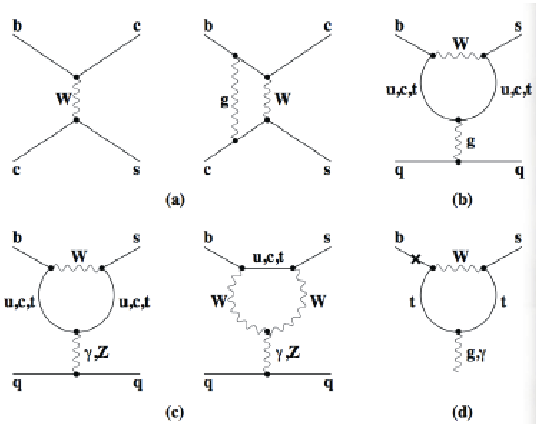
> Mary K. [Gaillard], Dimitri [Nanopoulos] and I first got interested in what are now called penguin diagrams while we were studying CP violation in the Standard Model in 1976... The penguin name came in 1977, as follows.
>
> In the spring of 1977, Mike Chanowitz, Mary K and I wrote a paper on GUTs predicting the b quark mass before it was found. When it was found a few weeks later, Mary K, Dimitri, Serge Rudaz and I immediately started working on its phenomenology. That summer, there was a student at CERN, Melissa Franklin who is now an experimentalist at Harvard. One evening, she, I, and Serge went to a pub, and she and I started a game of darts. We made a bet that if I lost I had to put the word penguin into my next paper. She actually left the darts game before the end, and was replaced by Serge, who beat me. Nevertheless, I felt obligated to carry out the conditions of the bet.
>
> For some time, it was not clear to me how to get the word into this b quark paper that we were writing at the time. Then, one evening, after working at CERN, I stopped on my way back to my apartment to visit some friends living in Meyrin where I smoked some illegal substance. Later, when I got back to my apartment and continued working on our paper, I had a sudden flash that the famous diagrams look like penguins. So we put the name into our paper, and the rest, as they say, is history.

Realistic B decay?

- previous diagrams are not realistic. More realistic ones include hadronization and huge number of soft gluons.



Effective Hamiltonian in one slide



- many scales (m_b,m_W,Λ_{QCD}) are involved in B decays
- large logarithms appear in the calculation

$$\ln\frac{M_W^2}{\mu^2},\left(\ln\frac{M_W^2}{\mu^2}\right)^2,\ldots$$

- perturbative calculation might be broken because of the large logarithms

- go to the M_W scale, where the logarithms disappers
- Physical process should be calculated at the m_b scale
- use the operator product expansion (OPE)
- the large logarithms are summed up in Wilson coefficients (RGE required)

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{cb} \left[C_1(\mu) O_1 + C_2(\mu) O_2\right]$$

$$\langle K^- D^+|\mathcal{H}_{\rm eff}|\bar{B}_d^0\rangle = \frac{G_F}{\sqrt{2}} V_{us}^* V_{cb}\Big[a_1 \langle K^- D^+|(\bar{s}_\alpha u_\alpha)_{\rm V\text{-}A}(\bar{c}_\beta b_\beta)_{\rm V\text{-}A}|\bar{B}_d^0\rangle + 2\, C_1 \langle K^- D^+|(\bar{s}_\alpha\, T^a_{\alpha\beta}\, u_\beta)_{\rm V\text{-}A}(\bar{c}_\gamma\, T^a_{\gamma\delta}\, b_\delta)_{\rm V\text{-}A}|\bar{B}_d^0\rangle\Big],$$

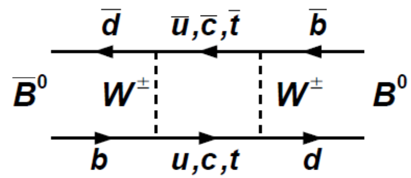
- short distance physics and long distance physics are well seperated

- how to calculate the matrix elements? →Naïve Factorization,QCDF,PQCD,SCET,...

$B^0 - \overline{B}^0$ Mixing

- Neutral meson systems: $K^0\overline{K}^0, D^0\overline{D}^0, B^0\overline{B}^0, B_s^0\overline{B}_s^0$

  - flavour mixing through box diagrams → coupled system



$$|\psi(t)\rangle = a(t)\,|B^0\rangle + b(t)\,|\overline{B}^0\rangle$$

  - time evolution can be described by a two-component Schrödinger equation with an effective Hamiltonian H

$$-i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = H \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

  - mesons can decay → unitarity not conserved → H is not Hermitian
  - decompose H into two Hermitian parts

$$\left.\begin{aligned} M &\equiv \frac{1}{2}\left(H + H^{\dagger}\right) \\ \frac{\Gamma}{2} &\equiv \frac{1}{2i}\left(H - H^{\dagger}\right) \end{aligned}\right\} \Rightarrow H = M - \frac{i}{2}\Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

"dispersive" part **"absorptive" part**

  - off-diagonal elements of M and Γ describes meson-antimeson mixing

$B^0 - \overline{B}^0$ Mixing

- M and Γ are Hermitian → $M_{21} = M_{12}^*$ and $\Gamma_{21} = \Gamma_{12}^*$

- Assume CPT conservation (i.e. meson and its antimeson have the same mass and lifetime)

$$H = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

$$\boxed{\begin{array}{ccccc} M & \equiv & M_{11} & = & M_{22} \\ \Gamma & \equiv & \Gamma_{11} & = & \Gamma_{22} \end{array}}$$

- diagonalize H to determine Eigenvalues and Eigenstates

$$\lambda_{L,H} = M_{L,H} - i\frac{\Gamma_{L,H}}{2} = \frac{H_{11} + H_{22}}{2} \pm \sqrt{H_{12}H_{21}}$$

$$|B_L\rangle = p|B^0\rangle + q|\overline{B}^0\rangle = \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}\left[(1+\tilde{\epsilon})|B^0\rangle + (1-\tilde{\epsilon})|\overline{B}^0\rangle\right]$$

$$|B_H\rangle = p|B^0\rangle - q|\overline{B}^0\rangle = \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}\left[(1+\tilde{\epsilon})|B^0\rangle - (1-\tilde{\epsilon})|\overline{B}^0\rangle\right]$$

$$|p|^2 + |q|^2 = 1$$

$$\frac{q}{p} = \frac{1-\tilde{\epsilon}}{1+\tilde{\epsilon}} = \sqrt{\frac{H_{21}}{H_{12}}} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}$$

$B^0 - \overline{B}^0$ Mixing

- B_H and B_L have well-defined masses and decay widths

$$|B_H(t)\rangle = \left(p \cdot |B^0\rangle - q \cdot |\overline{B}^0\rangle \right) \cdot e^{-i m_H t} \cdot e^{-\Gamma_H t/2}$$
$$|B_L(t)\rangle = \left(p \cdot |B^0\rangle + q \cdot |\overline{B}^0\rangle \right) \cdot e^{-i m_L t} \cdot e^{-\Gamma_L t/2}$$

note: B_H and B_L are not particle/antiparticle, can have different mass and lifetime

Home discussion

1. H is not Hermitian, which means that the probabilities are not conserved. In quantum mechanics, everything we measure is real and Hermitian matrices have real eigenvalues. Discuss why non Hermitian Hamilatonian is allowed in quantum field theory and why this does not conflict with quantum mechanics.

$B^0 - \bar{B}^0$ Mixing

- Time evolution of initially pure flavour states
  - mesons are not produced in mass eigenstates, but in pure flavour states

    $|B^0\rangle$ or $|\bar{B}^0\rangle$ at t=0

  - They are decomposed into a super position of mass eigenstates

    $$\left|B^0_{t=0}\right\rangle = \frac{1}{2p}\left(\left|B_H\right\rangle + \left|B_L\right\rangle\right) \quad \text{and} \quad \left|\bar{B}^0_{t=0}\right\rangle = \frac{1}{2q}\left(\left|B_L\right\rangle - \left|B_H\right\rangle\right)$$

    $$a_H(t) = a_H(0)e^{-iM_H t}e^{-\frac{1}{2}\Gamma_H t} \qquad a_L(t) = a_L(0)e^{-iM_L t}e^{-\frac{1}{2}\Gamma_L t}$$

  - propagate according to the solution of the Schrödinger equation

    $$\left|B^0_{t=0}(t)\right\rangle = \frac{1}{2p}\left(\left|B_H\right\rangle e^{-im_H t}e^{-\Gamma_H t/2} + \left|B_L\right\rangle e^{-im_L t}e^{-\Gamma_L t/2}\right)$$

    $$\left|\bar{B}^0_{t=0}(t)\right\rangle = \frac{1}{2q}\left(\left|B_L\right\rangle e^{-im_L t}e^{-\Gamma_L t/2} - \left|B_H\right\rangle e^{-im_H t}e^{-\Gamma_H t/2}\right)$$

  - time evolution of initially pure flavour states

    $$\left|B^0_{t=0}(t)\right\rangle = g_+(t)\cdot\left|B^0\right\rangle + \frac{q}{p}\cdot g_-(t)\cdot\left|\bar{B}^0\right\rangle$$

    $$\left|\bar{B}^0_{t=0}(t)\right\rangle = g_+(t)\cdot\left|\bar{B}^0\right\rangle + \frac{p}{q}\cdot g_-(t)\cdot\left|B^0\right\rangle$$

$$\begin{aligned} \bar{m} &\equiv (m_H + m_L)/2 \\ \bar{\Gamma} &\equiv (\Gamma_H + \Gamma_L)/2 \\ \Delta m &\equiv m_H - m_L > 0 \\ \Delta\Gamma &\equiv \Gamma_H - \Gamma_L \end{aligned}$$

with $g_\pm(t) = \frac{1}{2}e^{-i\bar{m}t}e^{-\frac{\bar{\Gamma}t}{2}}\left(e^{i\frac{\Delta m\, t}{2}}e^{+\frac{\Delta\Gamma\, t}{4}} \pm e^{-i\frac{\Delta m\, t}{2}}e^{-\frac{\Delta\Gamma\, t}{4}}\right)$

$B^0 - \overline{B}^0$ Mixing

- mixing probabilities

$$P(B^0 \to B^0, t) = \frac{1}{2} \cdot e^{-\Gamma t} \cdot \left\{ \cosh\left(\frac{\Delta\Gamma}{2} t\right) + \cos(\Delta m\, t) \right\}$$

$$P(\overline{B}^0 \to \overline{B}^0, t) = P(B^0 \to B^0, t)$$

$$P(B^0 \to \overline{B}^0, t) = \frac{1}{2} \cdot \left|\frac{q}{p}\right|^2 \cdot e^{-\Gamma t} \cdot \left\{ \cosh\left(\frac{\Delta\Gamma}{2} t\right) - \cos(\Delta m\, t) \right\}$$

$$P(\overline{B}^0 \to B^0, t) = \frac{1}{2} \cdot \left|\frac{p}{q}\right|^2 \cdot e^{-\Gamma t} \cdot \left\{ \cosh\left(\frac{\Delta\Gamma}{2} t\right) - \cos(\Delta m\, t) \right\}$$

- Time-dependent asymmetries

$$a_{mix}(t) \equiv \frac{N(B^0 \to B^0) - N(B^0 \to \overline{B}^0)}{N(B^0 \to B^0) + N(B^0 \to \overline{B}^0)} = \frac{\cos(\Delta m \cdot t) + \delta \cdot \cosh(\Delta\Gamma \cdot t/2)}{\cosh(\Delta\Gamma \cdot t/2) + \delta \cdot \cos(\Delta m \cdot t)}$$

$$\overline{a}_{mix}(t) \equiv \frac{N(\overline{B}^0 \to \overline{B}^0) - N(\overline{B}^0 \to B^0)}{N(\overline{B}^0 \to \overline{B}^0) + N(\overline{B}^0 \to B^0)} = \frac{\cos(\Delta m \cdot t) - \delta \cdot \cosh(\Delta\Gamma \cdot t/2)}{\cosh(\Delta\Gamma \cdot t/2) - \delta \cdot \cos(\Delta m \cdot t)}$$

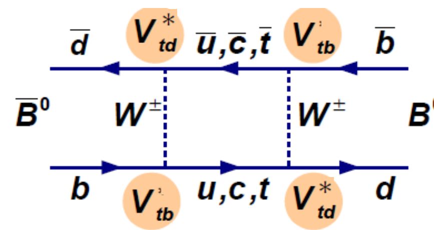
$\delta \neq 0 \Leftrightarrow$ ***CP* violation in mixing**

$$\delta \equiv \frac{1 - |q/p|^2}{1 + |q/p|^2}$$

$B^0 - \overline{B}^0$ Mixing

$$\delta = \frac{1 - |q/p|^2}{1 + |q/p|^2}$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}$$



- $B^0 - \overline{B}^0$ transition amplitude is described by Effective Hamiltonian

$$H_{12} = M_{12} - (i/2)\,\Gamma_{12}$$

  - M_{12}: transitions through off-shell intermediate states, $M_{12} \propto m_t^2 \cdot (V_{td} V_{tb}^*)^2$
  - Γ_{12}: transitions through on-shell intermediate states, $\Gamma_{12} \propto m_c^2 \cdot (V_{cd} V_{cb}^*)^2$
  - $\Gamma_{12} \ll M_{12} \Rightarrow$ interference term small $\Rightarrow$ *CP* violation in mixing small
- neglect CP violation for now (δ=0)

$$a_{mix}(t) = \overline{a}_{mix}(t) = \frac{\cos(\Delta m\, t)}{\cosh(\Delta\Gamma\, t/2)} = \frac{\cos(x \cdot \overline{\Gamma} t)}{\cosh(y \cdot \overline{\Gamma} t)}$$

$$x \equiv \Delta m/\overline{\Gamma}$$
$$y \equiv \Delta\Gamma/2\overline{\Gamma}$$

- oscillatory behavior with

frequency x: mass difference between two eigenstates
damping y: lifetime difference between two eigenstates

Mixing phenomenology

$K^0\overline{K}^0$

$x_K \approx 0.95$

$y_K \approx -1$

- strong damping, only K_L left after about one oscillation

$D^0\overline{D}^0$

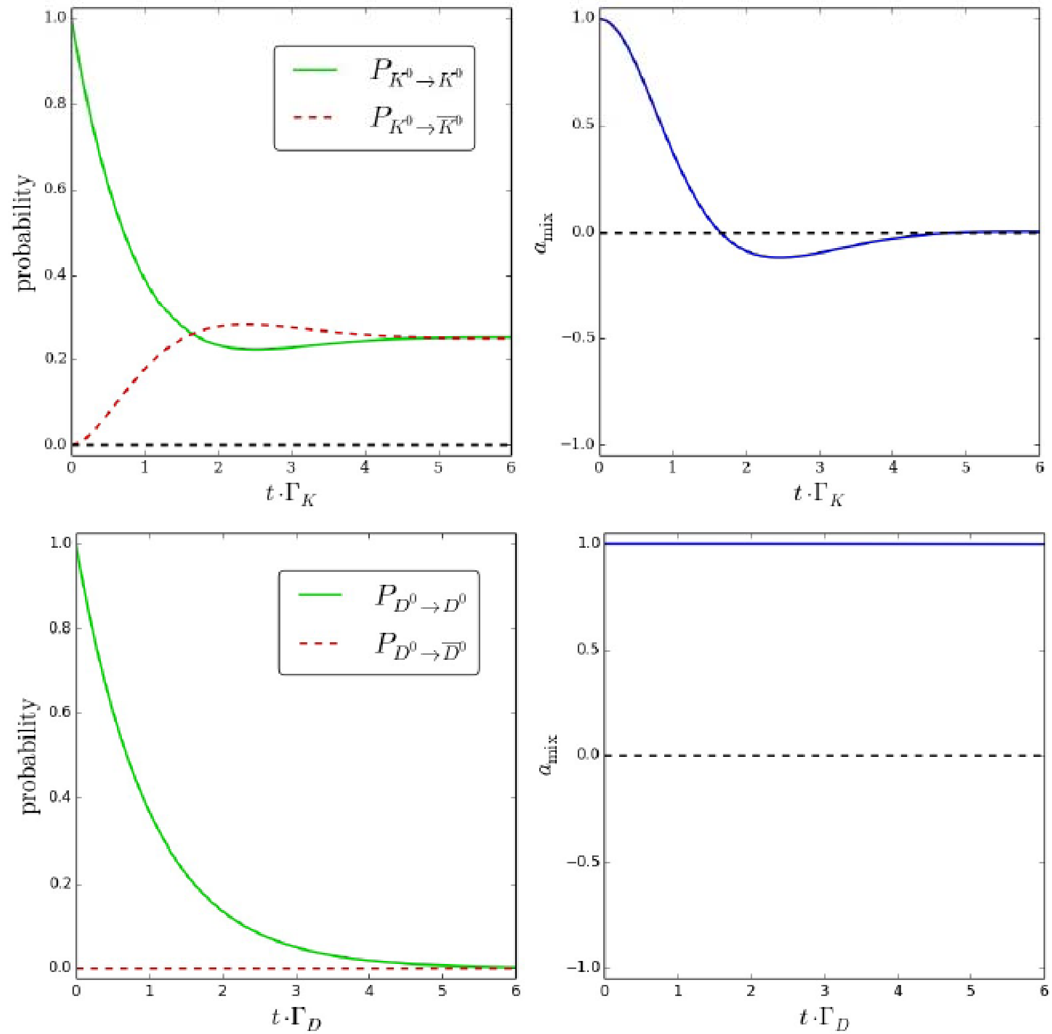
$x_D \approx 5 \times 10^{-3}$

$y_D = (7.15 \pm 0.09) \times 10^{-3}$

[PDG]

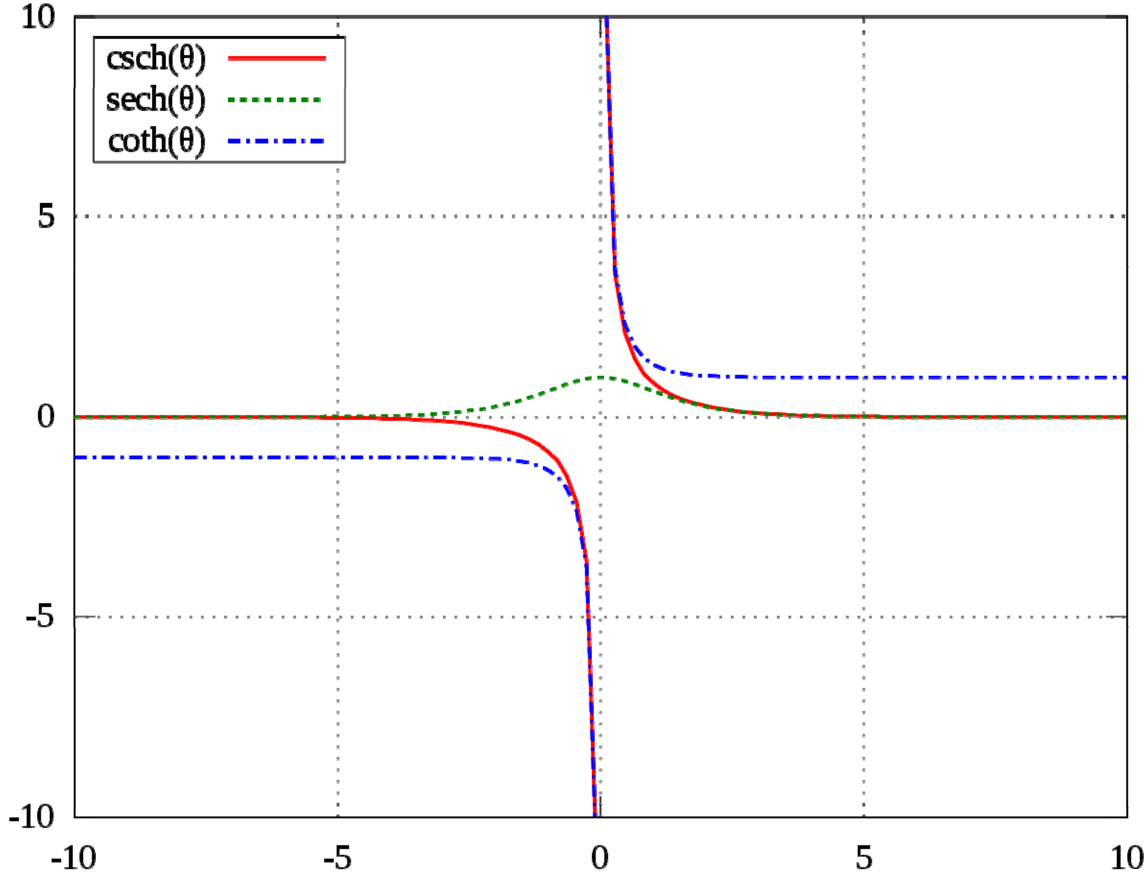
- mixing very small, time-integrated probability

$$\chi_D = \frac{x_D^2 + y_D^2}{2\left(1 + x_D^2\right)} \approx 3 \times 10^{-5}$$



- first evidence for $D^0\overline{D}^0$ mixing reported by B factories in 2007

Hyperbolic function



Mixing phenomenology

$B^0\overline{B}^0$

$x_d = 0.775 \pm 0.006$

$y_d = 0.007 \pm 0.009$

[PDG]

- **small damping, significant mixing:**

$$\chi_d = \frac{x_d^2 + y_d^2}{2\,(1 + x_d^2)} \approx 18\,\%$$

$B_s^0\overline{B}_s^0$

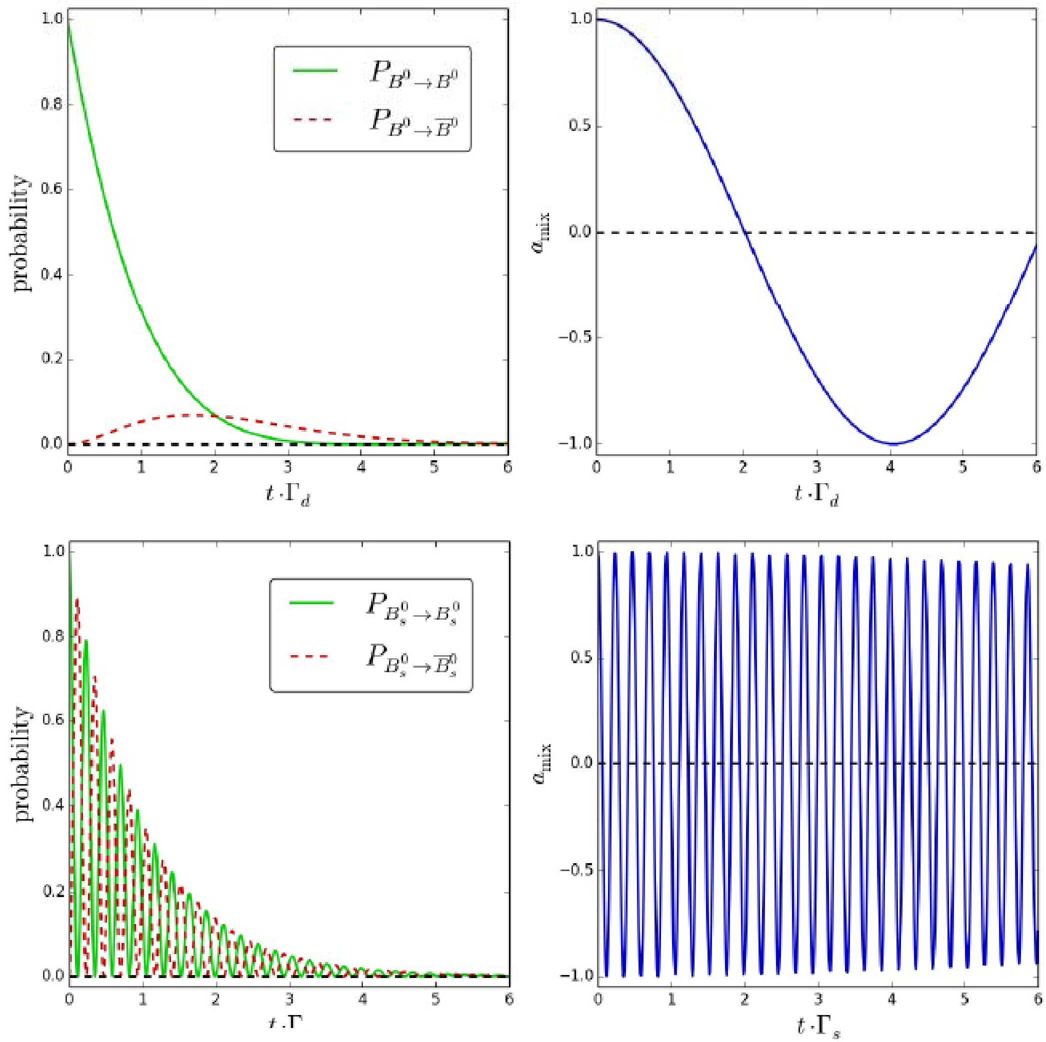
$x_s = 26.82 \pm 0.23$

$y_s = 0.058 \pm 0.010$

[PDG]

- **very fast oscillation and complete mixing:**

$$\chi_s = \frac{x_s^2 + y_s^2}{2\,(1 + x_s^2)} > 49.9\,\%$$



- **first measurement of oscillation frequency by CDF in 2006**

$B_s - \bar{B}_s$ mixing

- B_s oscillation is harder to be measured compared to B_d because
  - the production rate of B_s is smaller than B_d by1/2~1/3
  - the mixing effect is at its maximum
  - the oscillation period is ~40 times shorter

- The hadron collider has the advantage
  - the production of $B_s\bar{B}_s$ is more boosted
  - hence longer decay length which makes observation of the time variation easier
  - first measurement in the Tevatron and later LHC confirmed

Δm_d and Δm_s

S = Inami-Lim function → project
for the t-t contribution
(from perturbative calculations)

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_W^2 \eta_b S(x_t) m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{tb}|^2 |V_{td}|^2$$

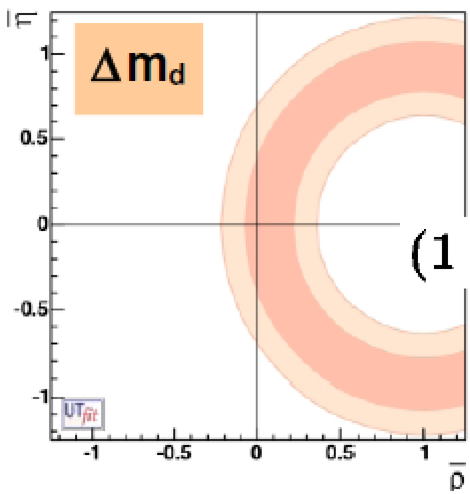
$$= \frac{G_F^2}{6\pi^2} m_W^2 \eta_b S(x_t) m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{cb}|^2 \lambda^2 ((1-\bar{\rho})^2 + \bar{\eta}^2)$$

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{td}|^2}{m_{B_s} f_{B_s}^2 \hat{B}_{B_s} |V_{ts}|^2}$$

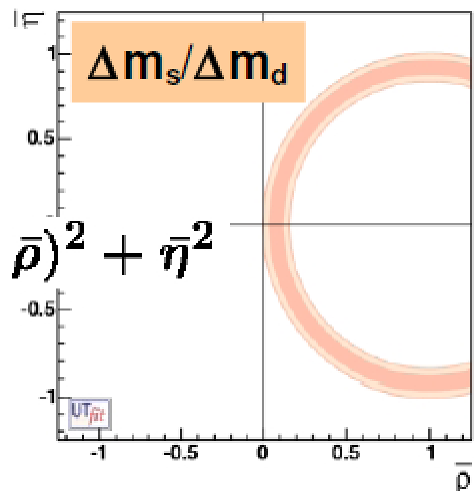
B_{B_d} and f_{B_d} from lattice QCD

We can use these results to extract possible values for fundamental SM parameters: 2 of the parameters of the CKM matrix:

ρ and η



$$(1-\bar{\rho})^2 + \bar{\eta}^2$$



Flavour physics beyond the SM

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}}(A_a, \psi_i) + \mathcal{L}_{\text{Higgs, Yukawa}}(\phi, A_a, \psi_i) + \sum_{d \geq 5} \frac{c_n}{\Lambda^{d-4}} O_n^{(d)}(\phi, A_a, \psi_i)$$

$$M(B_d - \bar{B}_d) \sim \frac{(y_t^2 V_{tb}^* V_{td})^2}{16\pi^2 m_t^2} + c_{NP} \frac{1}{\Lambda^2}$$

Λ = *effective scale of new physics*

c_{NP}:

- ~ 1 —(tree / strong + generic flavor)→ $\Lambda \gtrsim 2\times10^4$ TeV [K]
- $\sim 1/(16\pi^2)$ —(loop + generic flavor)→ $\Lambda \gtrsim 2\times10^3$ TeV [K]
- $\sim (y_t V_{ti}^* V_{tj})^2$ —(tree / strong + "alignment")→ $\Lambda \gtrsim 5$ TeV [K & B]
- $\sim (y_t V_{ti}^* V_{tj})^2/(16\pi^2)$ —(loop + "alignment")→ $\Lambda \gtrsim 0.5$ TeV [K & B]

Flavour physics beyond the SM

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum \frac{c_{ij}}{\Lambda^2} O_{ij}^{(6)}$$

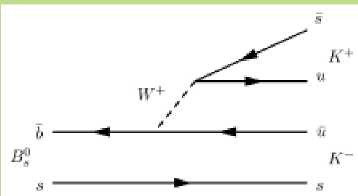
G.I, Nir, Perez '10

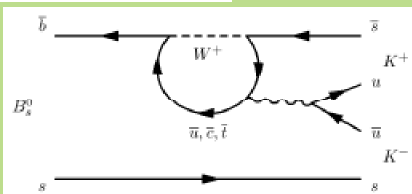
Operator	Bounds on Λ (TeV)		Bounds on c_{ij} ($\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L \gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ε_K
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ε_K
$(\bar{c}_L \gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p \rvert$, ϕ_D
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\lvert q/p \rvert$, ϕ_D
$(\bar{b}_L \gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{B_d \to \psi K}$
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{B_d \to \psi K}$
$(\bar{b}_L \gamma^\mu s_L)^2$	1.1×10^2	1.1×10^2	7.6×10^{-5}	7.6×10^{-5}	Δm_{B_s}
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	3.7×10^2	3.7×10^2	1.3×10^{-5}	1.3×10^{-5}	Δm_{B_s}

New flavor-breaking sources at the TeV scale (if any) are highly tuned

Three types of CP violation

CPV in decay
("direct *CP* violation")

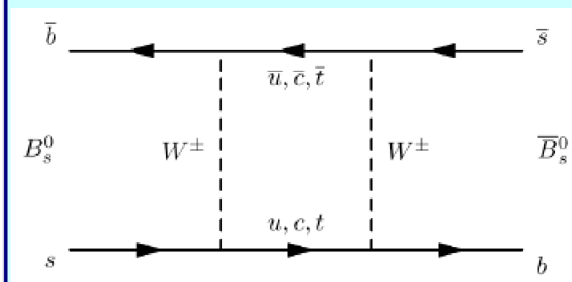




- interference of decay diagrams with different weak and strong phases
- different decay rates

  $B \to f$ vs $\overline{B} \to \overline{f}$
- beware of strong phases

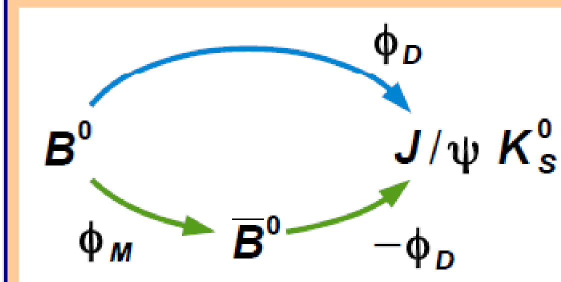
CPV in mixing
("indirect *CP* violation")



- interference of absorp-tive and dispersive part of mixing amplitude
- different mixing rate

  $B^0_{(s)} \to \overline{B}^0_{(s)}$ vs $\overline{B}^0_{(s)} \to B^0_{(s)}$
- small in Standard Model

CPV in interference of mixing and decay



- interference between direct decay and decay after mixing
- different decay rates

  $B^0_{(s)} \to f_{CP}$ vs $\overline{B}^0_{(s)} \to f_{CP}$
- "golden modes"

Direct CP violation

- CP violation in decays if $A(\bar{B} \to \bar{f}) \neq A(B \to f)$

  - requires interference of at least two decay amplitudes with different weak phase and different strong phase, which lead to the same final state

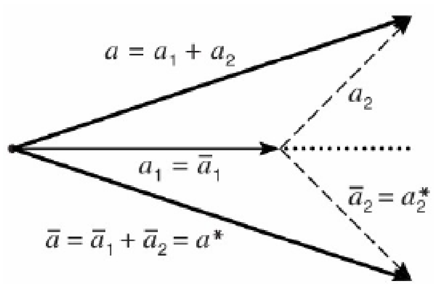
$$A_f \equiv A(B \to f) = \sum_i a_i \, e^{i(\delta_i + \phi_i)}$$

$$\bar{A}_{\bar{f}} \equiv A(\bar{B} \to \bar{f}) = \sum_i a_i \, e^{i(\delta_i - \phi_i)}$$

ϕ_i: weak phase, changes sign under *CP*

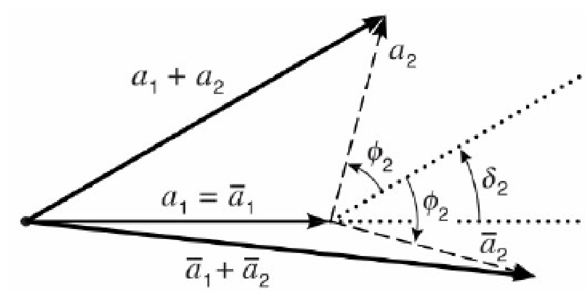
δ_i: strong phase, does not change sign under *CP*

$$|A_f|^2 - |\bar{A}_{\bar{f}}|^2 = -2 \sum_{ij} a_i a_j \cdot \sin(\phi_i - \phi_j) \cdot \sin(\delta_i - \delta_j)$$



$\phi_2 \neq \phi_1$
$\delta_2 = \delta_1$
$\Rightarrow |\bar{a}| = |a|$

$(\phi_1 = \delta_1 = 0)$



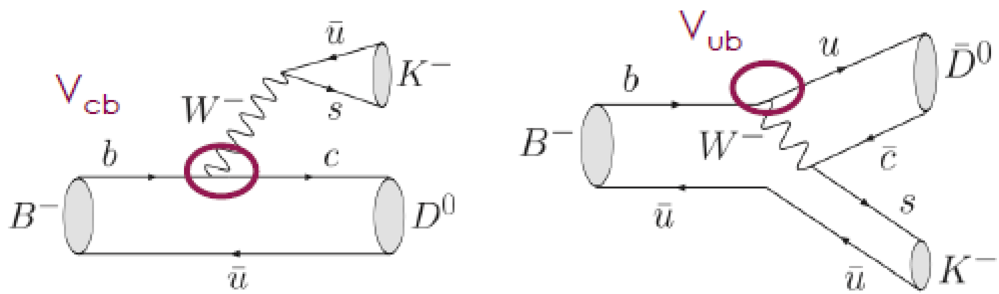
$\phi_2 \neq \phi_1$
$\delta_2 \neq \delta_1$
$\Rightarrow |\bar{a}| \neq |a|$

$(\phi_1 = \delta_1 = 0)$

- **interference and *CP* violation can be large**
  - New Physics can enter through loops if penguin diagrams involved
- **but have to battle large theoretical uncertainties due to the strong phases**

Direct CP violation

- CP violation in decays



$$\gamma = \arg\left(\frac{V_{ud}V_{ub}^*}{V_{cd}^*V_{cb}}\right)$$

D^0 and $\bar{D}^0$ decay into the same final state ($f_{CP} = KK, \pi\pi, \ldots$)

CP

$$A\left(B^- \to D^0\left(\to f_{CP}\right)K^-\right) = A_c \qquad A\left(B^- \to \bar{D}^0\left(\to f_{CP}\right)K^-\right) = A_u e^{i(\delta_B - \gamma)}$$

$$A\left(B^+ \to D^0\left(\to f_{CP}\right)K^+\right) = A_c \qquad A\left(B^+ \to D^0\left(\to f_{CP}\right)K^+\right) = A_u e^{i(\delta_B + \gamma)}$$

$$\Gamma\left(B^- \to f_{CP}K^-\right) = \left|A_c + A_u e^{i(\delta_B - \gamma)}\right|^2 = A_c^2 \times \left(1 + r_B^2 + 2r_B\cos\left(\delta_B - \gamma\right)\right)$$

$$\Gamma\left(B^+ \to f_{CP}K^+\right) = \left|A_c + A_u e^{i(\delta_B + \gamma)}\right|^2 = A_c^2 \times \left(1 + r_B^2 + 2r_B\cos\left(\delta_B + \gamma\right)\right)$$

$$r_B = \frac{A_u}{A_c}$$

$$A_{CP} = \frac{\Gamma\left(B^- \to f_{CP}K^-\right) - \Gamma\left(B^+ \to f_{CP}K^+\right)}{\Gamma\left(B^- \to f_{CP}K^-\right) + \Gamma\left(B^+ \to f_{CP}K^+\right)} = \frac{2r_B\sin\delta_B\sin\gamma}{1 + r_B^2 + 2r_B\cos\delta_B\cos\gamma}$$

Direct CP violation

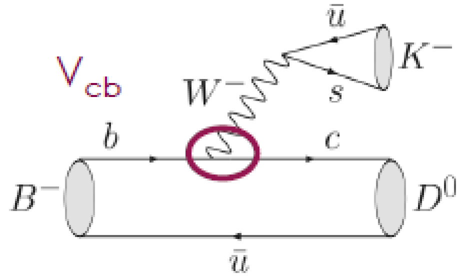
- at least two amplitudes with different weak phase but also strong phase are required.

$$\gamma = 0 \text{ or } \delta_B = 0 \;\Rightarrow\; \text{no CP violation}$$

- three unknowns: r_B, δ_B, γ

- Additional information is obtained from the Cabibbo-favored decay: $D^0 \to K^- \pi^+$

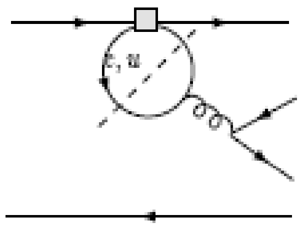
$$\Gamma\left(B^- \to D^0 K^-\right) = A_c^2 \simeq \frac{\Gamma\left(B^- \to D^0\left(\to K^-\pi^+\right)K^-\right)}{BF\left(D^0 \to K^-\pi^+\right)}$$



$$R_{CP} = \frac{\Gamma\left(B^- \to f_{CP}K^-\right) + \Gamma\left(B^+ \to f_{CP}K^+\right)}{2\Gamma\left(B^- \to D^0 K^-\right)} = 1 + r_B^2 + 2r_B \cos\delta_B \cos\gamma$$

Sources of strong phase

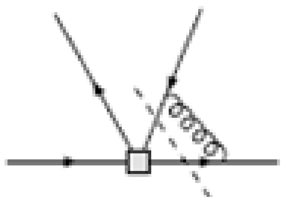
- The weak phase is the phase in the Lagrangian, but what is the origin of the strong phase?



Bander-Silverman-Soni (BSS) mechanism

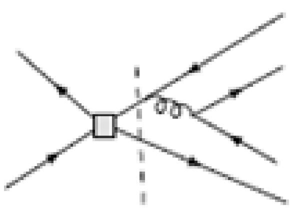
It gives a small phase.
Only source?
Important source?

⇒ Insufficient to explain large direct CP asymmetries



Vertex correctoins in QCDF

only account for perturbative strong phase

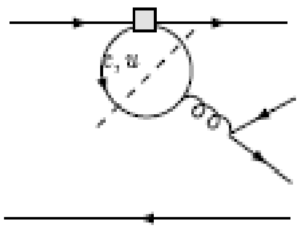


Annihilation diagram in PQCD

model-dependent

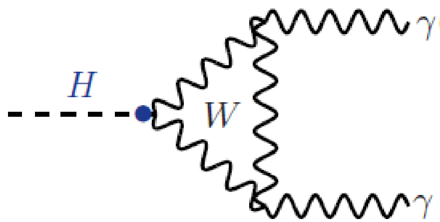
Final state interaction (long-range interaction)

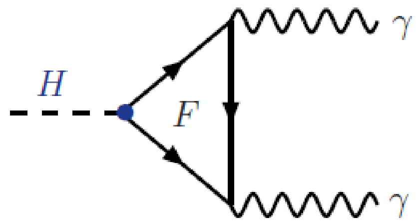
BSS mechanism



Bander-Silverman-Soni (BSS) mechanism

c.f. H→$\gamma\gamma$ decay





$$\Gamma\,(H \to \gamma\gamma) = \frac{G_\mu\,\alpha^2\,M_H^3}{128\,\sqrt{2}\,\pi^3}\left|\sum_f N_c Q_f^2 A_{1/2}^H(\tau_f) + A_1^H(\tau_W)\right|^2$$

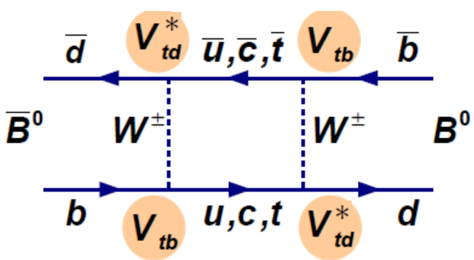
$$\begin{aligned} A_{1/2}^H(\tau) &= 2[\tau + (\tau-1)f(\tau)]\,\tau^{-2} \\ A_1^H(\tau) &= -[2\tau^2 + 3\tau + 3(2\tau-1)f(\tau)]\,\tau^{-2} \end{aligned}$$

$$f(\tau) = \begin{cases} \arcsin^2\sqrt{\tau} & \tau \le 1 \\ -\dfrac{1}{4}\left[\log\dfrac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi\right]^2 & \tau > 1 \end{cases}$$

$$\tau_i = M_H^2/4M_i^2 \text{ with } i = f, W$$

Indirect CP violation

- CP violation induced by mixing



$$H_{12} = M_{12} - (i/2)\,\Gamma_{12}$$

$\Gamma_{12} \ll M_{12} \Rightarrow$ interference term small

$\Rightarrow$ *CP* violation in mixing small

New physics can enter in box and may have significant effects

CP

$$P\left(B^0(0) \to \bar{B}^0(t)\right) = \left|\frac{q}{p}\right|^2 \left|g_-(t)\right|^2$$

$$P\left(\bar{B}^0(0) \to B^0(t)\right) = \left|\frac{p}{q}\right|^2 \left|g_-(t)\right|^2$$

CP violation is present if |q/p|≠1

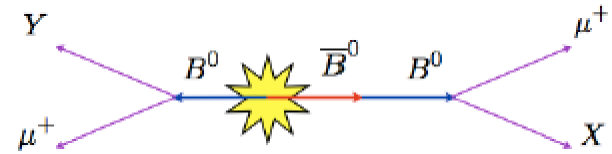
- In experiments, one can define

$$a_{sl} = \frac{P\left(\bar{B}^0(0) \to B^0(t)\right) - P\left(B^0(0) \to \bar{B}^0(t)\right)}{P\left(\bar{B}^0(0) \to B^0(t)\right) + P\left(B^0(0) \to \bar{B}^0(t)\right)} = \frac{1 - \left|\frac{q}{p}\right|^4}{1 + \left|\frac{q}{p}\right|^4}$$

$\Rightarrow$ Does not depend on time anymore

Indirect CP violation

- Possible experiments



$$A_{sl} = \frac{N(\mu^+\mu^+) - N(\mu^-\mu^-)}{N(\mu^+\mu^+) + N(\mu^-\mu^-)} = C_d a_{sl}^d + C_s a_{sl}^s$$ (for hadron colliders)

dimuon analysis from D0 (magnet polarity flip possible, symmetric initial state)

$$A_{sl}^b = (-0.787 \pm 0.172(\text{stat}) \pm 0.093(\text{syst}))\ \%$$

$$A_{sl} = C_d a_{sl}^d + C_s a_{sl}^s$$

-3.9 σ from SM predictions

changing the impact parameter cut changes the composition of the sample

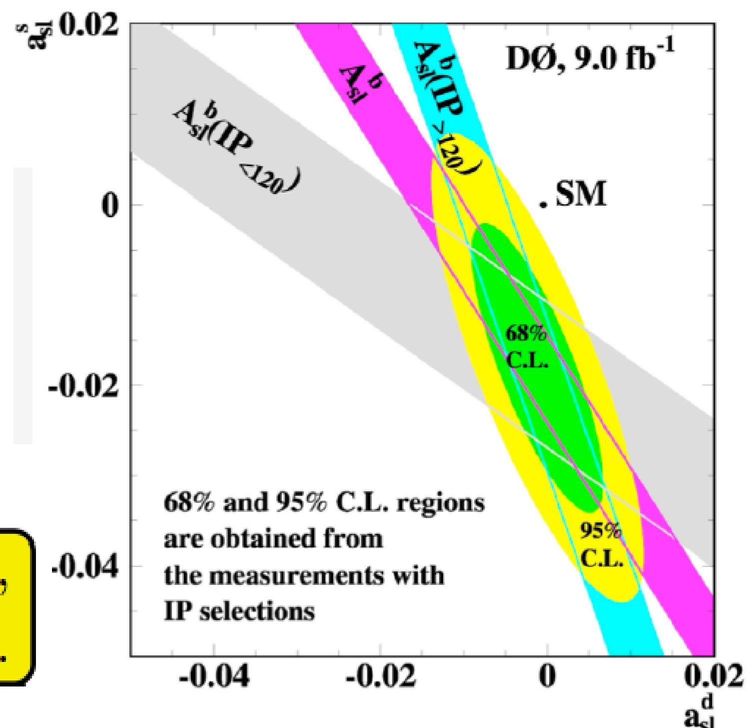
Extraction of a_{sl}^d and a_{sl}^s

Standard Model for B

$a_{sl}^s = (1.9 \pm 0.3) \times 10^{-5}$

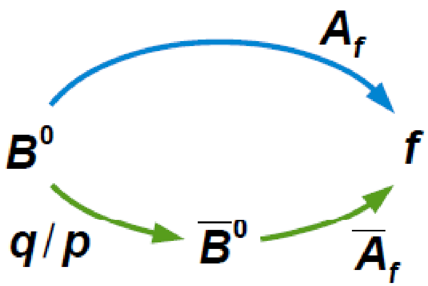
$a_{sl}^d = (-4.1 \pm 0.6) \times 10^{-4}$

$$a_{sl}^d = (-0.12 \pm 0.52)\%,$$
$$a_{sl}^s = (-1.81 \pm 1.06)\%.$$



CP violation in interference of mixing and decay

- For decay into a CP eigenstate f that is accessible to both B^0 and $\bar{B}^0$



$$\left|B^0_{t=0}\right\rangle = g_+(t)\left|B^0\right\rangle + \frac{q}{p} g_-(t)\left|\bar{B}^0\right\rangle$$

$$\left|\bar{B}^0_{t=0}\right\rangle = g_+(t)\left|\bar{B}^0\right\rangle + \frac{p}{q} g_-(t)\left|B^0\right\rangle$$

$$g_+(t) = e^{-iMt} e^{-\Gamma t/2} \cos(\Delta m_B t/2)$$

$$g_-(t) = e^{-iMt} e^{-\Gamma t/2} i \sin(\Delta m_B t/2)$$

N.B. $\Delta\Gamma \simeq 0$

- time-dependent decay rate asymmetry

$$a_f(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)}$$

- decay amplitudes are defined by

$$A_f = \left\langle f \,|\, H \,|\, B^0 \right\rangle \qquad A_{\bar{f}} = \left\langle \bar{f} \,|\, H \,|\, B^0 \right\rangle$$

$$\bar{A}_f = \left\langle f \,|\, H \,|\, \bar{B}^0 \right\rangle \qquad \bar{A}_{\bar{f}} = \left\langle \bar{f} \,|\, H \,|\, \bar{B}^0 \right\rangle$$

CP violation in interference of mixing and decay

$$\Gamma(B^0(t)\to f)\sim\left|g_+(t)\left\langle f\,|\,H\,|\,B^0\right\rangle+\frac{q}{p}g_-(t)\left\langle f\,|\,H\,|\,\bar{B}^0\right\rangle\right|^2$$

$$=\left|g_+(t)\right|^2\left|A_f\right|^2+\left|\frac{q}{p}\right|^2\left|g_-(t)\right|^2\left|\bar{A}_f\right|^2+2\,\mathrm{Re}\left(g_+(t)\left(\frac{q}{p}g_-(t)\right)^*A_f\bar{A}_f^*\right)$$

$$\Gamma(\bar{B}^0(t)\to f)\sim\left|g_+(t)\left\langle f\,|\,H\,|\,\bar{B}^0\right\rangle+\frac{p}{q}g_-(t)\left\langle f\,|\,H\,|\,B^0\right\rangle\right|^2$$

$$=\left|g_+(t)\right|^2\left|\bar{A}_f\right|^2+\left|\frac{p}{q}\right|^2\left|g_-(t)\right|^2\left|A_f\right|^2+2\,\mathrm{Re}\left(g_+(t)\left(\frac{p}{q}g_-(t)\right)^*\bar{A}_fA_f^*\right)$$

$$\Gamma(B^0(t)\to f)\sim\left|A_f\right|^2\left(1+\left|\lambda_f\right|^2\right)\left[1-S\sin\Delta m_Bt+C\cos\Delta m_Bt\right]$$

$$\Gamma(\bar{B}^0(t)\to f)\sim\left|A_f\right|^2\left|\frac{p}{q}\right|\left(1+\left|\lambda_f\right|^2\right)\left[1+S\sin\Delta m_Bt-C\cos\Delta m_Bt\right]$$

$$C_f=\frac{1-\left|\lambda_f\right|^2}{1+\left|\lambda_f\right|^2}\qquad S_f=\frac{2\,\mathrm{Im}\,\lambda_f}{1+\left|\lambda_f\right|^2}\qquad \lambda_f=\frac{q}{p}\frac{\bar{A}_f}{A_f}=\eta_f\frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_f}$$

CP violation in interference of mixing and decay

- In the SM, it is known that

$$1-\left|\frac{q}{p}\right|^2 \simeq \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \sim \begin{cases} O(10^{-3}) & \text{for } B_d^0-\overline{B}_d^0 \\ \lesssim O(10^{-4}) & \text{for } B_s^0-\overline{B}_s^0 \end{cases}$$

- time-dependent decay rate asymmetry

$$a_f(t) = \frac{\Gamma(B^0(t)\to f)-\Gamma(\overline{B}^0(t)\to f)}{\Gamma(B^0(t)\to f)+\Gamma(\overline{B}^0(t)\to f)} = C\cos(\Delta m_B t) - S\sin(\Delta m_B t)$$

- CP is violated if

$$\left|\lambda_f\right| \neq 1 \quad \text{and/or} \quad \mathrm{Im}\,\lambda_f \neq 0$$

- *C* is called sometimes "direct CP violation", but in this case no nontrivial strong phases are necessary, unlike CP violation in decays.

Determination of CKM angles

Belle notation:

$\phi_1 \equiv \beta$

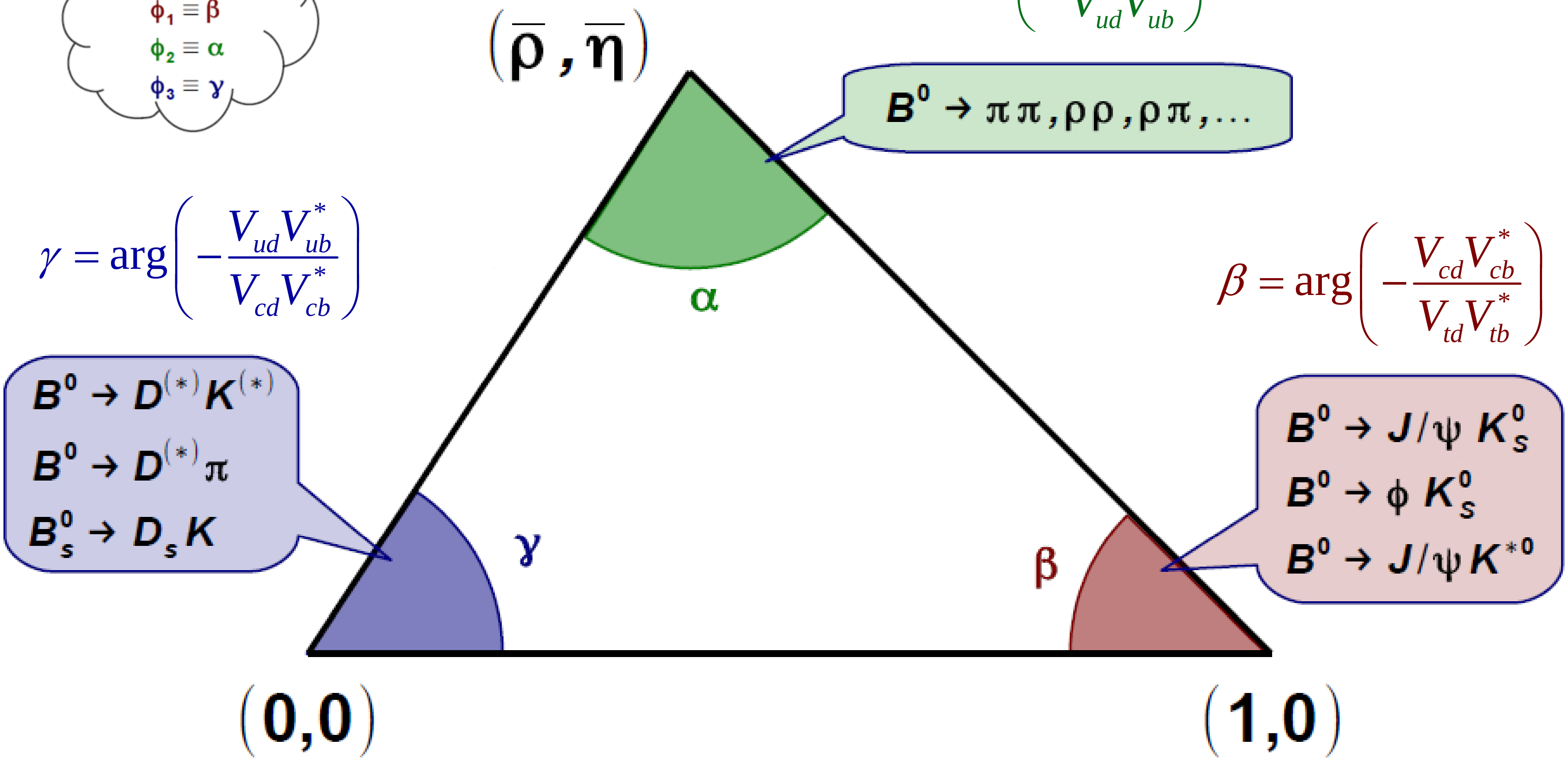
$\phi_2 \equiv \alpha$

$\phi_3 \equiv \gamma$

$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

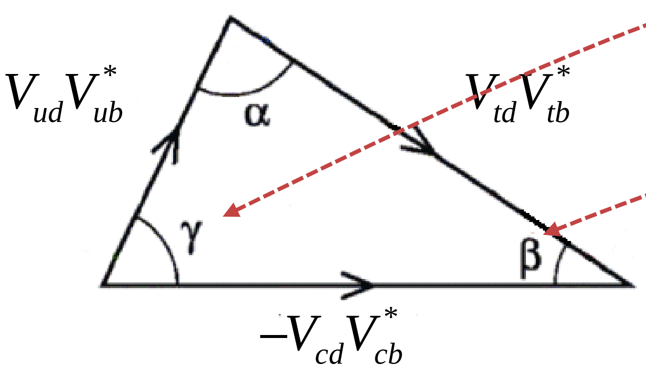
Useful notation for the CKM matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

complex

$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \qquad \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \qquad \gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



$$V_{\mathrm{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}| & |V_{tb}| \end{pmatrix}$$

$\sin 2\beta$

$b \to c\bar{c}s$

$B^0 \to J/\psi K_L^0$

$B^0 \to J/\psi K_S^0$

$B^0 \to \psi(2S) K_S^0$

$B^0 \to \chi_{1c} K_S^0$

$B^0 \to \eta_c K_S^0$

$B^0 \to J/\psi K^{*0}$

$B \to J/\psi \pi^0$

$B \to D^{(*)+} D^{(*)-}$

$B \to \eta' K^0$

$B \to \rho K^0$

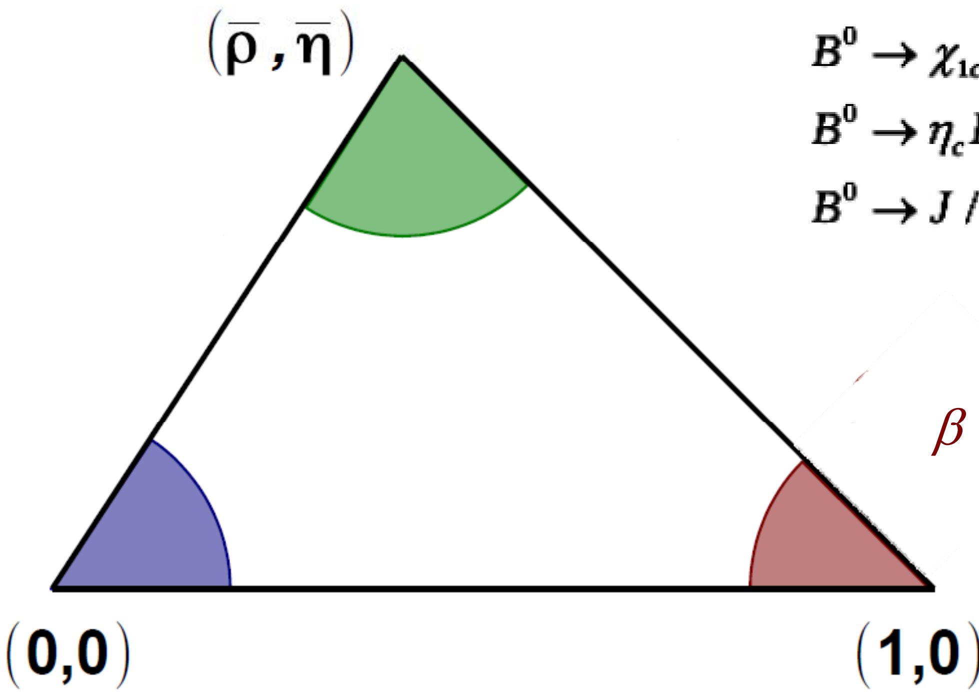
$B \to \omega K^0$

$B \to \pi^0 K^0$

$B \to \phi K^{(*)0}$

$B \to KKK^0$

$B \to f^0(980) K^0$



$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

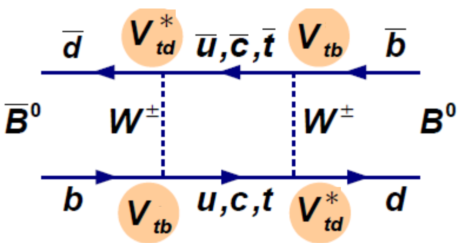
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

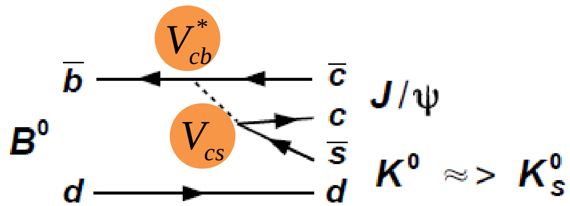
$\sin 2\beta$: Golden decay $B^0 \to J/\psi K_s^0$

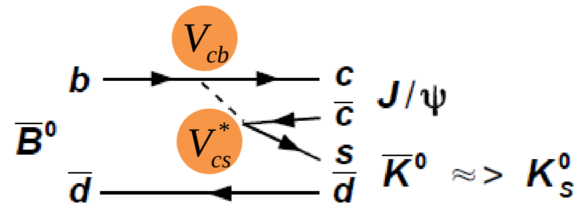
$J^P(K_s^0) = 0^-$

$J^{PC}(J/\psi) = 1^{--}$

- $CP(J/\psi K_s^0) = -1$ and both B^0 and $\bar{B}^0$ can decay to $J/\psi K_s^0$







- time-dependent decay rate asymmetry

$$a_f(t) = \frac{\Gamma(B^0(t)\to f) - \Gamma(\bar{B}^0(t)\to f)}{\Gamma(B^0(t)\to f) + \Gamma(\bar{B}^0(t)\to f)} = C\cos(\Delta m_B t) - S\sin(\Delta m_B t)$$

$$C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \qquad S_f = \frac{2\,\mathrm{Im}\,\lambda_f}{1+|\lambda_f|^2}$$

$$\lambda_{J/\psi K_s^0} = -\left(\frac{q}{p}\right)_{B^0}\left(\frac{\bar{A}_{J/\psi K_s^0}}{A_{J/\psi K_s^0}}\right)\left(\frac{q}{p}\right)_{K^0}$$

- define $\frac{\Gamma_{12}}{M_{12}} = re^{i\xi} \quad (r \sim 10^{-3})$

$$\left(\frac{q}{p}\right)_{B^0} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}} \simeq \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)$$

(leading order in r and t-t contribution is dominant in M_{12})

$\sin 2\beta$: Golden decay $B^0 \to J/\psi K_s^0$

$J^{P}(K_s^0) = 0^-$

$J^{PC}(J/\psi) = 1^{--}$

- $CP(J/\psi K_s^0) = -1$ and both B^0 and $\bar{B}^0$ can decay to $J/\psi K_s^0$

$$\lambda_{J/\psi K_s^0} = -\left(\frac{q}{p}\right)_{B^0}\left(\frac{\bar{A}_{J/\psi K_s^0}}{A_{J/\psi K_s^0}}\right)\left(\frac{q}{p}\right)_{K^0} = -\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)\left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*}\right)\left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}\right)$$

- In the SM,

$$1 - \left|\frac{q}{p}\right|^2 \simeq \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \sim \begin{cases} O(10^{-3}) & \text{for } B_d^0 - \bar{B}_d^0 \\ \lesssim O(10^{-4}) & \text{for } B_s^0 - \bar{B}_s^0 \end{cases} \quad \Rightarrow \quad \left|\frac{q}{p}\right| \sim 1$$

- Then we obtain

$$\lambda_{J/\psi K_s^0} \cong \eta_{J/\psi K_s^0} e^{-2i\beta} = -e^{-2i\beta}$$

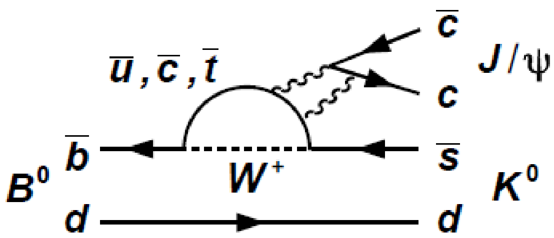
$$a_{J/\psi K_s^0}(t) \cong \eta_{J/\psi K_s^0} \sin 2\beta \sin \Delta m_B t$$

opposite oscillation
for CP even final states

- this holds if tree amplitude dominates

$\sin 2\beta$: Golden decay $B^0 \to J/\psi K_s^0$

- contamination from penguin amplitudes



From unitary condition

$$V_{tb}^* V_{ts} = -V_{cb}^* V_{cs} - V_{ub}^* V_{us}$$

$$A_{J/\psi K^0} = P_t \cdot (V_{tb}^* V_{ts}) + (T + P_c) \cdot (V_{cb}^* V_{cs}) + P_u \cdot (V_{ub}^* V_{us})$$

$$= \underbrace{(T + P_c - P_t)}_{\approx 0.1 \cdot T} \cdot \underbrace{(V_{cb}^* V_{cs})}_{\propto \lambda^2} + \underbrace{(P_u - P_t)}_{\approx 0.1 \cdot T} \cdot \underbrace{(V_{ub}^* V_{us})}_{\propto \lambda^4}$$

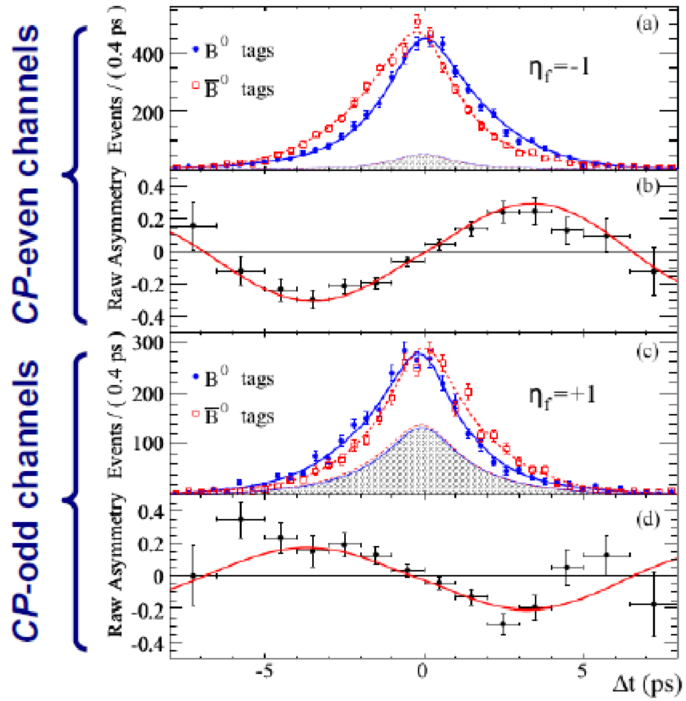
contamination is smaller than 1%

$\Longrightarrow$ Golden decay mode to measure $\sin 2\beta$

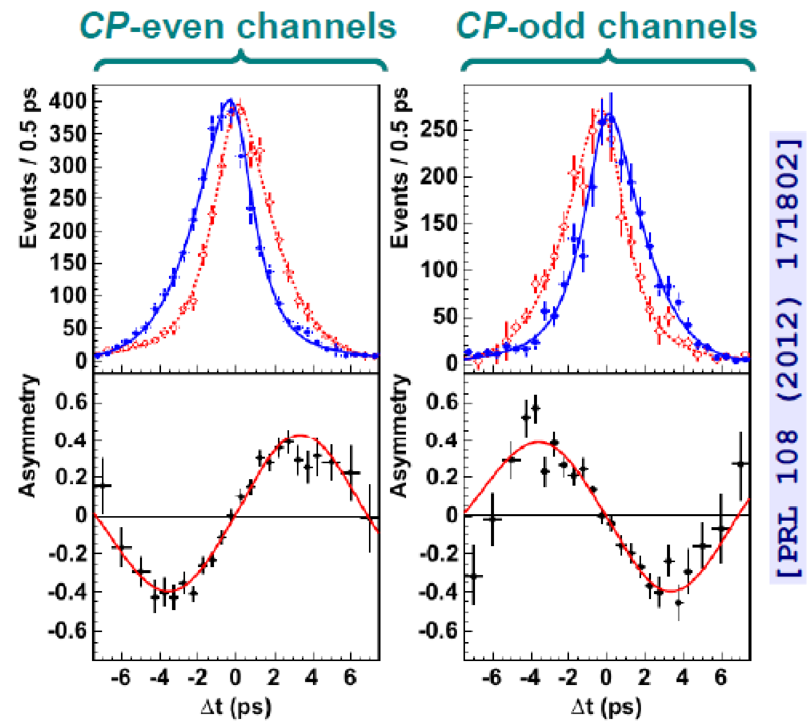
$B^0 \to J/\psi K_L^0$
$B^0 \to J/\psi K_S^0$
$B^0 \to \psi(2S) K_S^0$
$B^0 \to \chi_{1c} K_S^0$
$B^0 \to \eta_c K_S^0$
$B^0 \to J/\psi K^{*0}$

$\sin 2\beta$ from $b \to c\bar{c}s$

Babar (465M $B\bar{B}$ pairs):



Belle (772M $B\bar{B}$ pairs):



$\sin 2\beta = 0.687 \pm 0.028 \pm 0.012$

$\sin 2\phi_1 = 0.667 \pm 0.023 \pm 0.012$

- direct CP asymmetry is consistent with zero as expected

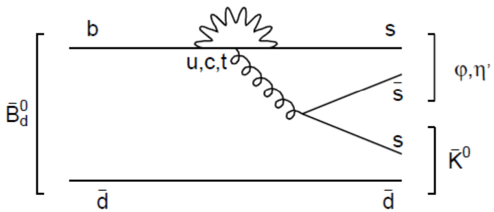
$C_f = 0.024 \pm 0.020 \pm 0.016$

$C_f = 0.006 \pm 0.019 \pm 0.012$

$\sin 2\beta_{\rm eff}$ from $b \to s\bar{s}s$

$B \to \eta' K^0$
$B \to \rho K^0$
$B \to \omega K^0$
$B \to \pi^0 K^0$
$B \to \phi K^{(*)0}$
$B \to K K K^0$
$B \to f^0(980) K^0$

$B^0 \to \phi K_s^0$



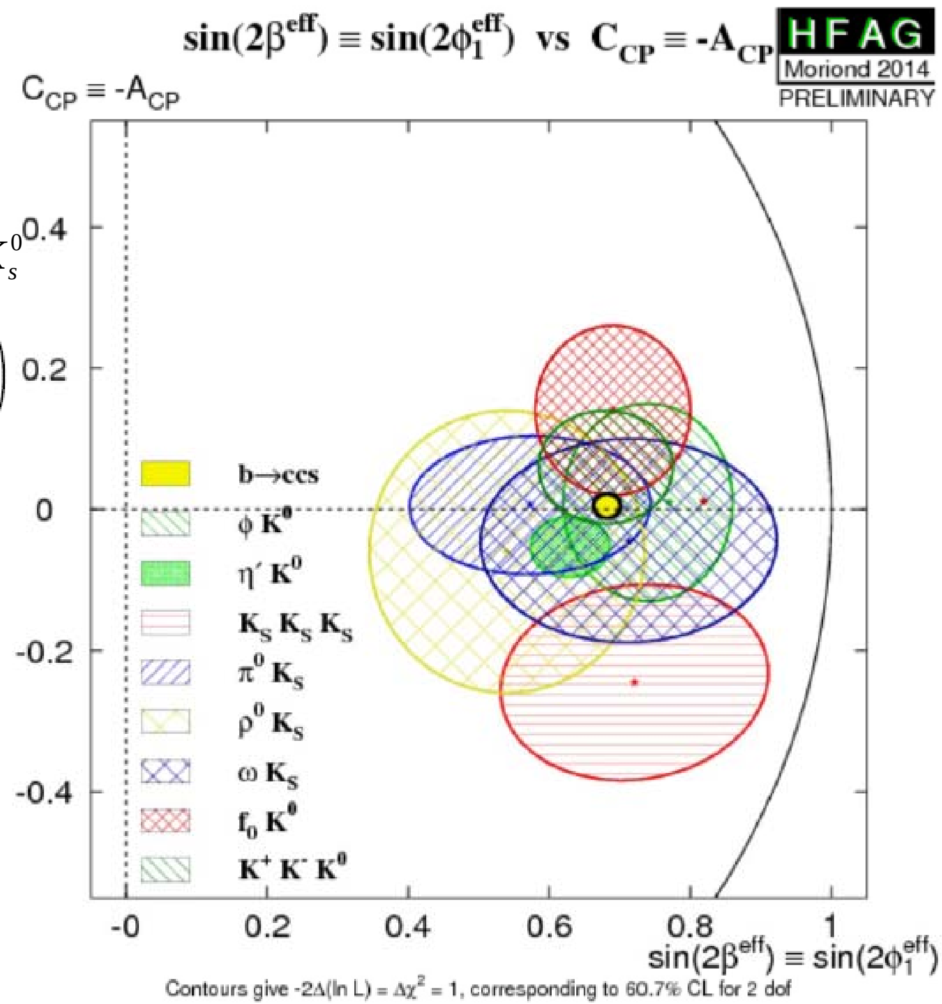
- almost same CKM phase as $B^0 \to J/\psi K_s^0$

$$\lambda(B_d \to \phi K_S) \cong -\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)\left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*}\right)\left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}\right)$$

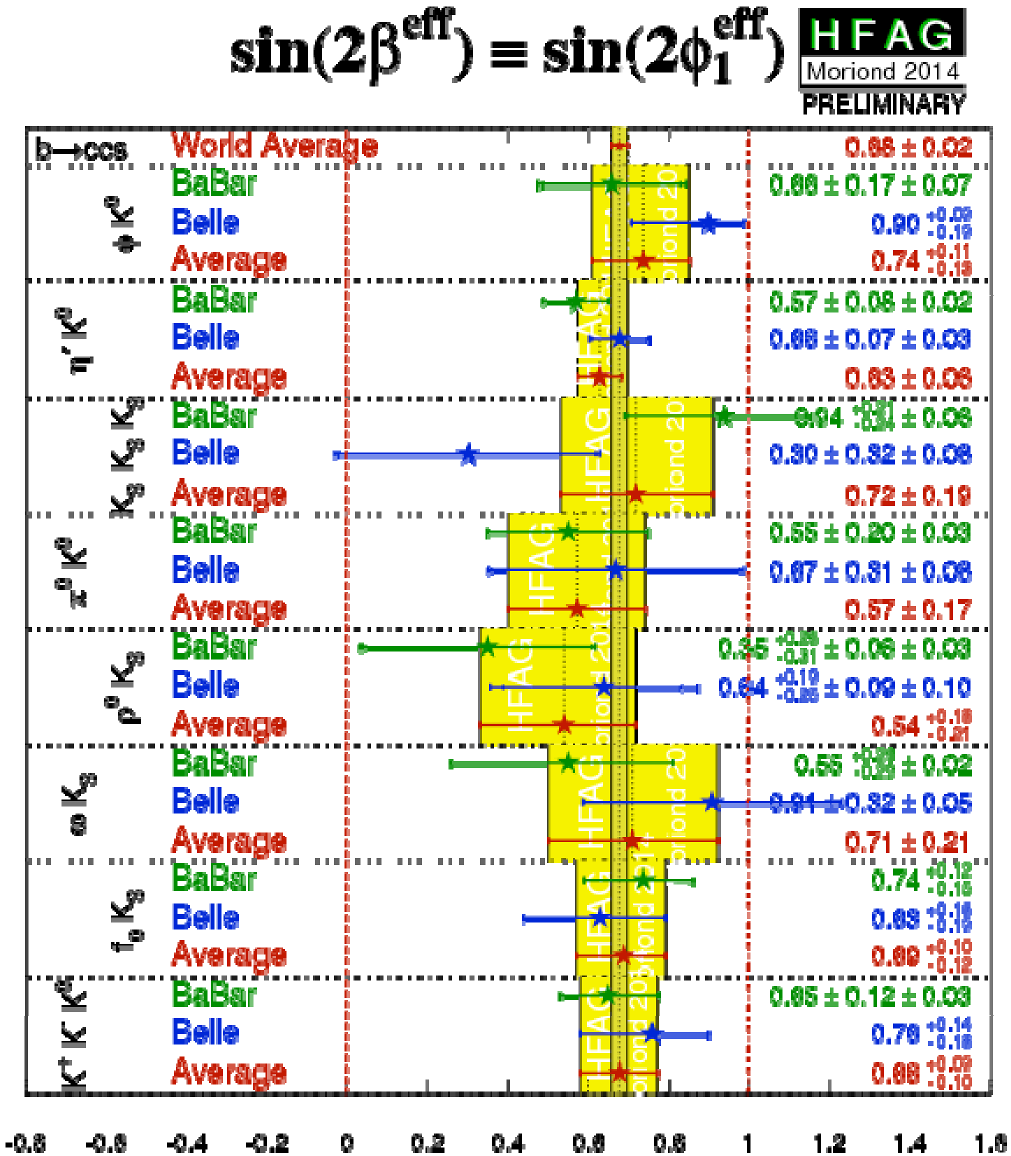
$$\mathrm{Im}\lambda(B_d \to \phi K_S) \cong \sin 2\beta$$

- decay dominated by penguin amp
~ sensitive to possible New Physics

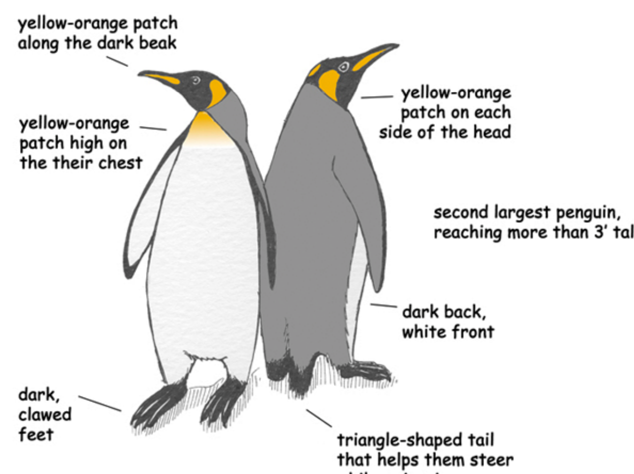
other $b \to s\bar{s}s$ modes may have contributions
from tree amp or non-resonant decay
$\Rightarrow$ contamination



Status of β measurement

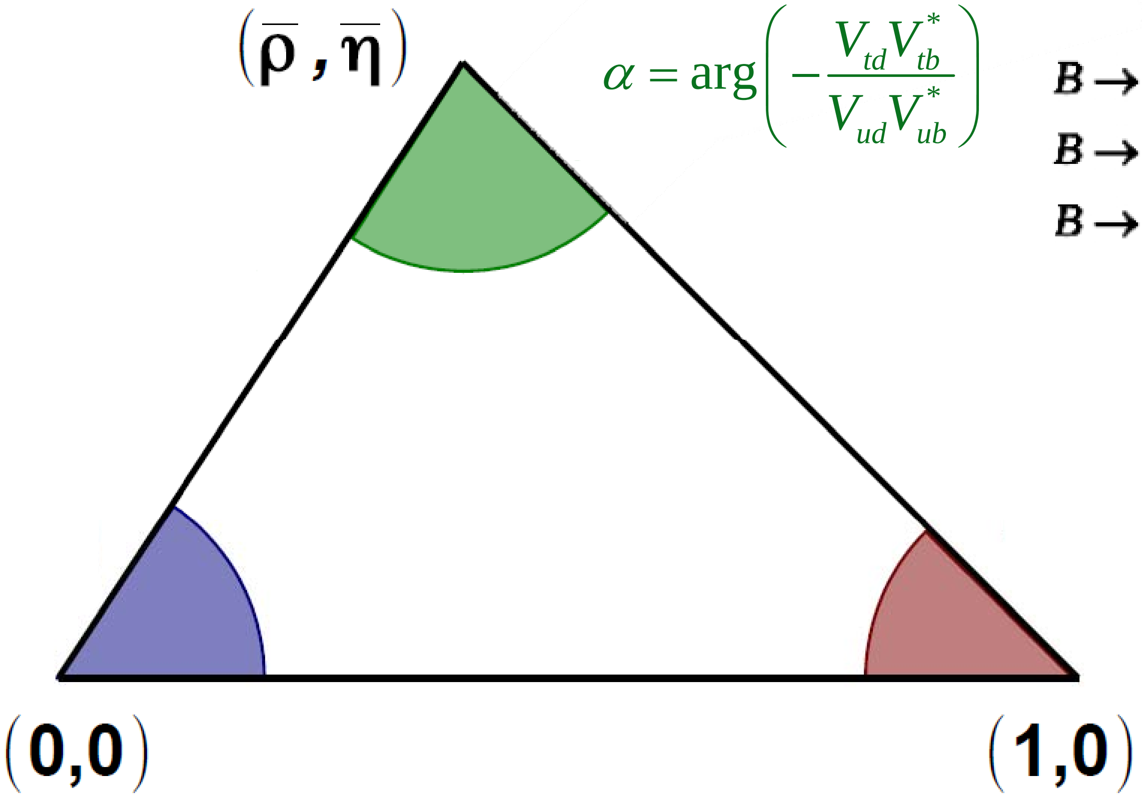


New Physics Phases in penguin b→s decays



<u>No evidence for NP at current level of sensitivity</u>

$\sin 2\alpha$



$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$u\bar{u}d$

$B \to \pi\pi$

$B \to \rho\pi$

$B \to \rho\rho$

$B \to a_1\pi$

$B \to a_1\rho$

$B \to b_1\pi$

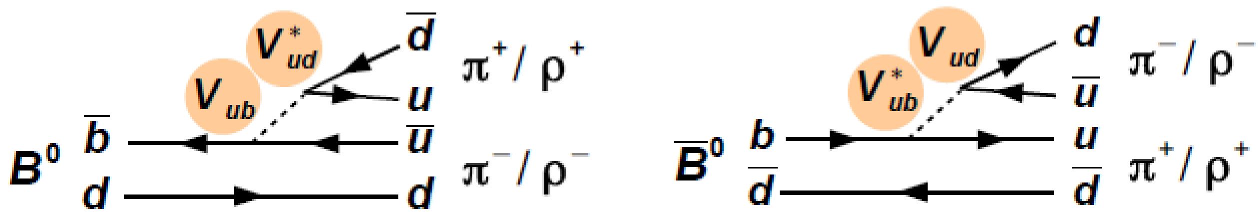
$B \to b_1\rho$

$B \to a_1a_1$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$\sin 2\alpha$ in $B^0 \to \pi^+\pi^-$

- $\pi^+\pi^-$ is a CP even eigenstate and B^0 and $\bar{B}^0$ can decay to $\pi^+\pi^-$

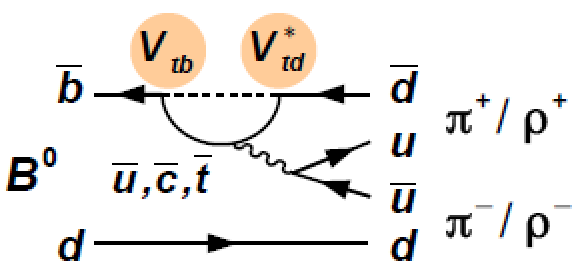


- If the tree-level amplitudes dominate, easy to measure α

$$\lambda_{\pi\pi} = \eta_{\pi\pi}\left(\frac{q}{p}\right)_{B^0}\left(\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}}\right) = \left(\frac{V^*_{tb}V_{td}}{V_{tb}V^*_{td}}\right)\left(\frac{V^*_{ud}V_{ub}}{V_{ud}V^*_{ub}}\right) = e^{-2i(\beta+\gamma)} = e^{2i\alpha}$$

$$a_{\pi\pi}(t) \sim \eta_{\pi\pi}\sin 2\alpha \sin \Delta m_B t$$

- but significant penguin contamination with different weak phase exists



$$A_{\pi\pi} = P_t \cdot (V^*_{tb}V_{td}) + P_c \cdot (V^*_{cb}V_{cd}) + (T + P_u)\cdot(V^*_{ub}V_{ud})$$

$$= (P_t - P_c)\cdot\underbrace{(V^*_{tb}V_{td})}_{\propto\lambda^3} + \underbrace{(T + P_u - P_c)}_{\approx T}\cdot\underbrace{(V^*_{ub}V_{ud})}_{\propto\lambda^3}$$

$$V^*_{cb}V_{cd} = -V^*_{ub}V_{ud} - V^*_{tb}V_{td}$$

$\Rightarrow$ measure effective angle $\alpha_{eff} = \alpha_{CKM} + \Delta\alpha$

$P/T \sim 30\ \%$

Isospin analysis

- $B \to \pi^+\pi^-, \pi^+\pi^0, \pi^0\pi^0$ are connected from isospin relations
- $\pi\pi$ states can have I=2 or I=0 because of Bose statistics
- isospins of B's are $\mathrm{I}(B) = \frac{1}{2}$
- the tree amplitude is $b \to u\bar{u}d$ and $\Delta\mathrm{I} = \frac{1}{2}$ or $\frac{3}{2}$
- the gluonic penguin contributes only to $\Delta\mathrm{I} = \frac{1}{2}$ because isospin is conserved in QCD

⟹ only $\mathrm{I} = 1$ state

- $\pi^+\pi^0$ is a pure $\mathrm{I} = 2$ state because $\mathrm{I}_z(\pi^+\pi^0) = 1$

⟹ only tree diagram (ΔI=3/2) contributes to $B^+ \to \pi^+\pi^0$

$$|\pi^+\pi^0\rangle = |\pi\pi, I=2\rangle$$

Isospin analysis

- isospin decomposition in the $\pi^+\pi^-$ state

$$|2,2\rangle = |1,1;1,1\rangle$$

$$|2,1\rangle = \frac{1}{\sqrt{2}}[|1,1;1,0\rangle + |1,1;0,1\rangle]$$

$$|2,0\rangle = \frac{1}{\sqrt{6}}[|1,1;1,-1\rangle + 2|1,1;0,0\rangle + |1,1;-1,1\rangle]$$

$$|2,-1\rangle = \frac{1}{\sqrt{2}}[|1,1;0,-1\rangle + |1,1;-1,0\rangle]$$

$$|2,-2\rangle = |1,1;-1,-1\rangle$$

$$|1,1\rangle = \frac{1}{\sqrt{2}}[|1,1;1,0\rangle - |1,1;0,1\rangle]$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}[|1,1;1,-1\rangle - |1,1;-1,1\rangle]$$

$$|1,-1\rangle = \frac{1}{\sqrt{2}}[|1,1;0,-1\rangle - |1,1;-1,0\rangle]$$

$$|0,0\rangle = \frac{1}{\sqrt{3}}[|1,1;1,-1\rangle - |1,1;0,0\rangle + |1,1;-1,1\rangle]$$

$\Rightarrow$

$$\left|\pi^+\pi^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\pi^+,\pi^0\right\rangle + \left|\pi^0,\pi^+\right\rangle\right) = \left|2,1\right\rangle$$

$$\left|\pi^+\pi^-\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\pi^+,\pi^-\right\rangle + \left|\pi^-,\pi^+\right\rangle\right) = \frac{1}{\sqrt{3}}\left|2,0\right\rangle + \frac{\sqrt{2}}{\sqrt{3}}\left|0,0\right\rangle$$

$$\left|\pi^0\pi^0\right\rangle = \frac{\sqrt{2}}{\sqrt{3}}\left|2,0\right\rangle - \frac{1}{\sqrt{3}}\left|0,0\right\rangle$$

Isospin analysis

- Hamiltonian for $\bar{b} \rightarrow \bar{u} u \bar{d}$

$$\mathcal{H} = A_{3/2}\left|\tfrac{3}{2}, +\tfrac{1}{2}\right\rangle + A_{1/2}\left|\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle$$

- Then, from Clebsh-Gordan coefficients

$$\mathcal{H}|B^+\rangle = \mathcal{H}\left|\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{3}{4}} A_{3/2}|2,1\rangle + \left(A_{1/2} - \tfrac{1}{2} A_{3/2}\right)|1,1\rangle$$

$$\mathcal{H}|B^0\rangle = \mathcal{H}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{2}} A_{3/2}|2,0\rangle + \sqrt{\tfrac{1}{2}}\left(A_{1/2} + A_{3/2}\right)|1,0\rangle + \sqrt{\tfrac{1}{2}} A_{1/2}|0,0\rangle$$

- the final $\pi\pi$ states are

$$|\pi^+\pi^0\rangle = |2,1\rangle$$

$$|\pi^+\pi^-\rangle = \sqrt{\tfrac{1}{3}}|2,0\rangle + \sqrt{\tfrac{2}{3}}|0,0\rangle$$

$$|\pi^0\pi^0\rangle = \sqrt{\tfrac{2}{3}}|2,0\rangle - \sqrt{\tfrac{1}{3}}|0,0\rangle$$

Includes hadronization and rescattering effects

- define $A^{ij} \equiv \langle \pi^i \pi^j | \mathcal{H} | B^{i+j} \rangle$

$$A_2 \equiv \tfrac{1}{2}\sqrt{\tfrac{1}{3}} A_{3/2,2}, \quad A_0 \equiv \sqrt{\tfrac{1}{6}} A_{1/2,0} \quad \Rightarrow$$

$$A^{+0} = 3A_2$$

$$\sqrt{\tfrac{1}{2}} A^{+-} = A_2 - A_0$$

$$A^{00} = 2A_2 + A_0$$

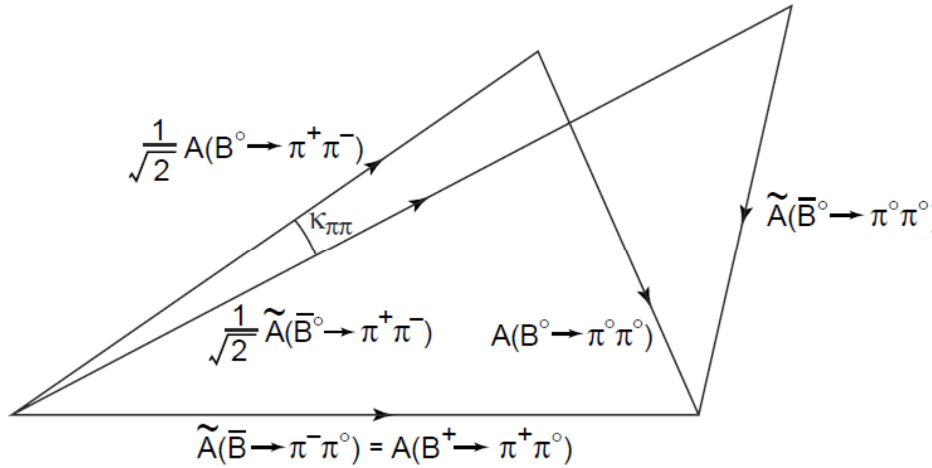
Isospin analysis

- two triangles

CP

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0}$$

$$\frac{1}{\sqrt{2}}\overline{A}^{+-} + \overline{A}^{00} = A^{-0}$$



- $B^{\pm} \to \pi^{\pm}\pi^{0}$ is a pure tree decay

$$\Rightarrow \quad |A^{+0}| = |A^{-0}|$$

- define $\tilde{A}^{ij} \equiv e^{2i\phi_T}\overline{A}^{ij}$ where ϕ_T is the CKM phase of the tree diagram

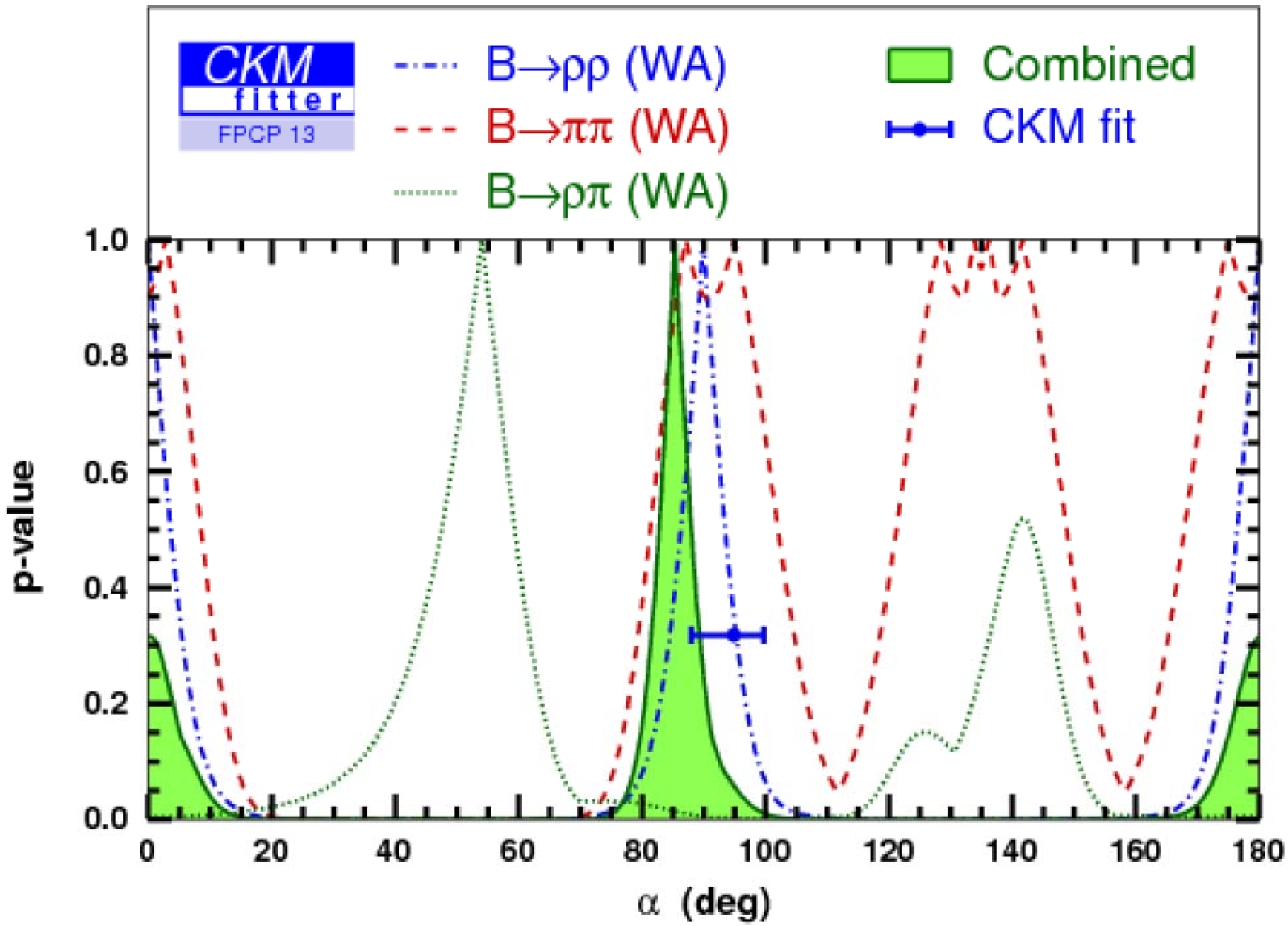
$$\Rightarrow \quad \tilde{A}^{-0} \equiv e^{2i\phi_T}A^{-0} = A^{+0}$$

$$\kappa_{\pi\pi} = 2\Delta\alpha$$

α can be determined with an eight-fold ambiguity

(4 from orientation × 2 from the measurement of sin2(α+$\Delta\alpha$)

Status of α measurement



Direct	CKM fit
$\alpha = \left(88.8^{+4.5}_{-4.3}\right)^o$	$\left(93.6^{+3.2}_{-2.9}\right)^o$

$K \to \pi\pi$ and $\Delta I = 1/2$ rule

- $s \to u\bar{u}d$ transition → ΔI=1/2 or ΔI=3/2

$$A^{+-} = A_0 + \frac{A_2}{\sqrt{2}} \qquad A^{00} = A_0 - \sqrt{2}A_2 \qquad A^{+0} = \frac{3}{2}A_2$$

- Naïve dimensional analysis tells us that A_0 and A_2 should be the same order of magnitude

- for A_2=0, $A^{+-} = A^{00}$.

- in experiments,

	Hadronic modes	
Γ_1	$\pi^0\pi^0$	(30.69±0.05) %
Γ_2	$\pi^+\pi^-$	(69.20±0.05) %

- from a global fit, $\left|\frac{A_2}{A_0} \sim \frac{1}{22}\right|$

- ΔI=1/2 is more important than ΔI=3/2. (known as ΔI=1/2 rule) Why?

- partial answer(?): penguin (contributes only to A_0), but not sufficient, unknown non-perturbative effects?

- another example, $\frac{\Gamma(K^+ \to \pi^+\pi^0\,(I=2))}{\Gamma(K_S^0 \to \pi\pi\,(I=0))} = 4 \times 10^{-2}$

γ

$B \to D^{(*)} K^{(*)}$

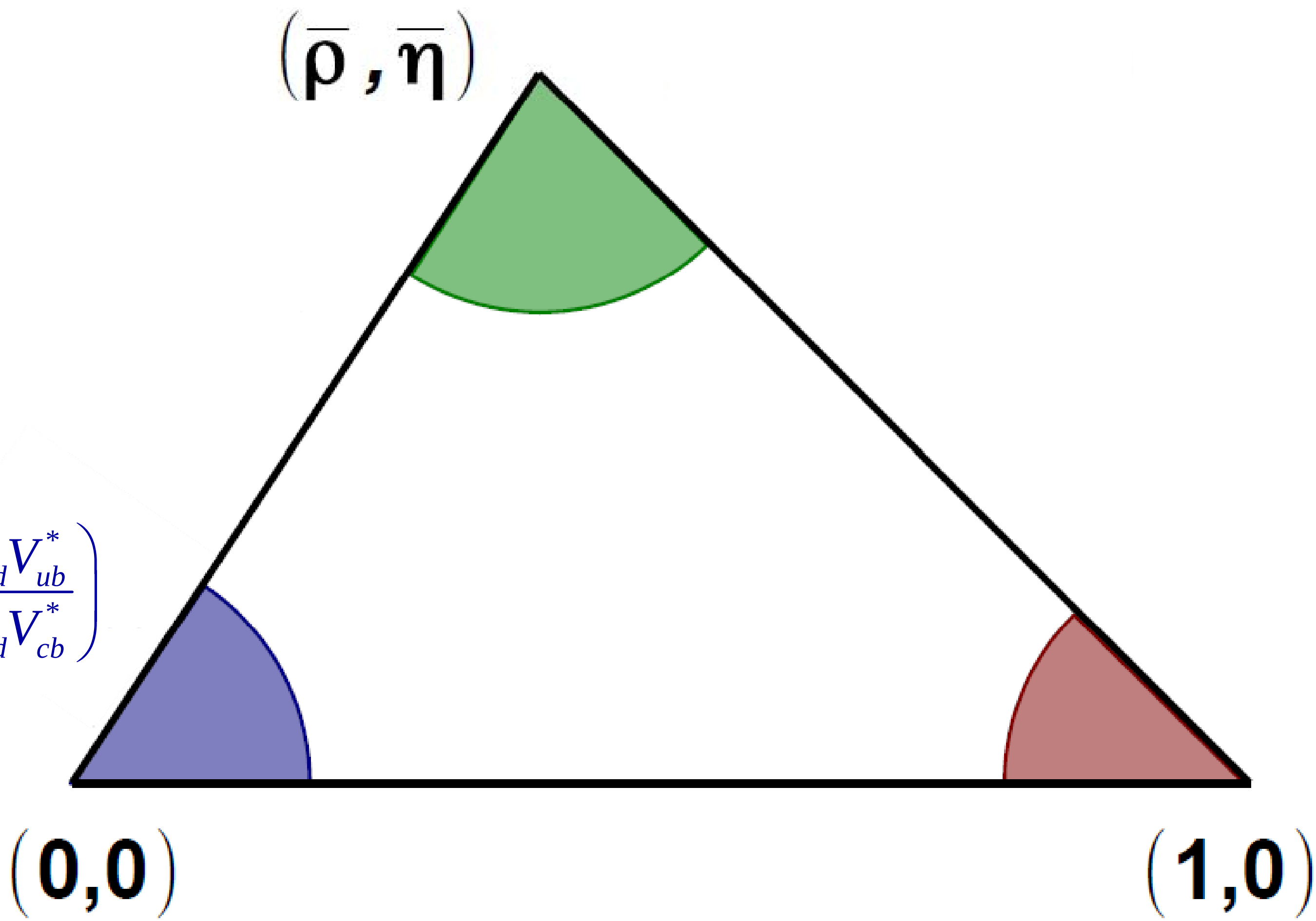
$B^0 \to D^- K^0 \pi^+$

$B^0 \to D^{(*)} \pi$

$B^0 \to D^{(*)} \rho$

$+\ charmless$

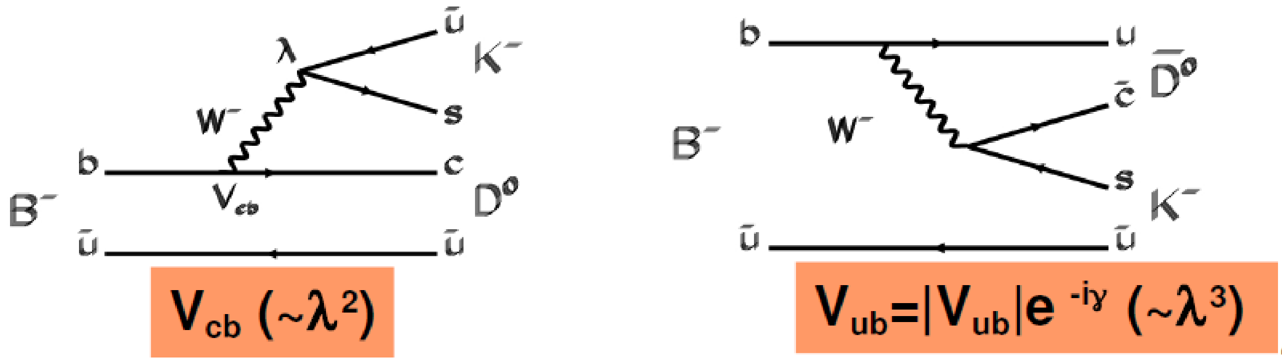
$$\gamma = \arg\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right)$$



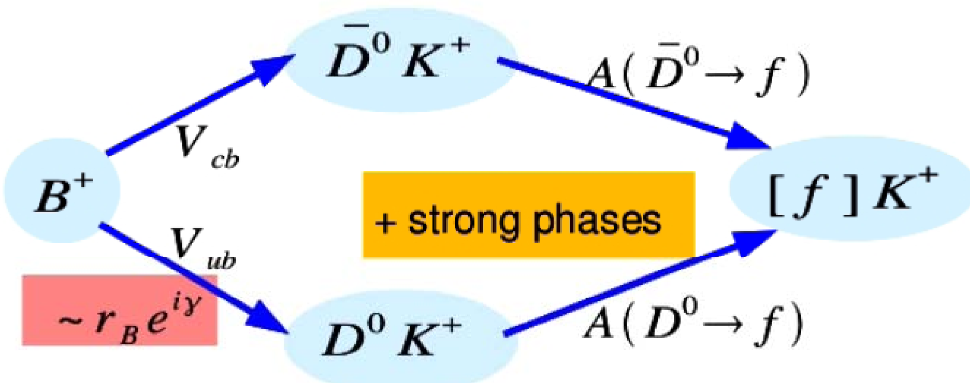
$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

γ measurement in $B \to D^{(*)} K^{(*)}$

- interference of tree diagrams: theoretically clean (no penguin pollution)

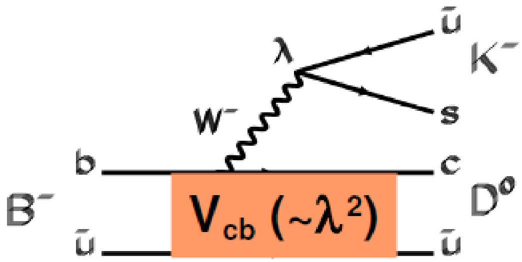


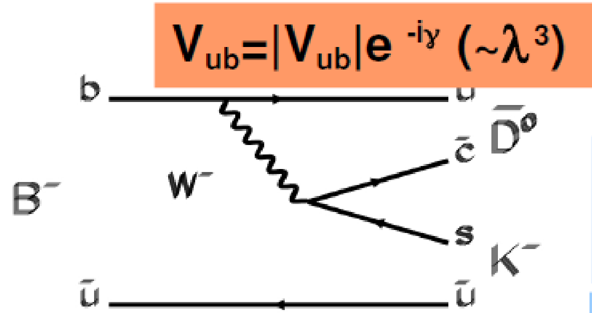
- direct CP violation used ~ no time-dependent CP asymmetry, just rates



- not sensitive to New Physics ~ theoretically clean determination of γ
- theory uncertainties from hadronic parameters

γ measurement in $B^{\pm} \rightarrow D^{(*)}K^{\pm(*)}$





δ_B = strong phase diff.

r_B = amplitude ratio

$$A(B^- \rightarrow D^0 K^-) = A_B \quad A(B^- \rightarrow \bar{D}^0 K^-) = A_B r_B e^{i(\delta_B - \gamma)}$$

$$A(B^+ \rightarrow \bar{D}^0 K^+) = A_B \quad A(B^+ \rightarrow D^0 K^+) = A_B r_B e^{i(\delta_B + \gamma)}$$

$r_B \sim 0.1$

$$A_{CP} \propto A_B r_B \sin\gamma \sin\delta_B$$

- different methods to determine four parameters

1) Use a CP mode for the D⁰

GLW (Gronau, London, Wyler)
CP+ and CP- modes
$(K^+K^-, \pi^+\pi^-)$ ← → $(K_S\pi^0, \phi K_S, \eta K_S, \rho K_S, \omega K_S)$

(Very) small Branching Ratios

CP- probably not accessible at LHC

2) Use CA($K^-\pi^+$) mode for the V_{ub} decay and DCS($K^-\pi^+$) for the V_{cb} decay

ADS (Atwood, Dunietz, Soni)
$D^0 \rightarrow K^-\pi^+$
$D^0 \rightarrow K^-\pi^+\pi^0$
$D^0 \rightarrow K^-\pi^+\pi^-\pi^+$

(Very) small Branching Ratios

Strong phase between the D⁰ decays

3) Use the $D^0 \rightarrow K_S\,\pi\,\pi$ decay

Dalitz GGSZ (Giri, Grossman, Soffer, Zupan)

3 body decay : 2D plane (Dalitz plot) analysis

Dalitz plot description

Only analysis giving information on γ

GLW method: $B^\pm \to DK^\pm \to [f]_D K^\pm$

[PLB 253 (1991) 483]
[PLB 265 (1991) 172]

- proposed by Gronau, London, Wyler
- measure decay rates to CP eigenstates and flavour-specific states
  - ✓ CP eigenstates: $[f]_D = \text{(CP even) } K^+K^-, \pi^+\pi^-, \text{(CP odd) } K_s\pi^0, K_s\omega, K_s\phi$
  - ✓ flavour-specific eigenstates: $D^0 \to K^+\pi^-, \bar{D}^0 \to K^-\pi^+$
- CP eigenstates $\left|D^0{}_{CP+}\right\rangle = \frac{1}{2}\left(\left|D^0\right\rangle + \left|\bar{D}^0\right\rangle\right)$ → two triangles in the complex plane

$$\sqrt{2} \cdot A(B^+ \to D^0_{CP+} K^+) = A(B^+ \to D^0 K^+) + A(B^+ \to \bar{D}^0 K^+)$$

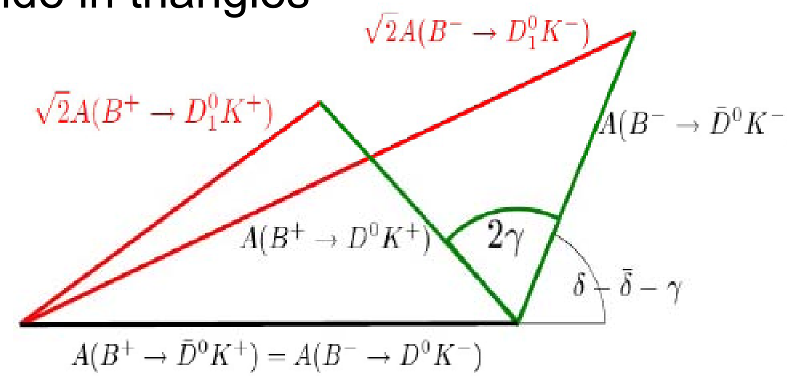
$$\sqrt{2} \cdot A(B^- \to D^0_{CP+} K^-) = A(B^- \to \bar{D}^0 K^-) + A(B^- \to D^0 K^-)$$

- $b \to c$ transition is real $\Longrightarrow$ one common side in triangles

$$A(B^+ \to \bar{D}^0 K^+) = A(B^- \to D^0 K^-)$$

- γ is extracted from $b \to u$ transition

$$A(B^+ \to D^0 K^+) = e^{2i\gamma} \cdot A(B^- \to \bar{D}^0 K^-)$$



- experimentally challenge due to small r_B ~ squashed triangle

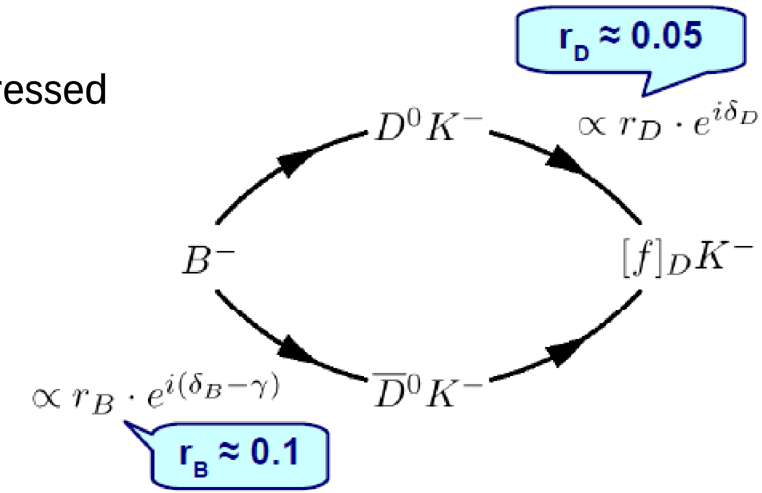
ADS method: $B^\pm \to DK^\pm \to [f]_D K^\pm$

[PRL 78 (1997) 3257]
[PRD 63 (2001) 036005]

- proposed by Atwood, Dunietz, Soni
- interference of amplitudes with similar magnitude → large interference and $\not{CP}$

$$[f]_D = K^+\pi^- \begin{cases} \overline{D}^0 \to K^+\pi^- : \text{Cabibbo - favored} \\ D^0 \to K^+\pi^- : \text{doubly Cabibbo - suppressed} \end{cases}$$

$$r_D = \frac{|A(D^0 \to K^+\pi^-)|}{|A(\overline{D}^0 \to K^+\pi^-)|} \approx \frac{|V^*_{cd} V_{us}|}{|V^*_{ud} V_{cs}|} = \lambda^2 \approx 0.05$$

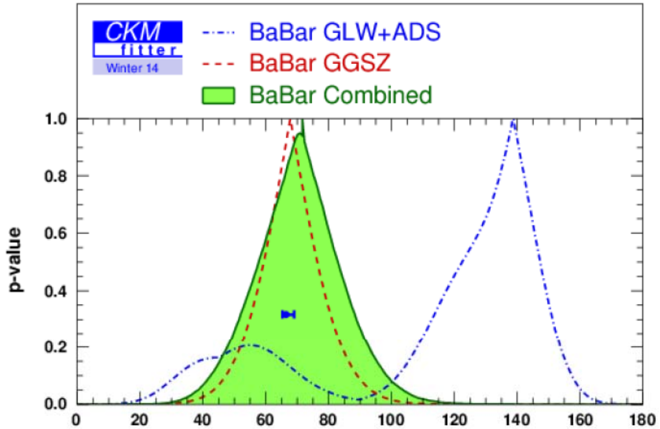


- γ extracted from ratio and asymmetry of decay rates

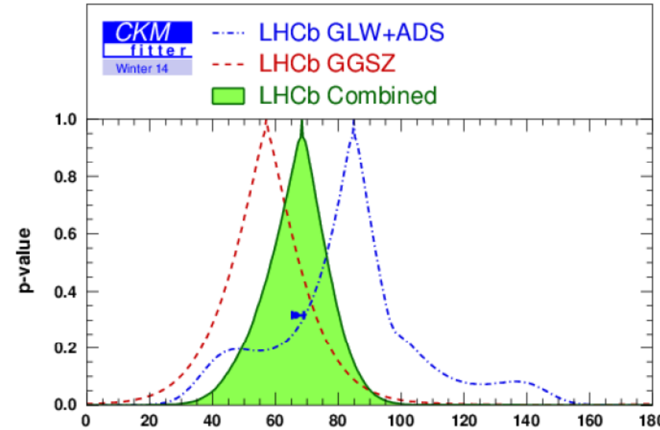
$$R_{ADS} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^-\pi^+]_D K^-) + \Gamma(B^+ \to [K^+\pi^-]_D K^+)} = r_B^2 + r_D^2 + 2\, r_B r_D \cos(\delta_B + \delta_D) \cdot \cos\gamma$$

$$A_{ADS} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) - \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)} = \frac{2 \cdot r_B r_D \sin(\delta_B + \delta_D) \cdot \sin\gamma}{R_{ADS}}$$

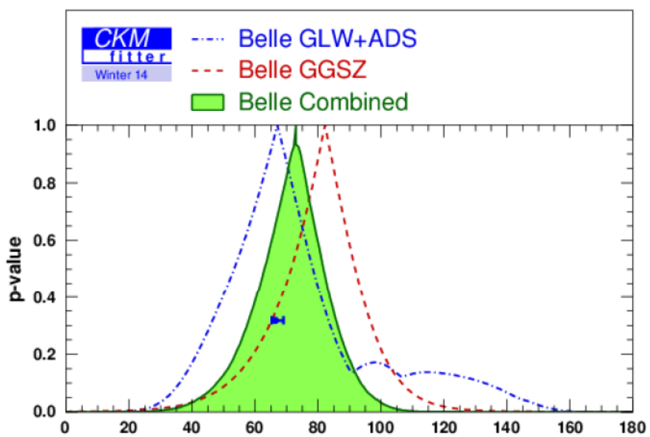
Status of γ measurement



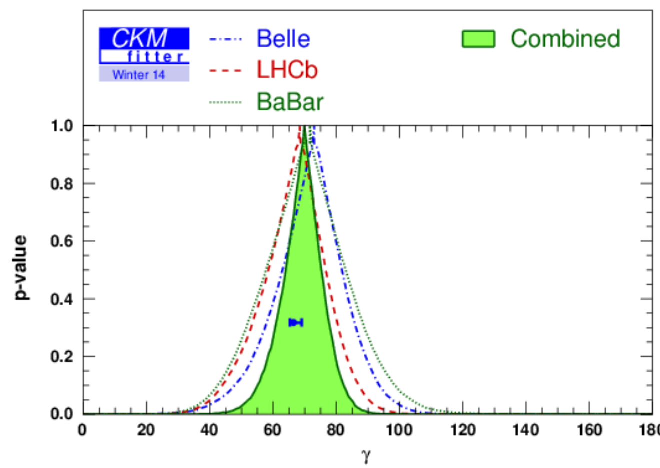
$\gamma[\text{BaBar}] = 72^{+17}_{-19}{}^{\circ}$



$\gamma[\text{LHCb}] = 68^{+12}_{-15}{}^{\circ}$



$\gamma[\text{Belle}] = 73^{+13}_{-15}{}^{\circ}$



$\gamma[\text{combined}] = 70.0^{+7.7}_{-9.0}{}^{\circ}$

$\gamma[\text{fit}] = 67.2^{+4.4}_{-4.6}{}^{\circ}$ (blue bar)