Flavour Physics 1

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Outline

- Flavour Physics 1

  - Flavour Physics in the Standard Model

  - A brief history of Flavour Physics

  - CKM matrix

- Flavour Physics 2

  - CP violation in K decays

  - CP violation in B decays

  - Determination of α, β, γ

- Flavour Physics 3, 4 by S.W. Baek

  - FCNC, rare B decays, lepton flavour violation, minimal flavour violation, Flavour Physics in SUSY, etc.

Structure of the Standard Model

- Gauge group - $SU(3)_\mathrm{c} \times SU(2)_L \times U(1)_Y$
- Matter representation

$$\begin{array}{cc} Q_i & (\mathbf{3},\mathbf{2})_{1/6} \\ U_i^c & (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \\ D_i^c & (\bar{\mathbf{3}},\mathbf{1})_{1/3} \\ L_i & (\mathbf{1},\mathbf{2})_{-1/2} \\ E_i & (\mathbf{1},\mathbf{1})_{1}, \end{array} \qquad \phi \ \ (\mathbf{1},\mathbf{2})_{1/2}.$$

- Spontaneous symmetry breaking - $SU(2)_\mathrm{L} \times U(1)_\mathrm{Y} \to U(1)_\mathrm{EM}$

  for SSB, a negative dim-2 term is required in the Higgs potential.

$$-\mu^2|\phi|^2$$

- The most renormalizable Lagrangian

$$\mathcal{L}_\mathrm{SM} = \mathcal{L}_\mathrm{gauge}(A_a,\psi_i) + \mathcal{L}_\mathrm{Higgs}(\phi, A_a) + \mathcal{L}_\mathrm{Yukawa}(\phi,\psi_i)$$

Structure of the Standard Model

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm gauge}(A_a, \psi_i) + \mathcal{L}_{\rm Higgs}(\phi, A_a) + \mathcal{L}_{\rm Yukawa}(\phi, \psi_i)$$

$$\mathcal{L}_{\rm gauge}(A_a, \psi_i) = -\sum_a \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \sum_{\psi,i} \bar{\psi}_i i D \psi_i$$

- Natural
- Experimentally tested with high accuracy
- stable with respect to quantum corrections
- Highly symmetric: $SU(3)_c \times SU(2)_L \times U(1)_Y$ local symmetry
  + global flavour symmetry
- 3 identical replica of the basic fermion family $\Longrightarrow$ huge flavour-degeneracy

Structure of the Standard Model

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}}(A_a, \psi_i) + \mathcal{L}_{\text{Higgs}}(\phi, A_a) + \mathcal{L}_{\text{Yukawa}}(\phi, \psi_i)$$

$$\mathcal{L}_{\text{Yukawa}}(\phi, \psi_i) = y_{ij}^e \overline{L}^i \phi E^j + y_{ij}^d \overline{Q}^i \phi D^j + y_{ij}^u \overline{Q}^i \tilde{\phi} U^j + \text{h.c.}$$

- Ad hoc
- necessary to describe data
- origin of the flavour structure of the model
- flavour degeneracy is broken

Parameters in the (B)SM

- 3 gauge couplings: α, G_F, α_s
- 2 Higgs parameters: μ^2, λ

Flavour parameters

- 6 quark masses: $m_d, m_u, m_s, m_c, m_b, m_t$
- 3 quark mixing angles + 1 phase: λ, A, ρ, η
- 3 charged lepton masses: m_e, m_μ, m_τ
- 3 neutrino masses
- 3 lepton mixing angles + 1 phase + 2 Majorana phases

BSM

+ strong CP

What is Flavour Physics?

- Flavours = Several copies of the same gauge quantum charges
- quarks and leptons come in three flavours in the Standard Model.

$$(u, c, t), \ (d, s, b), \ (e, \mu, \tau), \ (\nu_1, \nu_2, \nu_3)$$

- Flavour physics = Interactions that distinguish among flavours
- Flavour parameters = $Y_i \ (m_i), \ V_{ij} \ (W\text{-couplings})$
- Flavour changing processes: $B \to \psi K (b \to c\bar{c}s), \ \mu \to e\ell\nu ...$
- Flavour changing neutral current (FCNC):

$$B^0 \leftrightarrow \bar{B}^0 (\bar{b}d \leftrightarrow b\bar{d}), \ \mu \to e\gamma, \ K \to \pi\nu\bar{\nu}, ...$$

- Flavour factories: Belle, BaBaR, MEG, LHCb, CDF, D0, …

Aspects of Flavour Physics

- Families / generations
  - 3 pairs of quarks (are we sure?)
  - 3 pairs of leptons (are we sure?)
- Hierarchies

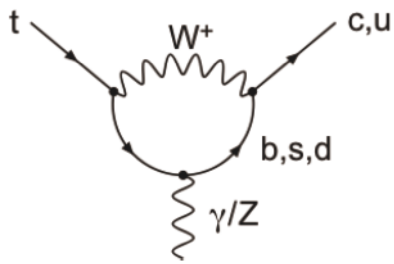
$$m(t) > m(c) > m(u) \qquad m(b) > m(s) > m(d)$$

$$m(\tau) > m(\mu) > m(e) \qquad m(\nu_\tau) > m(\nu_\mu) > m(\nu_e)\ ?$$

- Mixing & couplings
  - hierarchy in quark mixings
  - what about lepton mixings?

Why is (future) Flavour Physics interesting?

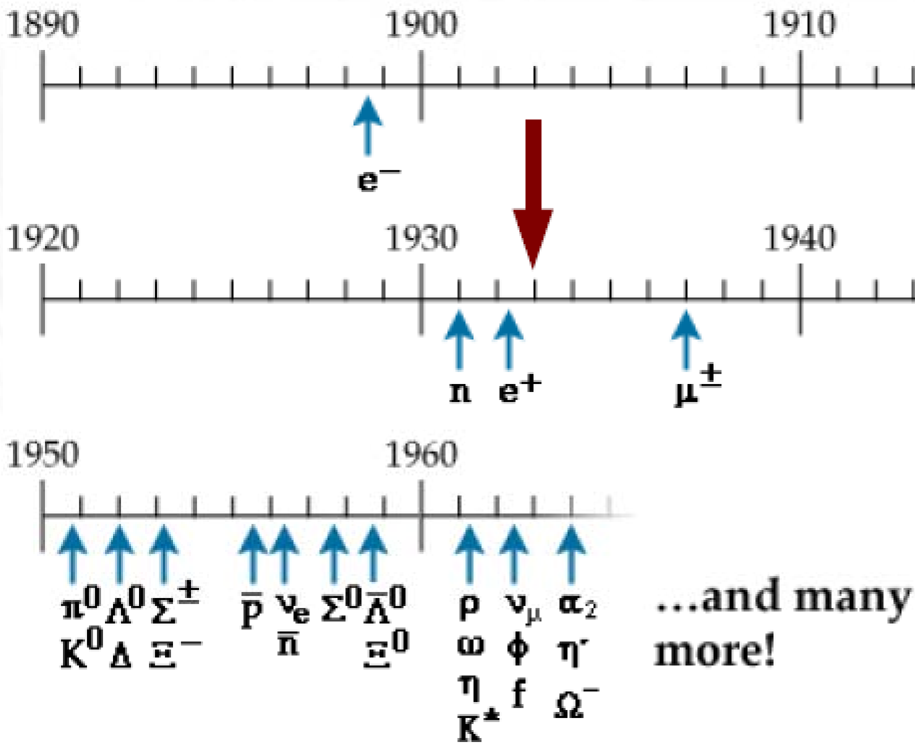
- Source of CP violation (CPV) in the SM: V_{ij}
  - but SM CPV cannot explain the baryon asymmetry of our universe
- Flavour physics is sensitive to new physics a $\Lambda_{\rm NP} \gg E_{\rm experiment}$
  - FCNC suppressed within the SM by $\alpha_W^n, |V_{ij}|, m_f$



$\mathcal{B}(t \to Xq) \sim 10^{-17}$-$10^{-12}$, X=H,γ,Z,g

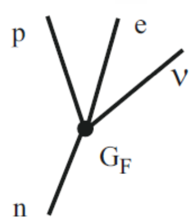
- Flavour puzzle in the SM
  - Why are the Flavour parameters small and hierarchical?
- Flavour puzzle in New Physics
  - If there is NP at the TeV scale, why are FCNC so small?

A brief history of Flavour Physics

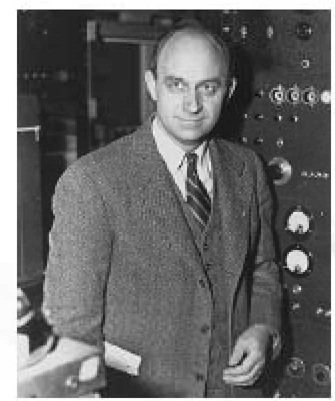


Fermi theory of beta decay

$n \to p + e^- + \overline{\nu}_e$

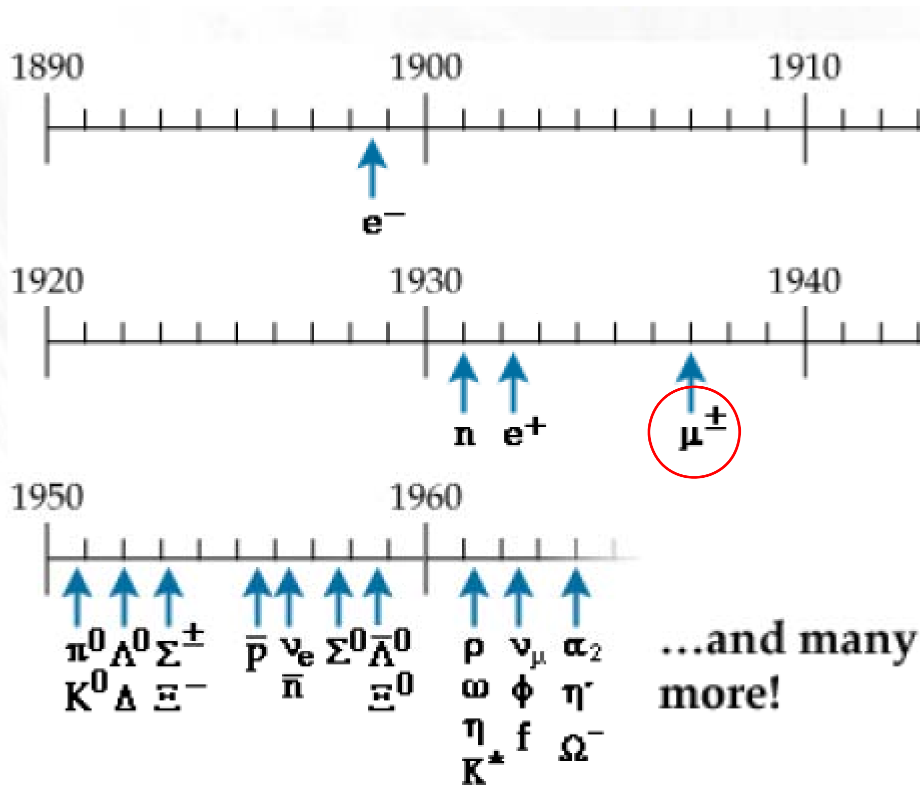


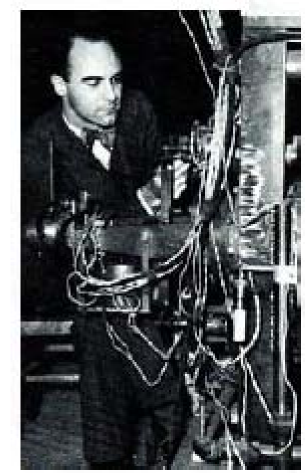
$G_F(\overline{p}\gamma^{\mu}n)(\overline{e}\gamma_{\mu}\nu)$



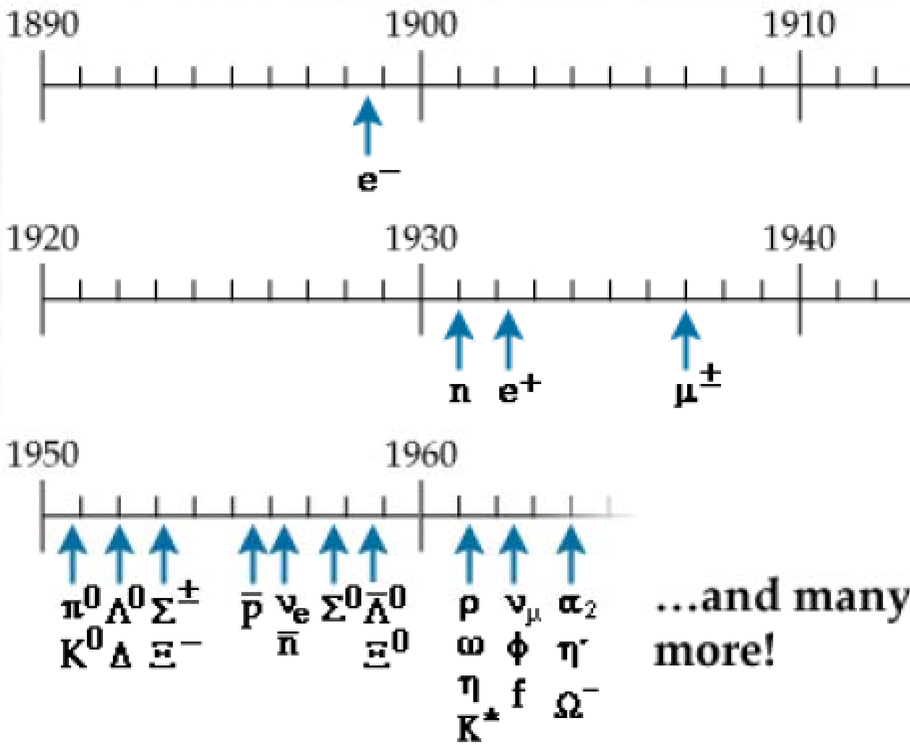
E. Fermi

A brief history of Flavour Physics




C.D. Anderson

A brief history of Flavour Physics



<u>**Observe "strangely behaved" particles**</u>

large production cross sections

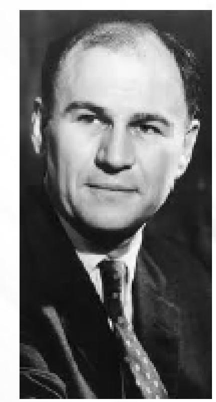
typical for strong interaction

but long lifetimes of order 10^{-10}s

typical for weak decays

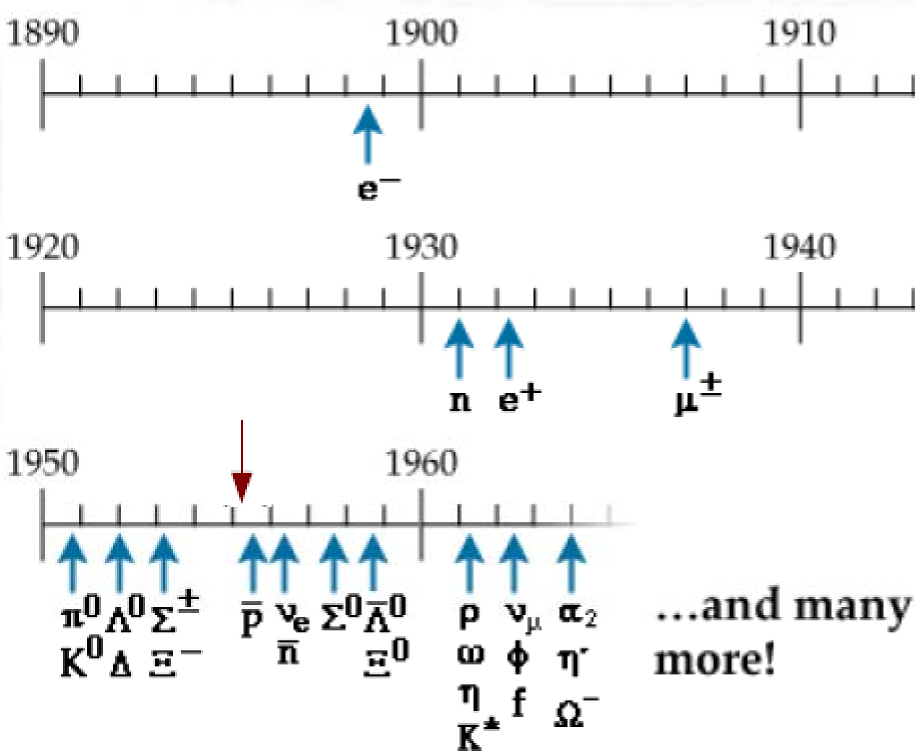
always produced in pairs

G.D. Rochester



C.C. Butler

A brief history of Flavour Physics



Observe "strangely behaved" particles

large production cross sections

typical for strong interaction

but long lifetimes of order 10^{-10}s

typical for weak decays

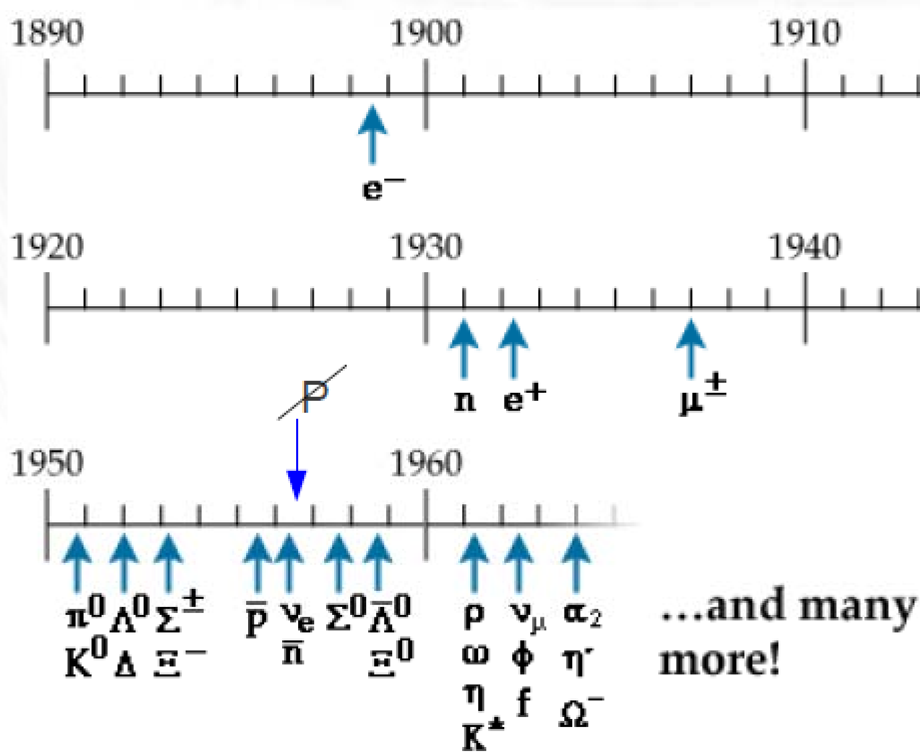
always produced in pairs

$\Rightarrow$

Strangeness
Gell-Mann, Nishijima
1954-55

- conserved in production (strong interaction)
- not conserved in decay (weak interaction)

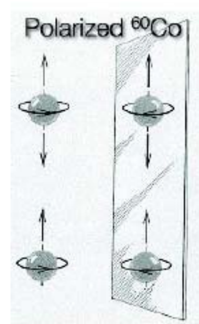
A brief history of Flavour Physics



1956 – Parity Violation

TD Lee

CN Yang

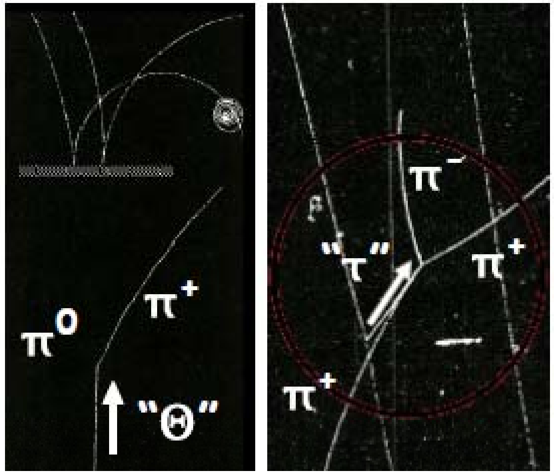


CS Wu (1956)

Parity Violation

Θ/τ-Puzzle: observe two charged, strange, spin-0 mesons

- same mass (~ 500 MeV) and same lifetime, but:
  - one ("Θ") decays into $\pi^+\pi^0$ (even parity)
  - the other ("τ") decays into $\pi^+\pi^+\pi^-$ (odd parity)



❑ $\Theta \rightarrow \pi^+\pi^0$

- spin of Θ and τ is known to be s=0
- spin parity of π $\quad J^P(\pi^\pm) = 0^-$ $\quad J^{PC}(\pi^0) = 0^{-+}$
- the orbital angular momentum of $\pi\pi$, $l = 0$

$$\mathrm{P}(\pi^+\pi^0) = (-1)^{l+2} = +1$$

❑ $\tau \rightarrow \pi^+\pi^+\pi^-$

l : the orbital angular momentum of $\pi\pi$

L : the orbital angular momentum of π and $(\pi\pi)$

- $\boldsymbol{l} + \boldsymbol{L} = \boldsymbol{J} = 0$, which means $l = L$

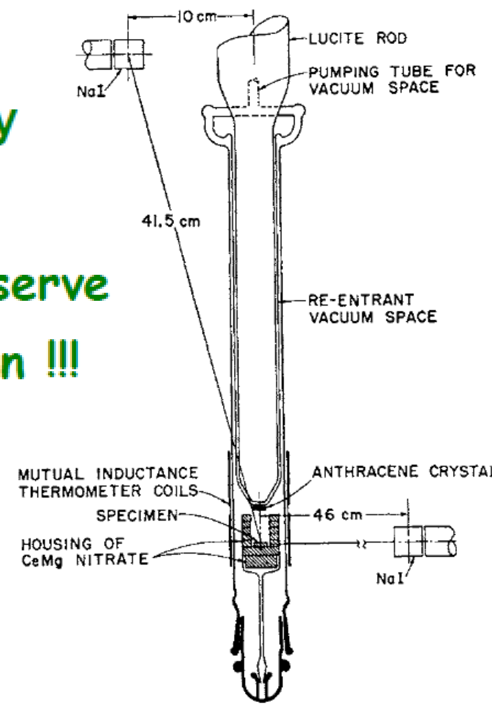
$$\mathrm{P}(\pi^+\pi^+\pi^-) = (-1)^{3+l+L} = -1$$

Parity Violation

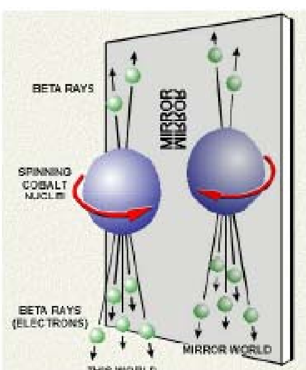
- Lee and Yang investigated past experiments and found that there was ample evidence of parity conservation in the strong and electromagnetic interactions but no evidence in the weak interaction.
  - **parity is not conserved in weak interactions**
  - **"Θ" and "τ" are in fact the same particle (K^+)**

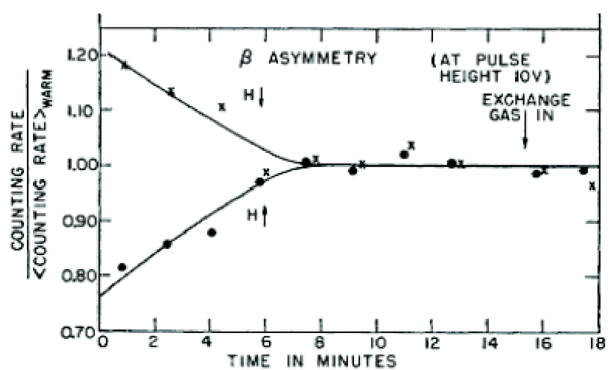
[PR 104 (1956) 254]

- Wu et al. (1956): direct observation of parity violation

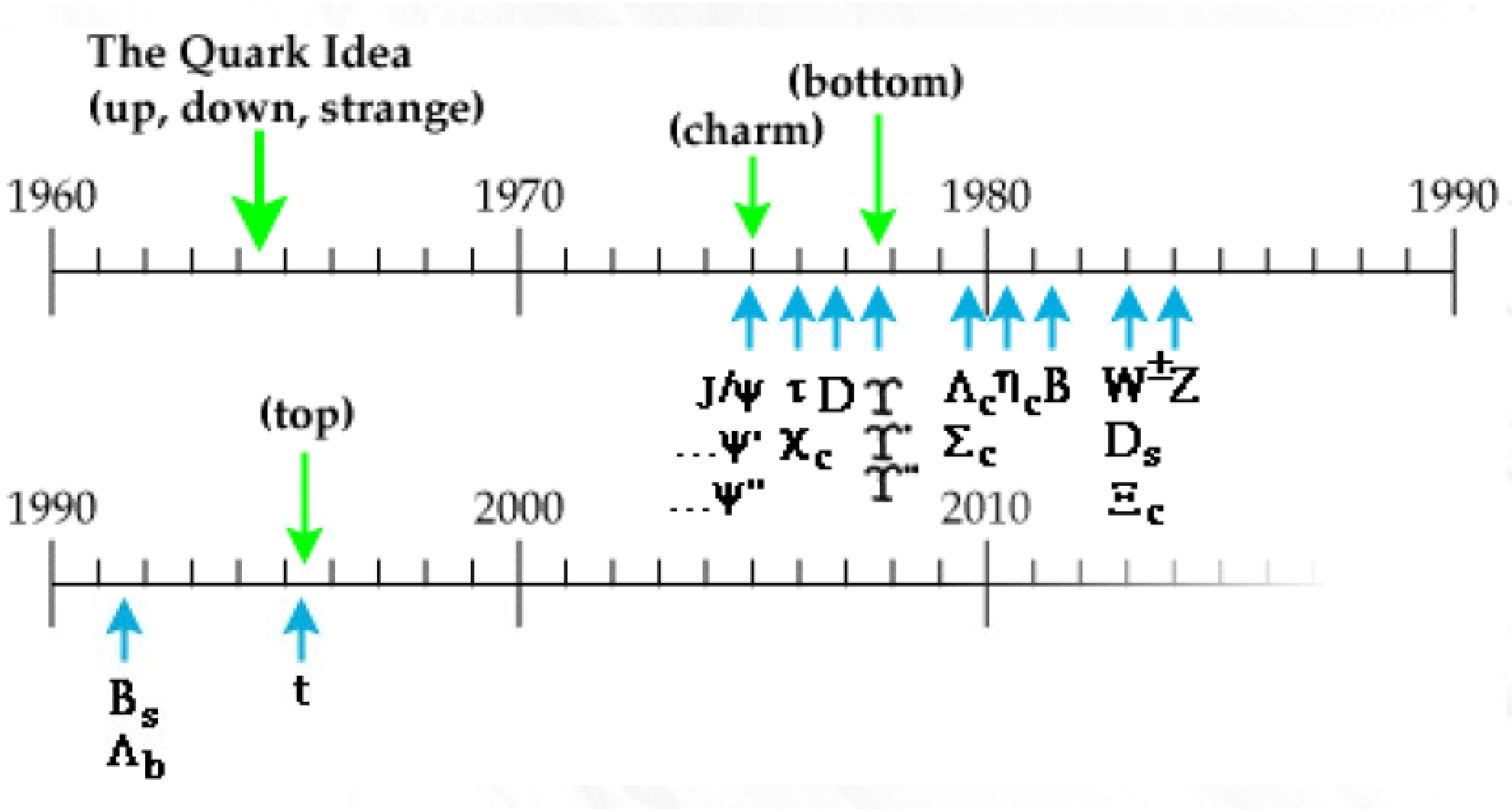


- **measure angular distribution of electrons from β-decay of polarized ^{60}Co (spin=5⁺) to $^{60}Ni^*$ (spin=4⁺)**
- **must be symmetric if parity is conserved → but: observe electrons emitted predominantly opposite to ^{60}Co-spin !!!**





A brief history of Flavour Physics



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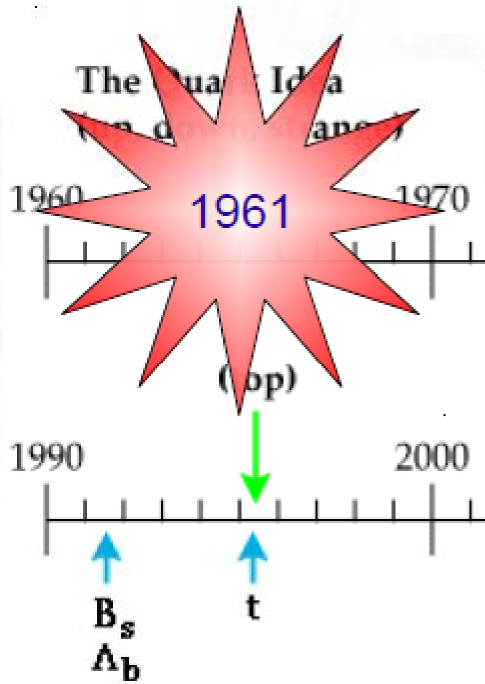


- **Electroweak theory SU(2)xU(1)**
- Quark model $SU_f(3)$
- Weak universality (Cabibbo angle)
- CP-violation in neutral Kaons
- ....

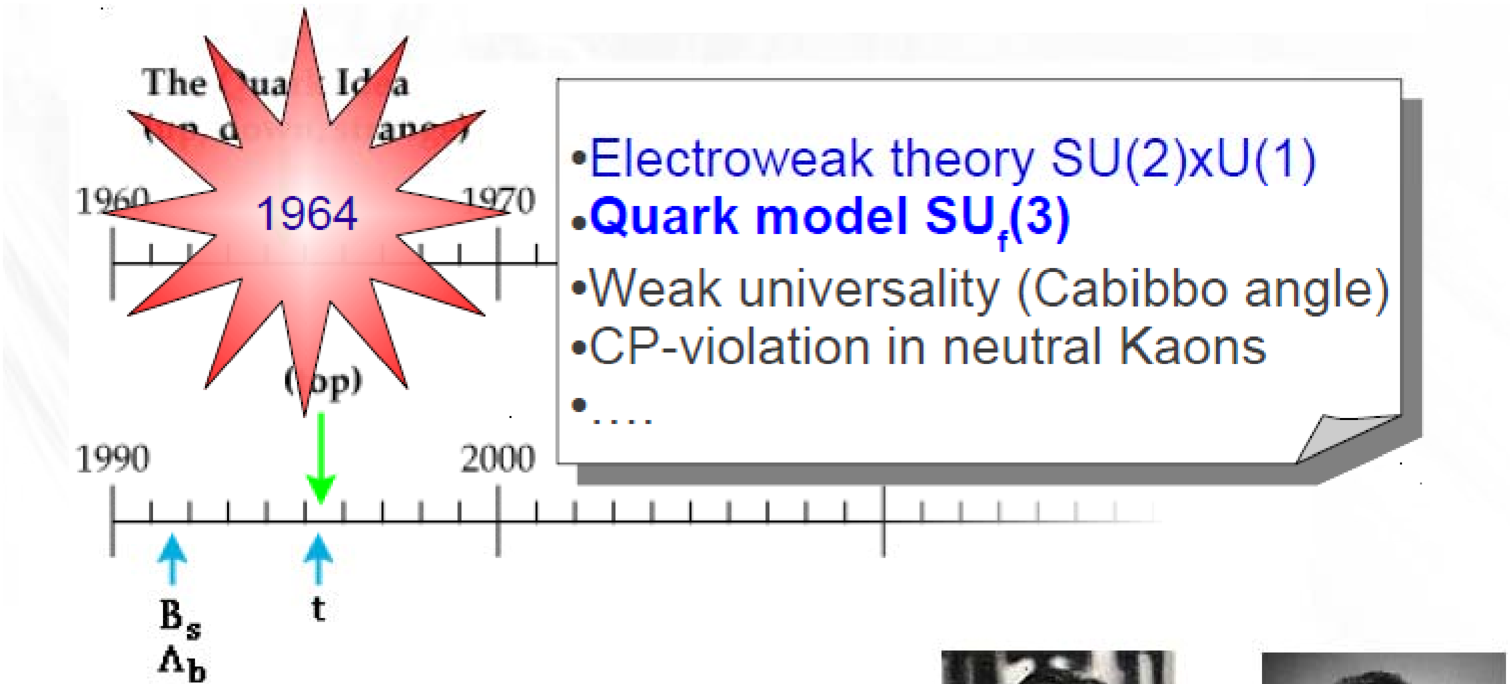
Massive
W,Z bosons
(Higgs mechanism)

S. Weinberg

S. Glashow

A. Salam

A brief history of Flavour Physics



- Electroweak theory SU(2)xU(1)
- **Quark model $SU_f(3)$**
- Weak universality (Cabibbo angle)
- CP-violation in neutral Kaons
- ….

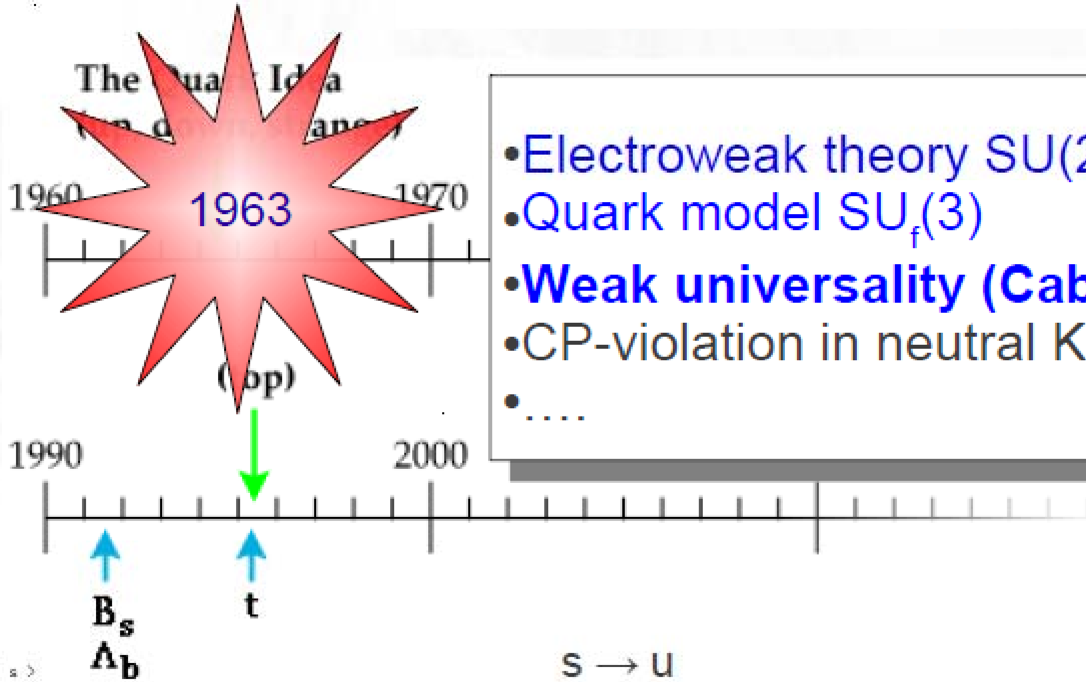
SU(3) flavor symmetry
Classification of hadrons

QUARKS!

G. Zweig

M. Gellman

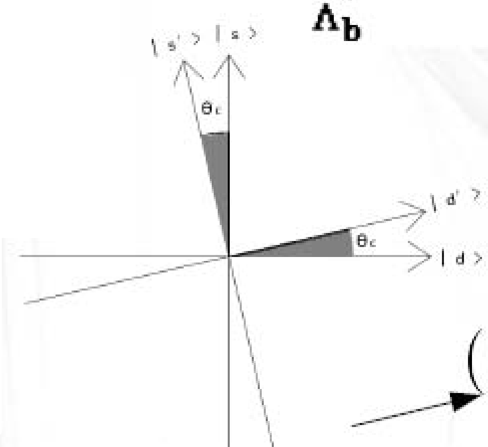
A brief history of Flavour Physics



- Electroweak theory SU(2)xU(1)
- Quark model $SU_f(3)$
- **Weak universality (Cabibbo angle)**
- CP-violation in neutral Kaons
- ….

N. Cabbibo

s → u
is suppressed
by a factor of ~20
in comparison with
d → u

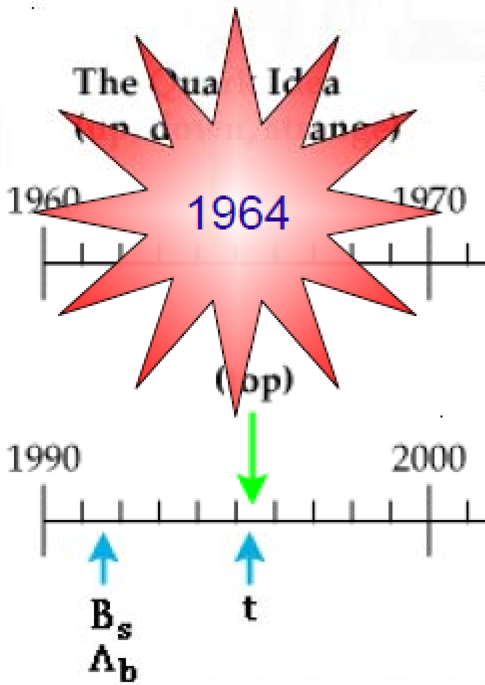


$$(u, d') = (u, d\cos\theta_c + s\sin\theta_c)$$

SU(2) doublet that couples to W

Ex: $\sin\theta_c \simeq ?$

A brief history of Flavour Physics

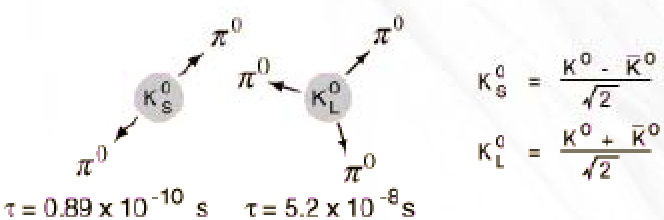


- Electroweak theory SU(2)xU(1)
- Quark model $SU_f(3)$
- Weak universality (Cabibbo angle)
- **CP-violation in neutral Kaons**
- ....

Christenson, Cronin, Fitch, Turlay

$$K^0(d\bar{s}), \qquad \bar{K}^0(\bar{d}s)$$

$$\mathrm{CP}\, K^0 = -\bar{K}^0$$



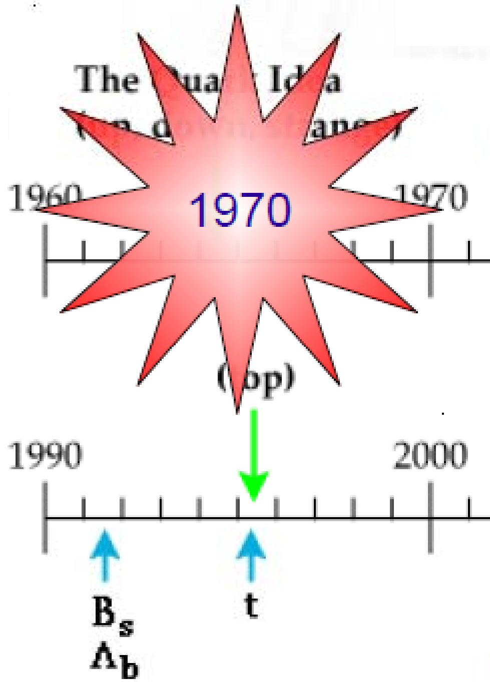
$$\mathrm{CP}|\pi\pi\rangle = |\pi\pi\rangle$$

$$\mathrm{CP}|\pi\pi\pi\rangle = -|\pi\pi\pi\rangle$$

$$\mathrm{CP} : K_L \not\to \pi\pi$$

$$\not{\mathrm{CP}} : K_L \to \pi\pi$$

A brief history of Flavour Physics



- ....
- GIM mechanism
- Renormalizability of EW theory

Suppression of Flavor-Changing Neutral Current (FCNC) interactions

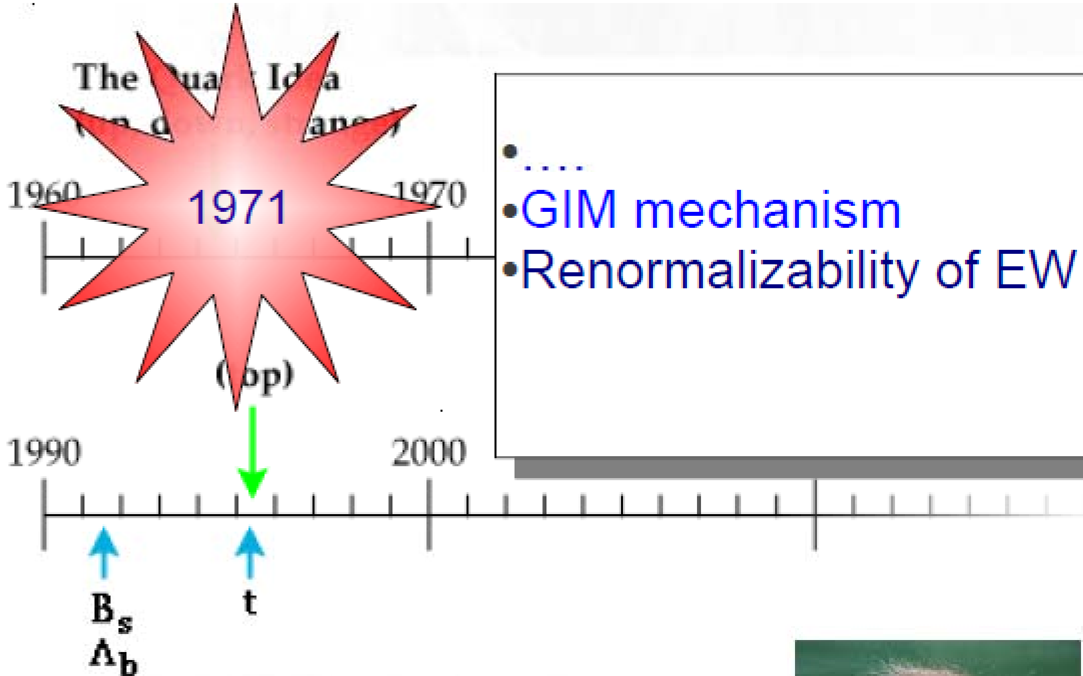
charm prediction

S. Glashow

J. Iliopoulos

L. Maniani

A brief history of Flavour Physics

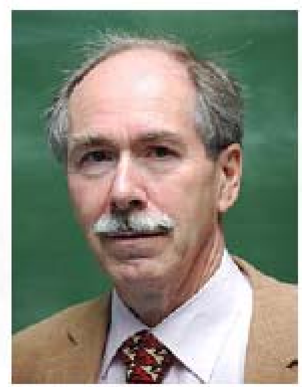


- ….
- GIM mechanism
- Renormalizability of EW theory

Invention of a powerful tool to perform Perturbation theory in gauge theories

$$\int d^4k \to \mu^{2\epsilon} \int d^{4-2\epsilon}k$$

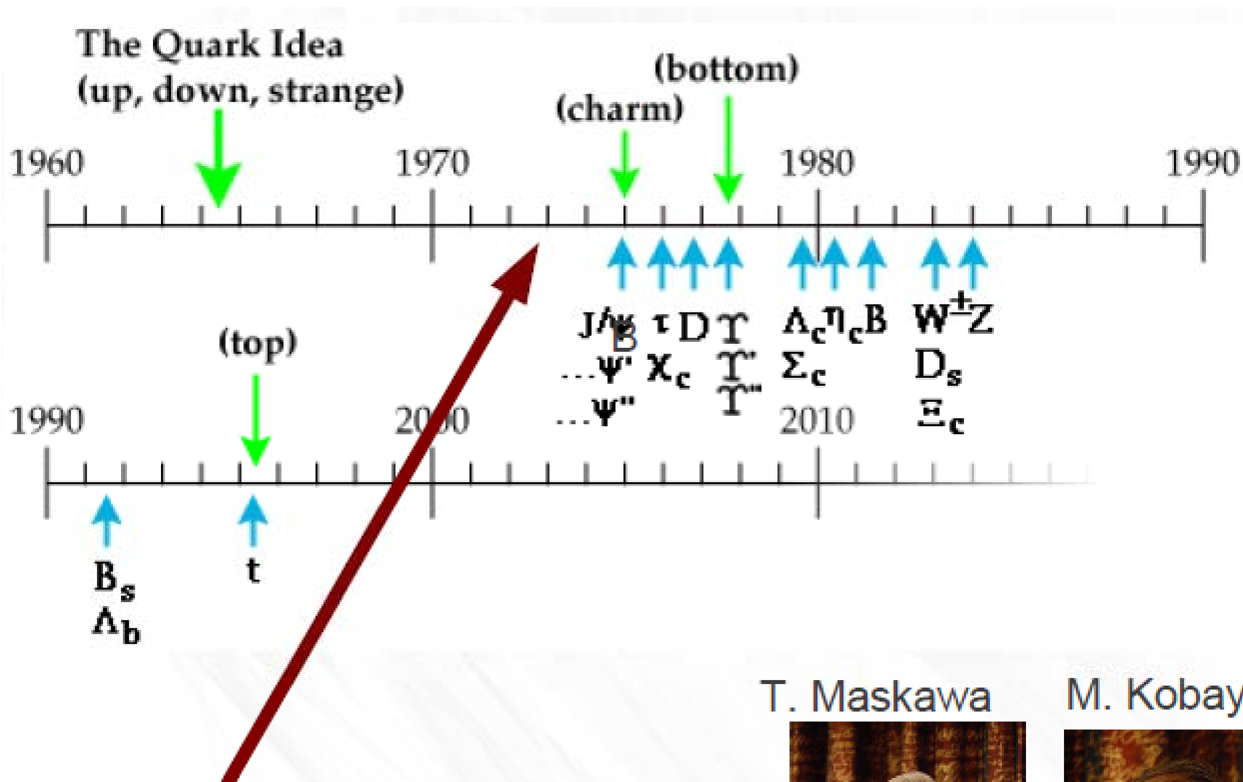
Dimensional regularization



G. 't Hooft

M. Veltman

A brief history of Flavour Physics



CP violation requires at least three generations

Prediction of 3rd generation!

T. Maskawa

M. Kobayashi

Charm Quark

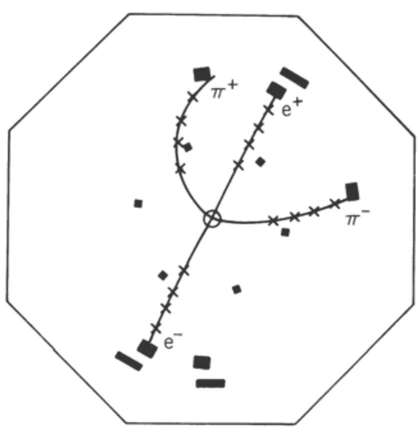
❑ November revolution (1974)

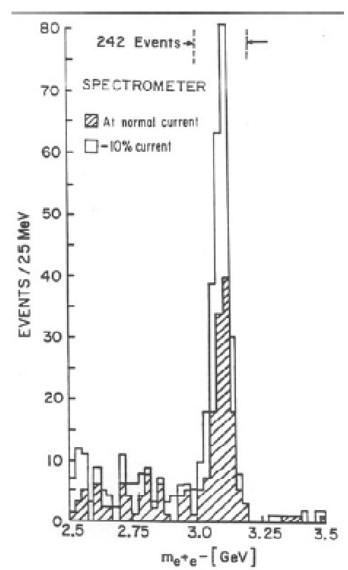
- observation of a narrow resonance at a mass of 3.1 GeV
  - in $p + Be \rightarrow e^+e^- + X$ at BNL (Ting et al.) → "*J*"
  - in $e^+e^- \rightarrow e^+e^-$, $\mu^+\mu^-$, hadrons at SLAC (Richter et al.) → "Ψ"

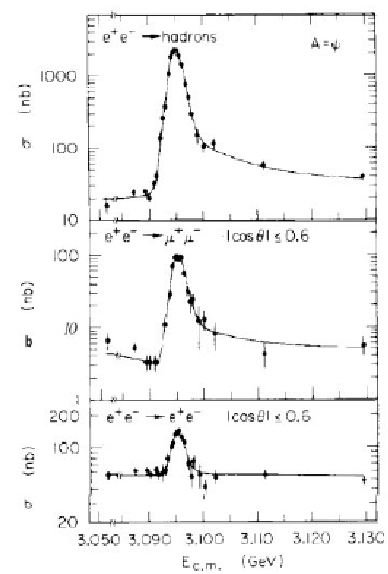
  } *J*/Ψ

- narrow width → long lifetime → cannot be an excited state of u,d,s
- a bound state of $c\bar{c}$

$$\psi(2S) \rightarrow J/\psi\pi^+\pi^-$$

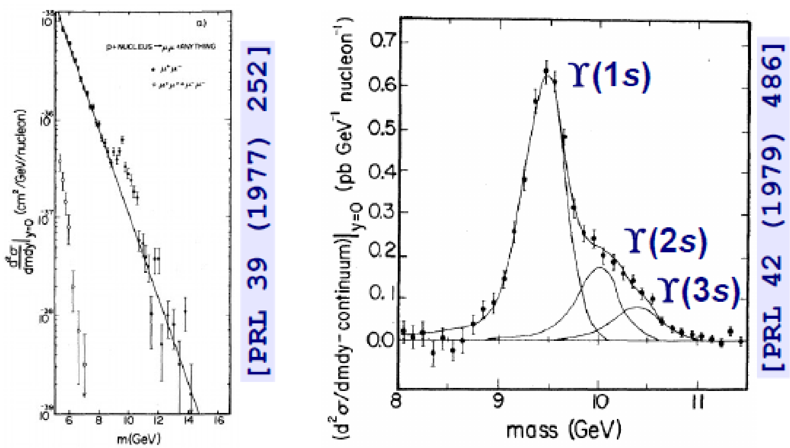






Bottom Quark

- Lederman et al. (1997): p+Cu→ $\mu^+\mu^-$+X
  - observation of an excess of $\mu^+\mu^-$ pairs at 9.4-10.4 GeV
  - resolved into three resonances, interpreted as bound states of $b\bar{b}$



  - 1980: Υ(4S) discovered in e^+e^- collisions by CLEO experiment at Cornell
  - first observation of B^0 and $B^{\pm}$ mesons by CLEO in 1983

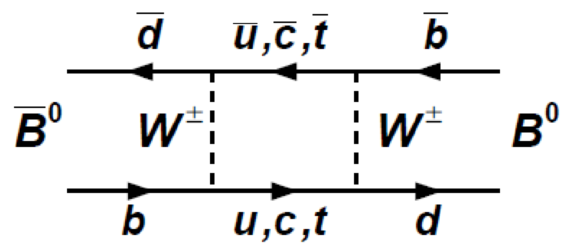
$B-\bar{B}$ Mixing

- Argus experiment at DESY(1987)

  - e^+e^- collider operating at the $\Upsilon(4s)$ resonance
  - produce $B^0\bar{B}^0$ pairs through

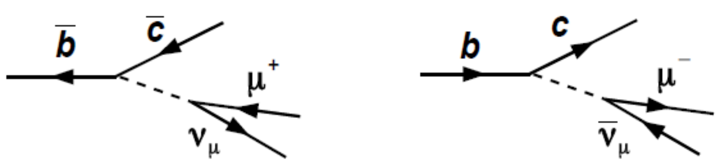
$$e^+e^- \rightarrow \Upsilon(4s) \rightarrow B^0\bar{B}^0$$

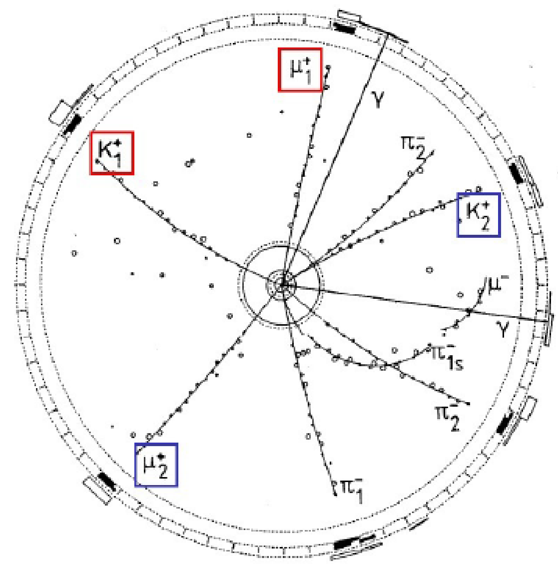
  - B^0–$\bar{B}^0$ mixing through box diagrams



  - can be measured in semi-leptonic decays

$$B^0 \rightarrow D^{*-}\mu^+\nu_\mu \qquad \bar{B}^0 \rightarrow D^{*+}\mu^-\bar{\nu}_\mu$$

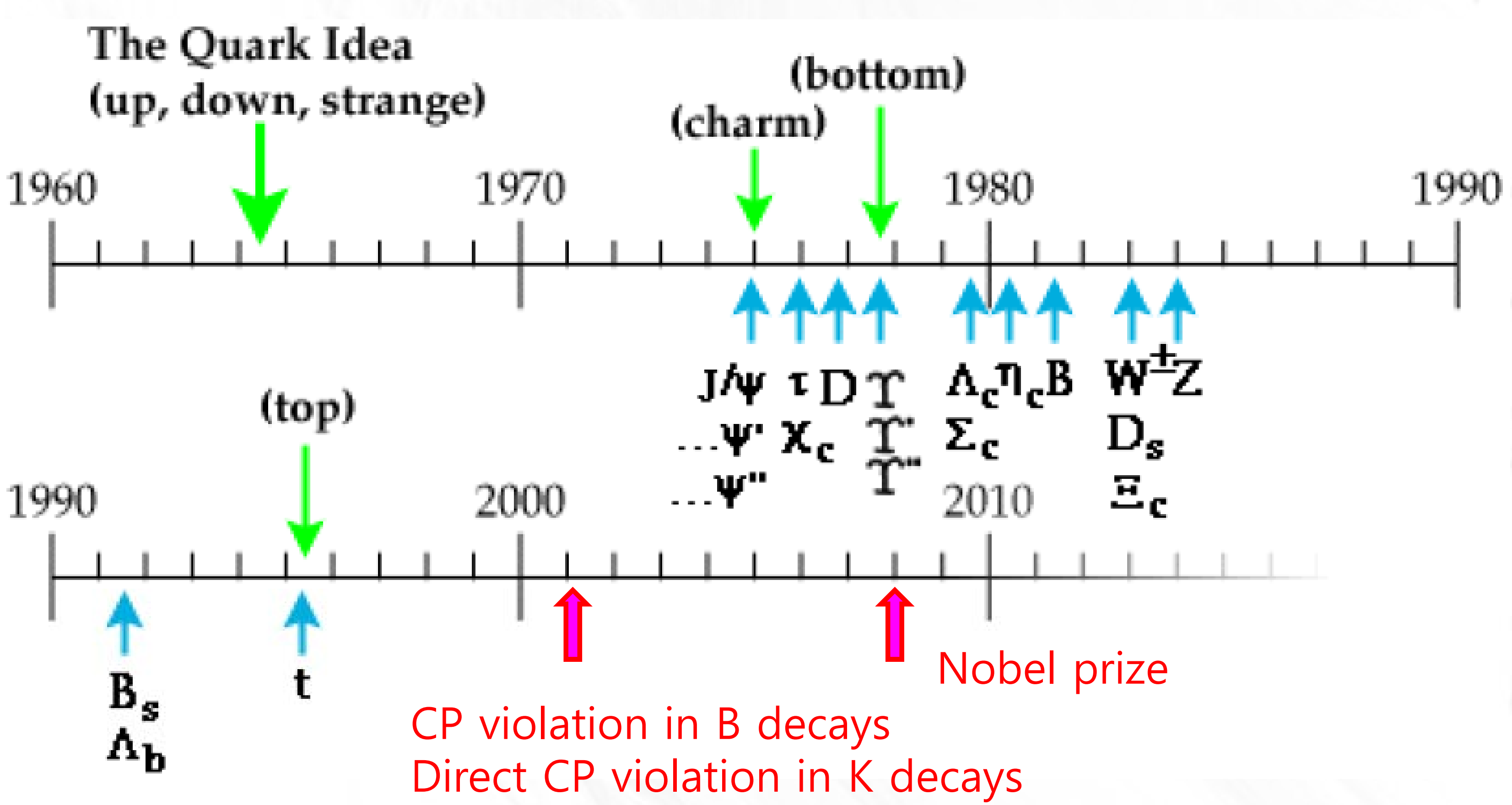




  - observe "like-sign event" with two μ^- or two μ^+
    → B^0 or $\bar{B}^0$ must have mixed before it decayed

  - observe stronger mixing than expected → top quark mass must be large

A brief history of Flavour Physics



Cabibbo Angle

- Observed different strength of weak interaction in different processes

  - about 4% smaller in neutron decay $n \to p e^- \bar{\nu}_e$ than in muon decay $\mu^- \to \nu_\mu e^- \bar{\nu}_e$

  - a factor of 20 smaller in decays of strange particles

    e.g. $\Sigma^- \to n e^- \bar{\nu}_e$

- Cabibbo: weak interaction couples to linear combination (1963)

$$j_W^{+\mu} = \bar{u}\gamma^\mu(1-\gamma^5)d'$$

$$d' = \cos\theta_c \cdot d + \sin\theta_c \cdot s$$

$$\lambda = \sin\theta_c \approx 0.22$$

d′ = weak eigenstate d, s = mass eigenstates

- the suppressions can be understood in today's language

$$\frac{d \to u\,W^-}{\mu^- \to \nu_\mu W^-} = \cos^2\theta_c \approx 0.96 \qquad \frac{s \to u\,W^-}{d \to u\,W^-} = \frac{\sin^2\theta_c}{\cos^2\theta_c} \approx \frac{1}{20}$$

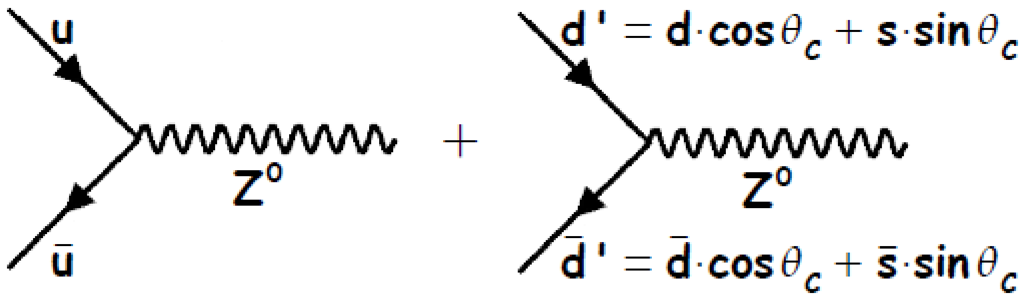
- G_F: the strength of the universal four-Fermi interactions.

Flavour changing neutral current (FCNC)

- Flavour change neutral currents observed to be strongly suppressed.

CC	BR	NC	BR
$K^+ \to \mu^+ \nu_\mu$	$63.54 \pm 0.14\%$	$K^0 \to \mu^- \mu^+$	$6.84 \pm 0.11 \times 10^{-9}$
$K^+ \to \pi^0 e^+ \nu_e$	$5.08 \pm 0.05\%$	$K^+ \to \pi^+ \overline{\nu}_e \nu_e$	$1.5^{+1.3}_{-0.9} \times 10^{-10}$ [a]

- FCNC amplitude should be sizable if weak interaction couples to u and d′.



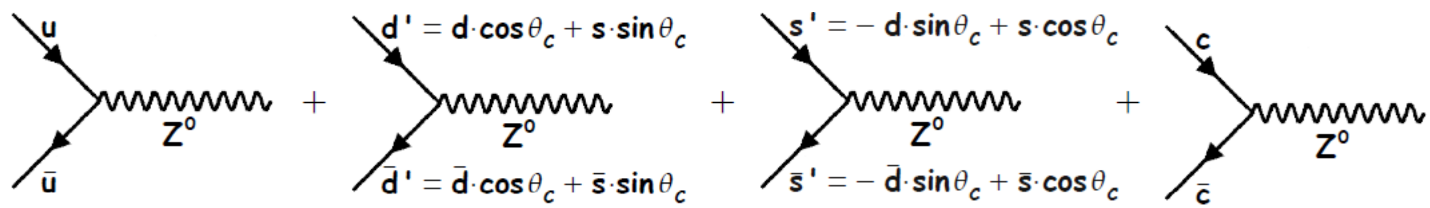
$$u\overline{u} + d'\overline{d}' \Rightarrow u\overline{u} + d\overline{d}\cos^2\theta_c + s\overline{s}\sin^2\theta_c + (d\overline{s} + \overline{d}s)\cos\theta_c \sin\theta_c$$

GIM mechanism

Glashow, Ilioupolis, Maiani (1970): quark doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \cdot \begin{pmatrix} d \\ s \end{pmatrix}$$

- Cancellation of FCNC at the tree level



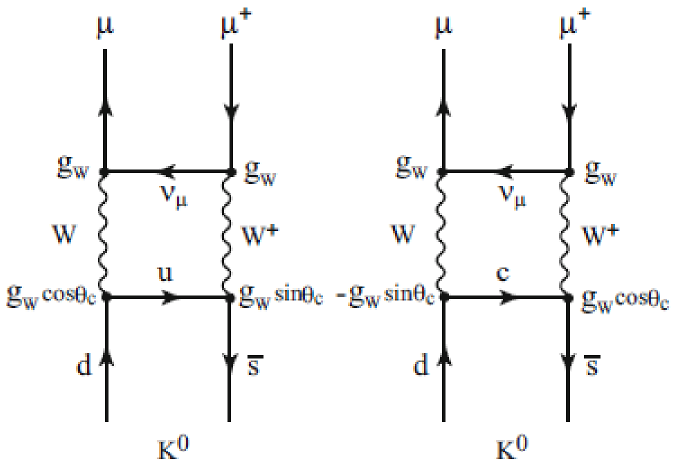
$$u\bar{u} + c\bar{c} + d'\bar{d}' + s'\bar{s}' \quad \Rightarrow \quad u\bar{u} + c\bar{c} + (d\bar{d} + s\bar{s})\cos^2\theta_c + (d\bar{d} + s\bar{s})\sin^2\theta_c$$

$$+(d\bar{s} + \bar{d}s)\cos\theta_c \sin\theta_c - (d\bar{s} + \bar{d}s)\sin\theta_c \cos\theta_c$$

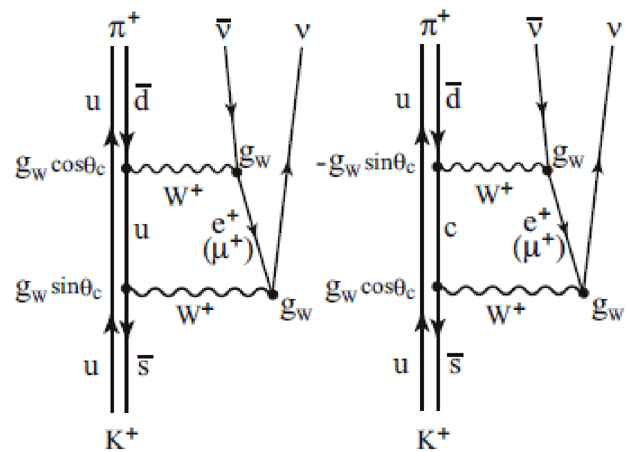
$$= u\bar{u} + c\bar{c} + d\bar{d} + s\bar{s}$$

GIM mechanism

- Why does FCNC decay exist? And why is it much smaller?
  - due to the higher order charged current process



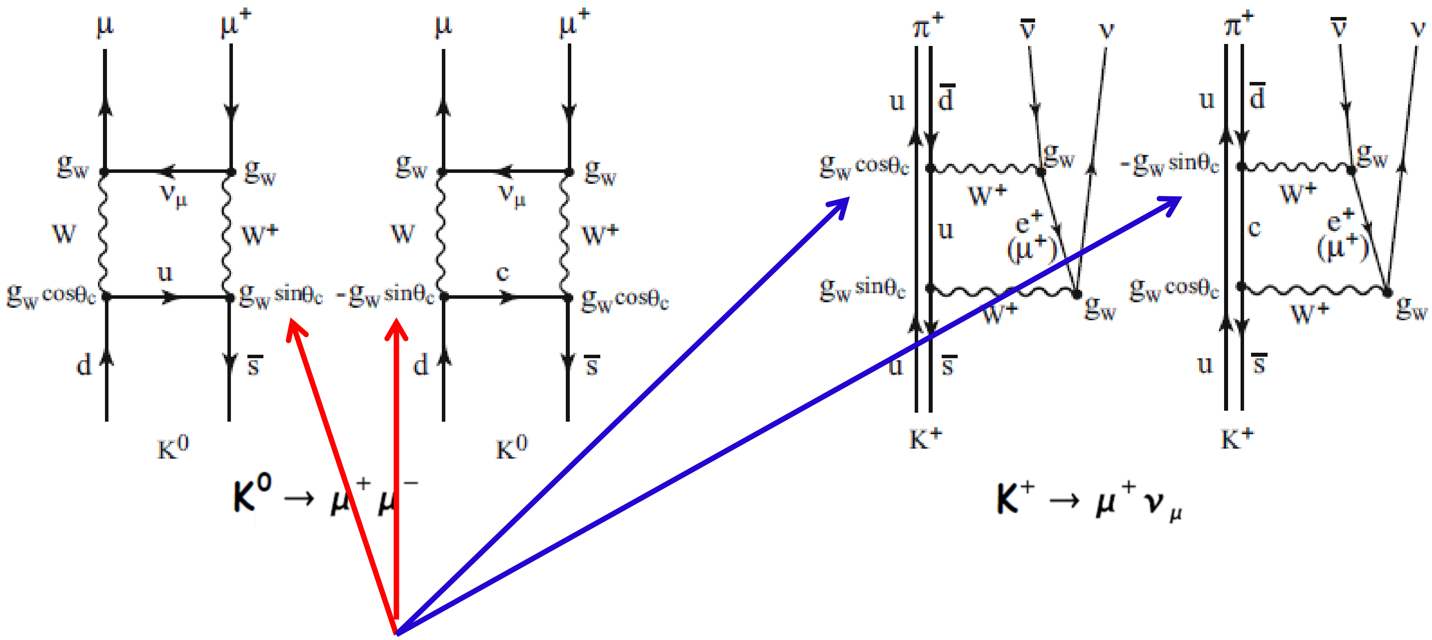
$K^0 \rightarrow \mu^+ \mu^-$



$K^+ \rightarrow \mu^+ \nu_\mu$

GIM mechanism

- Why does FCNC decay exist? And why is it much smaller?
  - due to the higher order charged current process



opposite signs $\Longrightarrow$ exact cancellation if degenerate quark masses

  - mass difference induces $\mathcal{M}(K^0 \to \mu\overline{\mu}) \sim \alpha\, G_F \left(\frac{m_u^2}{m_c}\right)^2 \Rightarrow$ $m_c \sim 1.5$ GeV

Kobayashi-Maskawa mechanism

- CP violation can be generated by complex phase in quark-mixing if three quark doublets exist (Kobayashi, Maskawa, 1972)

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

CKM matrix

- Introduction of the third family (before the discovery of charm quark)

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix} \quad \textbf{with} \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

mass eigenstates

- generalization of Cabibbo rotation ~ GIM extended
- V_{CKM} = a 3×3 unitary matrix

Origin of the CKM matrix

- SU(2) invariant Yukawa interactions

$$-\mathcal{L}_I(\Psi,\Phi) = \sum_{j,k}\left[Y_{jk}\overline{\Psi}'_{jL}\Phi\, d'_{kR} + Y'_{jk}\overline{\Psi}'_{jL}\Phi^c u'_{kR}\right] \qquad \Psi_{kL} = \begin{bmatrix} u'_{kL} \\ d'_{kL}\end{bmatrix} \quad \Phi = \begin{bmatrix}\phi^+ \\ \phi^0\end{bmatrix}$$

  - Terms with j≠k appear because there is no reason to forbid them.

- After symmetry breaking $\langle\phi^0\rangle = \frac{v}{\sqrt{2}}, \langle\phi^\pm\rangle = 0$

$$-\mathcal{L}_I(\Psi,\Phi) = \sum_{j,k}\left[\overline{u}'_{jL}M'_{jk}u'_{kR} + \overline{d}'_{jL}M_{jk}d'_{kR} + \text{h.c.}\right] + \cdots \equiv \left[\overline{U}'_L M^U U'_R + \overline{D}'_L M^D D'_R + \text{h.c.}\right] + \cdots$$

$$M_{jk} = Y_{jk}\frac{v}{\sqrt{2}}, \quad M'_{jk} = Y'_{jk}\frac{v}{\sqrt{2}} \qquad M^U = [M'_{jk}], \quad M^D = [M_{jk}] \qquad U' = \begin{bmatrix} u' \\ c' \\ t'\end{bmatrix}, \quad D' = \begin{bmatrix} d' \\ s' \\ b'\end{bmatrix}$$

  - M_{jk} and M'_{jk} are not diagonal in general.

  - Need to diagonalize them and express the Lagrangian in terms of the mass eigenstates to describe physical processes.

Origin of the CKM matrix

- Any N×N matrix can be diagonalized using two unitary matrices.

$$M^U_{\text{dia}} = A_L M^U A_R^\dagger\,, \quad M^D_{\text{dia}} = B_L M^D B_R^\dagger$$

$$M^U_{\text{dia}} = \begin{bmatrix} m_u & & \\ & m_c & \\ & & m_t \end{bmatrix}, \quad M^D_{\text{dia}} = \begin{bmatrix} m_d & & \\ & m_s & \\ & & m_b \end{bmatrix}$$

  - All masses can become real.

- The fermion mass Lagrangian is given by

$$\begin{aligned} -\mathcal{L}_M &= \overline{U}'_L M^U U'_R + \overline{D}_L M^D D'_R + \text{h.c.} \\ &= \overline{U}'_L A_L^\dagger A_L M^U A_R^\dagger A_R U'_R + \overline{D}'_L B_L^\dagger B_L M^D B_R^\dagger B_R D'_R + \text{h.c.} \\ &= (\overline{A_L U'_L}) M^U_{\text{dia}} (A_R U'_R) + (\overline{B_L D'_L}) M^D_{\text{dia}} (B_R D'_R) + \text{h.c.} \\ &= \sum \left[m_j (\overline{u}_{jL} u_{jR} + \overline{u}_{jR} u_{jL}) + m'_j (\overline{d}_{jL} d_{jR} + \overline{d}_{jR} d_{jL}) \right] \end{aligned}$$

with the unitary transformation

$$U_L = A_L U'_L\,, \quad U_R = A_R U'_R\,, \quad D_L = B_L D'_L\,, \quad D_R = B_R D'_R$$

  - Free Lagrangian and NC Lagrangian do not change under the unitary transformation.

Origin of the CKM matrix

- The charged current Lagrangian changes its form

$$
\begin{aligned}
-\mathcal{L}_{cc} &= (g_W/\sqrt{2}) \sum \left[\overline{U}'_L \gamma^\mu D'_L W_\mu^+ + \text{h.c.} \right] \\
&= (g_W/\sqrt{2}) \sum \left[\overline{U}_L \gamma^\mu \left(A_L^\dagger B_L \right) D_L W_\mu^+ + \text{h.c.} \right] \\
&= (g_W/\sqrt{2}) \sum \left[\overline{U}_L \gamma^\mu V D_L W_\mu^+ + \text{h.c.} \right] \\
&= (g_W/\sqrt{2}) \sum \left[\overline{u}_{jL} \gamma^\mu V_{jk} d_{kL} W_\mu^+ + \overline{d}_{kL} \gamma^\mu V_{jk}^* u_{jL} W_\mu^- \right]
\end{aligned}
$$

  - $V = A_L^\dagger B_L$ ~ the Cabibbo-Kobayashi-Maskawa (CKM) matrix

- CP invariance requires that the CKM matrix be real.

$$L \propto V_{ij} \overline{U}_i \gamma^\mu (1-\gamma_5) D_j W_\mu^\dagger + V_{ij}^* \overline{D}_i \gamma^\mu (1-\gamma_5) U_j W_\mu$$

$\updownarrow$ CP conjugation

$$L_{CP} \propto V_{ij} \overline{D}_i \gamma^\mu (1-\gamma_5) U_j W_\mu + V_{ij}^* \overline{U}_i \gamma^\mu (1-\gamma_5) D_j W_\mu^\dagger$$

If $V_{ij}^* = V_{ij} \rightarrow L = L_{CP}$: i.e. CP conservation

- Mixing of down-type quarks ~ a (historical) convention.

Why three generations?

- How may generation is required for CP violation?
  - A N×N matrix has $2N^2$ parameters
  - Unitary condition removes N^2 degree of freedoms.

$$\sum_j V_{ij}V_{ji}^* = 1,\ \sum_j V_{ij}V_{jk}^* = 0\ (N + 2\,{}_N C_2 = N^2)$$

  - $(2N^2\text{-}N^2)=N^2$ d.o.f. can be divided into

$${}_N C_2 = \frac{N(N-1)}{2} \text{ angles and } \frac{N(N+1)}{2} \text{ phases}$$

  - Each quark field can change their phase.

$$q_j \rightarrow q_j' = e^{i\phi_j} q_j$$

q_R should have an opposite phase to q_L to make the mass term invariant.

but, one of them corresponds to an overall phase. (V does not change)
This remove (2N-1) degree of freedoms. $q_j \rightarrow q_j' = e^{i\phi} q_j$

- N(N-1)/2 angles + (N-1)(N-2)/2 phases
- CP violation requires at least N>3.

δ_{KM}

n	$\frac{(n-1)(n-2)}{2}$
1	0
2	0
3	1
4	3

Homework

1. Show that there is no CP violation for N=3 if any two of the quarks are mass degenerate.

2. Explain why there is no CKM-like matrix in the lepton sector of the SM.

Parametrization of the CKM matrix

- Original KM matrix

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3 + \sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3 - \cos\theta_2\cos\theta_3 e^{i\delta} \end{pmatrix}.$$

- Standard CKM matrix

$$V_{\rm CKM} \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = R_{23}(I_{\delta_D} R_{13} I^{\dagger}_{\delta_D}) R_{12}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$I_{\delta_D} = {\rm diag}(1, 1, e^{i\delta}) \qquad 0 \le \theta_{ij} \le \pi/2, \quad 0 \le \delta \le 2\pi$$

$$s_{ij} \equiv \sin\Theta_{ij},\ c_{ij} \equiv \cos\Theta_{ij}$$

Wolfenstein parametrization

- V_{CKM} in Nature is hierarchical $\theta_{13} \ll \theta_{23} \ll \theta_{12} \ll 1$.
  - Also in experiments, $c_{13} - 1 < 10^{-5}$

$\lambda = \sin\theta_c \approx 0.22$

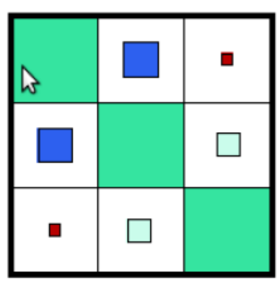
Cabibbo rotation

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \\ -\lambda & 1-\frac{1}{2}\lambda^2 & \\ & & \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- define $s_{12} = \lambda\,, \quad s_{23} = A\lambda^2\,, \quad s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$

$A, \rho, \eta \sim \mathcal{O}(1)$

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$



$$V_{\rm CKM}^\dagger V_{\rm CKM} = 1$$

Wolfenstein parametrization

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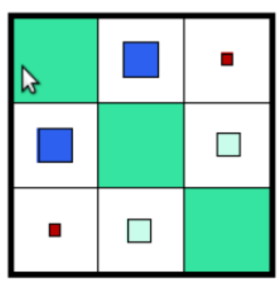
$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$



$$V_{\rm CKM}^\dagger V_{\rm CKM} = 1$$

Only V_{ub} and V_{td} are complex

Wolfenstein parametrization

- higher orders in λ

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2-\lambda^4/8 & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta})+A\lambda^5(\bar{\rho}-i\bar{\eta})/2 \\ -\lambda+A^2\lambda^5[1-2(\bar{\rho}+i\bar{\eta})]/2 & 1-\lambda^2/2-\lambda^4(1+4A^2)/8 & A\lambda^2 \\ A\lambda^3[1-\bar{\rho}-i\bar{\eta}] & -A\lambda^2+A\lambda^4[1-2(\bar{\rho}+i\bar{\eta})]/2 & 1-A^2\lambda^4/2 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

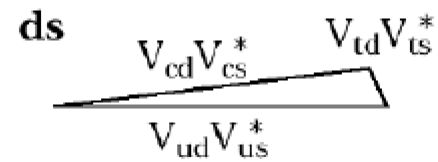
$$\overline{\rho}+i\overline{\eta} \equiv -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = (\rho+i\eta)\left(1-\frac{\lambda^2}{2}\right)+O(\lambda^4)$$

$$\rho+i\eta = \sqrt{\frac{1-A^2\lambda^4}{1-\lambda^2}}\,\frac{\overline{\rho}+i\overline{\eta}}{1-A^2\lambda^4(\overline{\rho}+i\overline{\eta})}$$

Hierarchy in Unitary relations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\underset{\mathcal{O}(\lambda)}{V_{ud}\,V_{us}^*} + \underset{\mathcal{O}(\lambda)}{V_{cd}\,V_{cs}^*} + \underset{\mathcal{O}(\lambda^5)}{V_{td}\,V_{ts}^*} = 0$$



Hierarchy in Unitary relations

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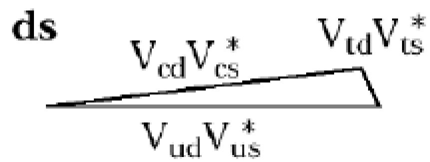
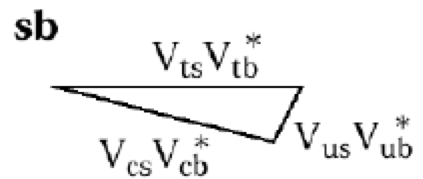


$$\underset{\mathcal{O}(\lambda^4)}{V_{us}\,V_{ub}^*} + \underset{\mathcal{O}(\lambda^2)}{V_{cs}\,V_{cb}^*} + \underset{\mathcal{O}(\lambda^2)}{V_{ts}\,V_{tb}^*} = 0$$

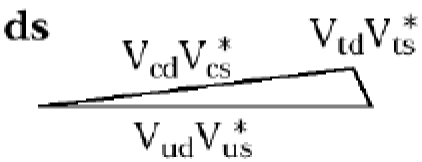


Hierarchy in Unitary relations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

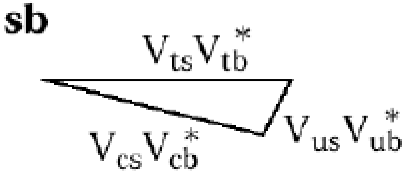
$$\underset{\mathcal{O}(\lambda)}{V_{ud}V_{us}^*} + \underset{\mathcal{O}(\lambda)}{V_{cd}V_{cs}^*} + \underset{\mathcal{O}(\lambda^5)}{V_{td}V_{ts}^*} = 0$$

ds



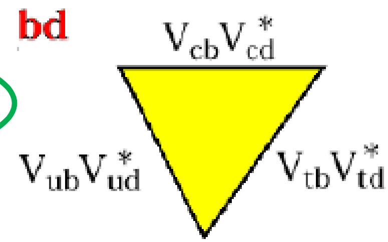
$$\underset{\mathcal{O}(\lambda^4)}{V_{us}V_{ub}^*} + \underset{\mathcal{O}(\lambda^2)}{V_{cs}V_{cb}^*} + \underset{\mathcal{O}(\lambda^2)}{V_{ts}V_{tb}^*} = 0$$

sb



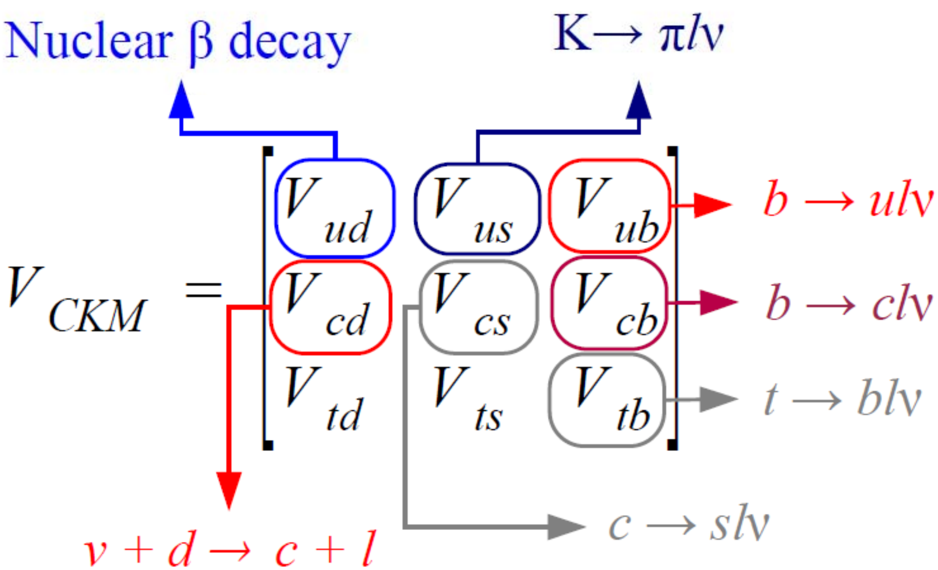
$$\underset{\mathcal{O}(\lambda^3)}{V_{ud}V_{ub}^*} + \underset{\mathcal{O}(\lambda^3)}{V_{cd}V_{cb}^*} + \underset{\mathcal{O}(\lambda^3)}{V_{td}V_{tb}^*} = 0$$

bd

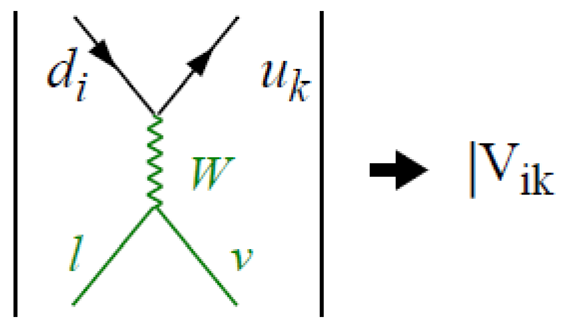


The CKM triangle !

Determination of CKM matrix elements



Once we assume unitarity, the CKM matrix can be <u>completely determined</u> using only exp. info from processes mediated <u>by tree-level</u> c.c. amplitudes



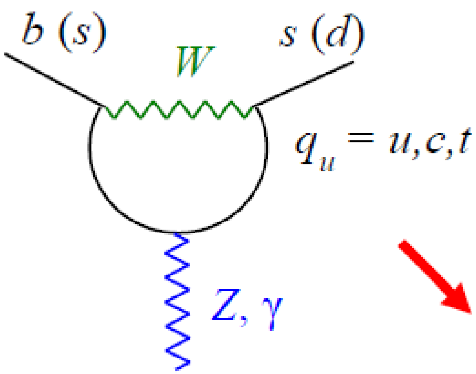
Excellent determination (error ~ 0.5 %)
Very good determination (error ~ 0.1%)
Good determination (error ~ 2 %)
Sizable error (5-15 %)
Not competitive with unitarity constraints

Determination of CKM matrix elements

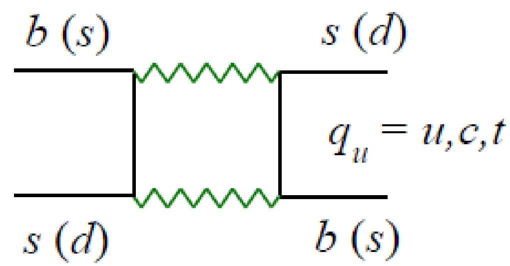
The only CKM elements we cannot access via tree-level processes are V_{ts} & V_{td}

Loop-induced amplitudes:

ΔF =1 FCNC



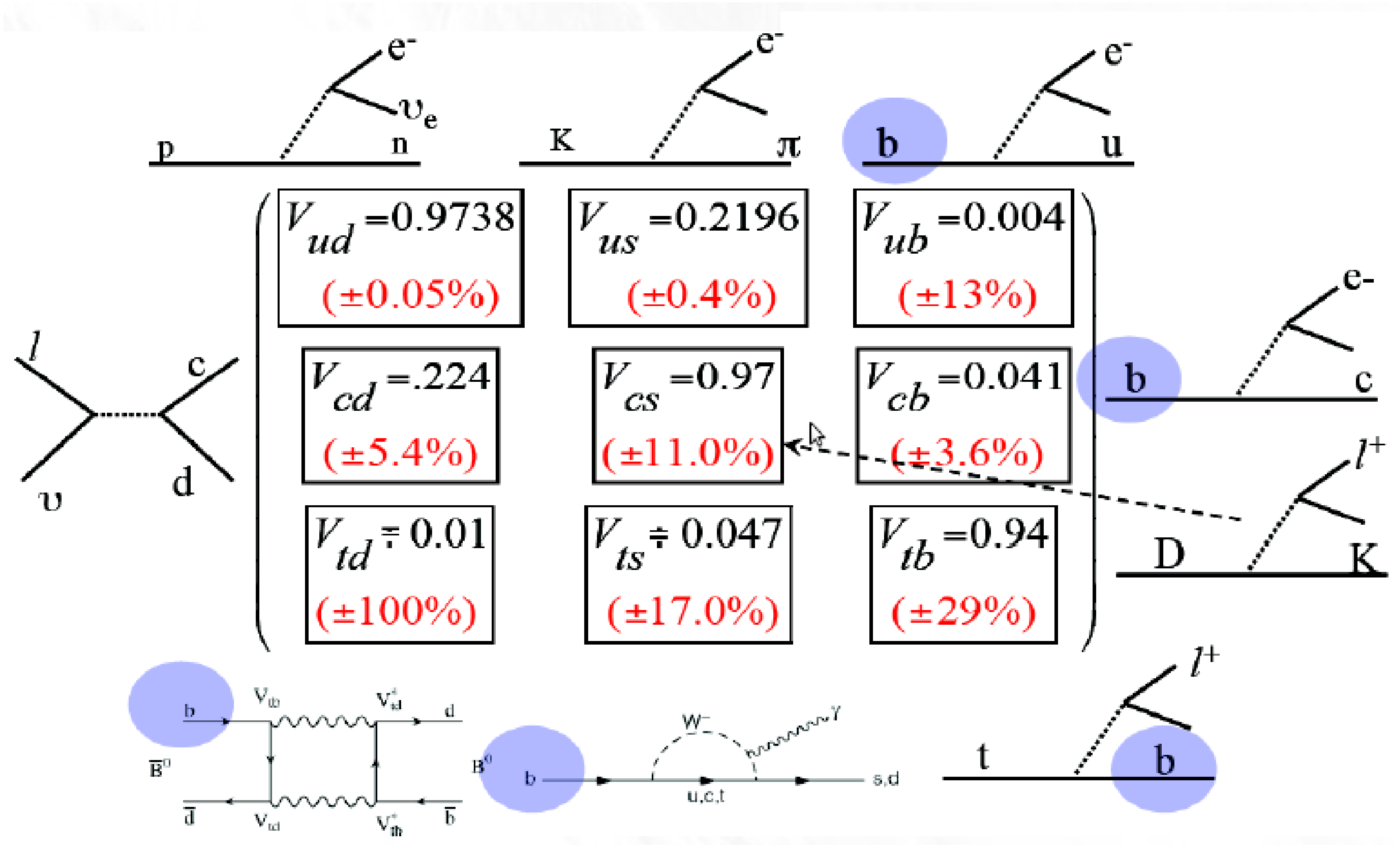
ΔF=2 (neutral-meson mixing)



GIM mechanism

[large top-quark contribution: $A \sim m_t^2 \, V_{tq}^* V_{tb}$]

Determination of CKM matrix elements



The Unitarity Triangle

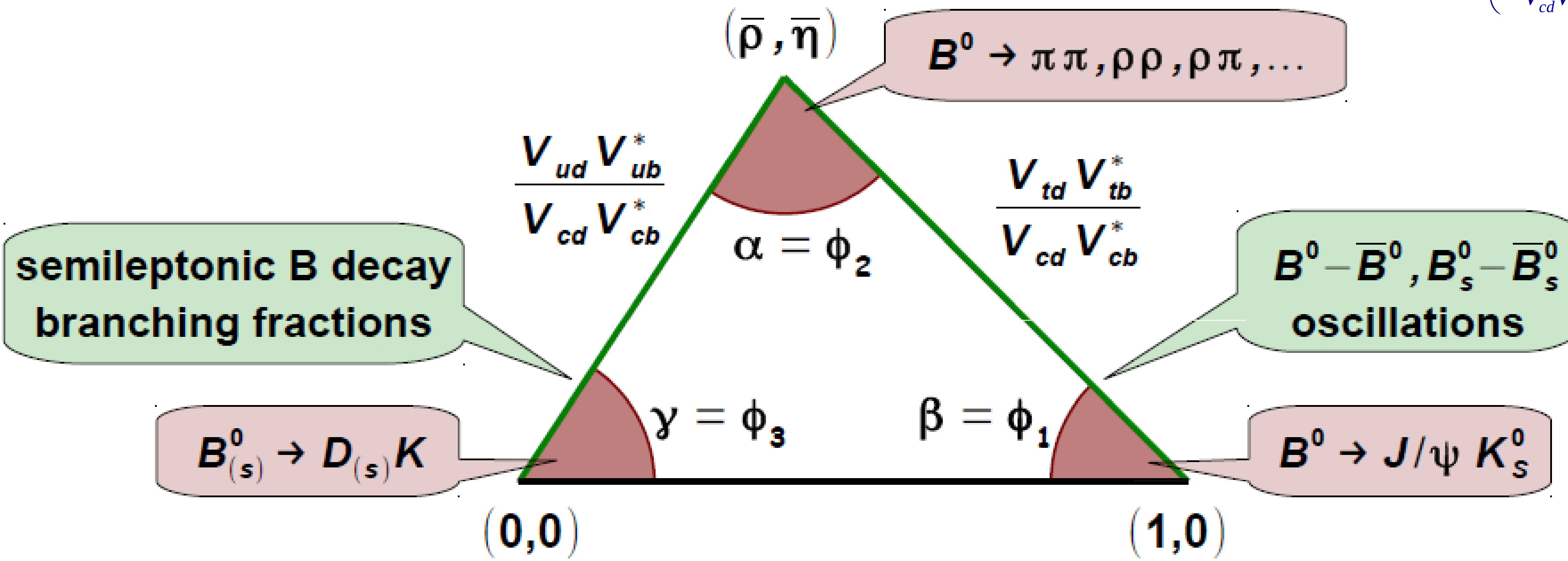
$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \Rightarrow 1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

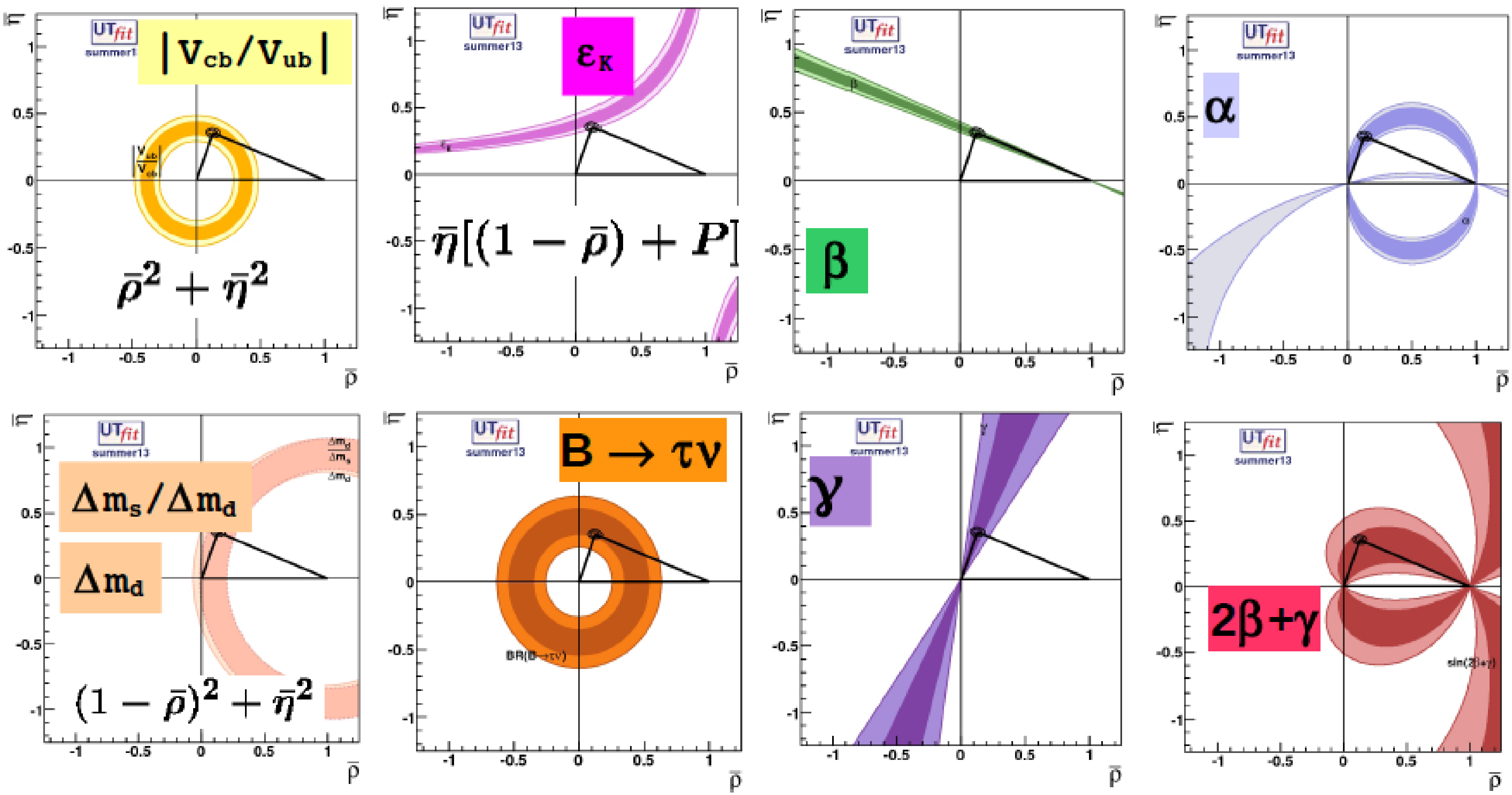
$A\lambda^3(\rho - i\eta)$ $\quad -A\lambda^3$ $\quad A\lambda^3(1-\rho-i\eta)$



- measure the lengths of the two sides: *CP* conserving quantities
- measure all three angles: *CP* violating quantities (angles = phases !)
- many observables → overconstraint determination of triangle

consistency check of Standard Model !

Unitarity Triangle analysis in the SM



Unitarity Triangle analysis in the SM

Observables	Accuracy
$\|V_{ub}/V_{cb}\|$	~ 13%
ε_K	~ 0.5%
Δm_d	~ 1%
$\|\Delta m_d/\Delta m_s\|$	~ 1%
$\sin 2\beta$	~ 3%
α	~ 8%
γ	~ 10%
$BR(B \to \tau\nu)$	~ 19%

The Unitarity Triangle

Unitarity implies that the weak couplings and phases form a triangle in the complex plane.

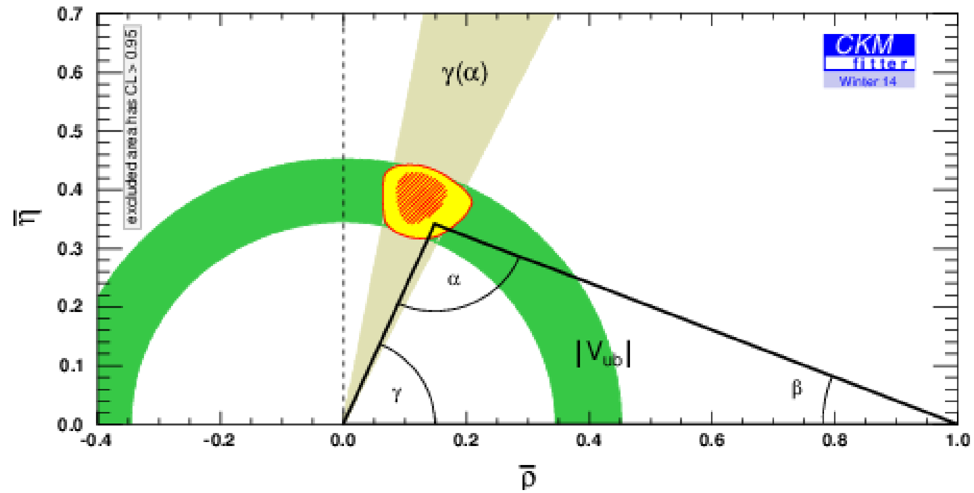
Big Questions:

Are determinations of angles consistent with determi nations of the sides of the triangle (CP conserving vs CP violating) ?

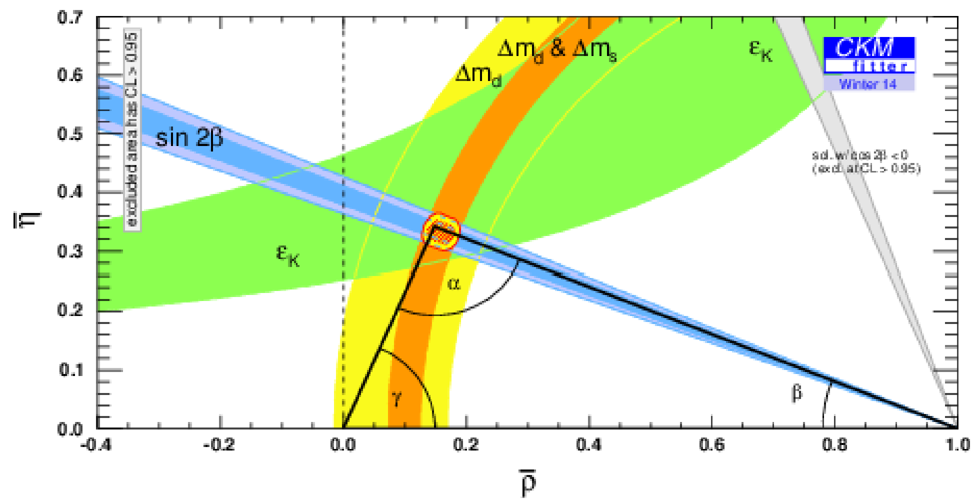
Are angle determinations from loop and tree decays consistent ?

The Unitarity Triangle : Tree vs Loop

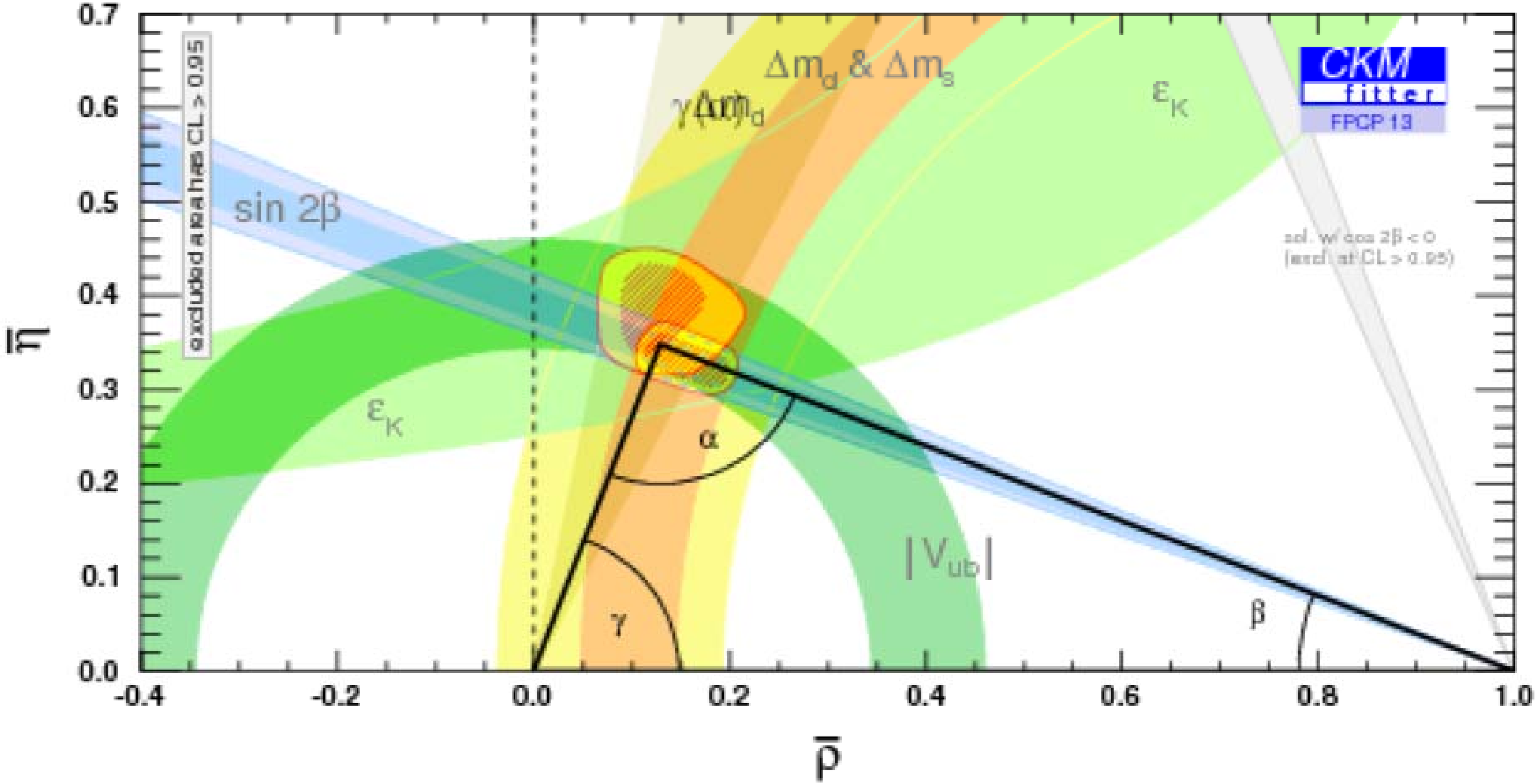
Unitarity triangle from "tree observables"



Unitarity triangle from "loop observables"

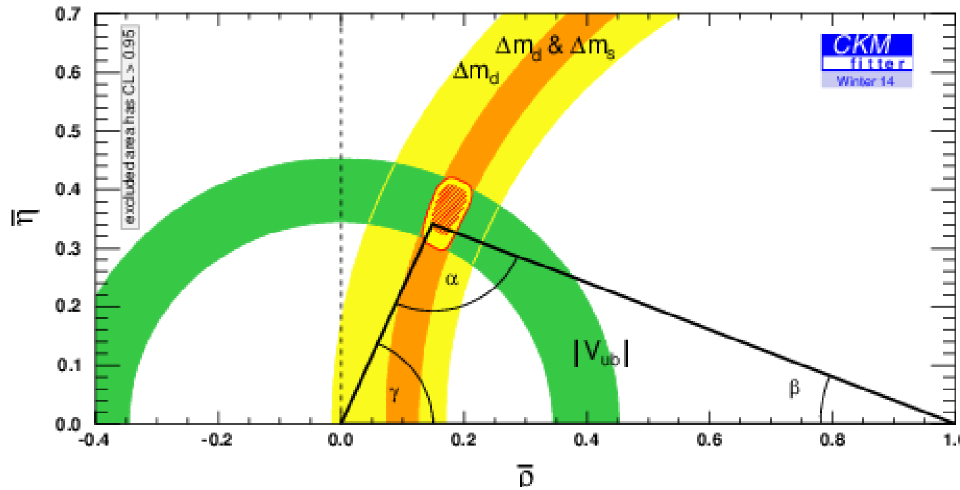


The Unitarity Triangle : Tree vs Loop

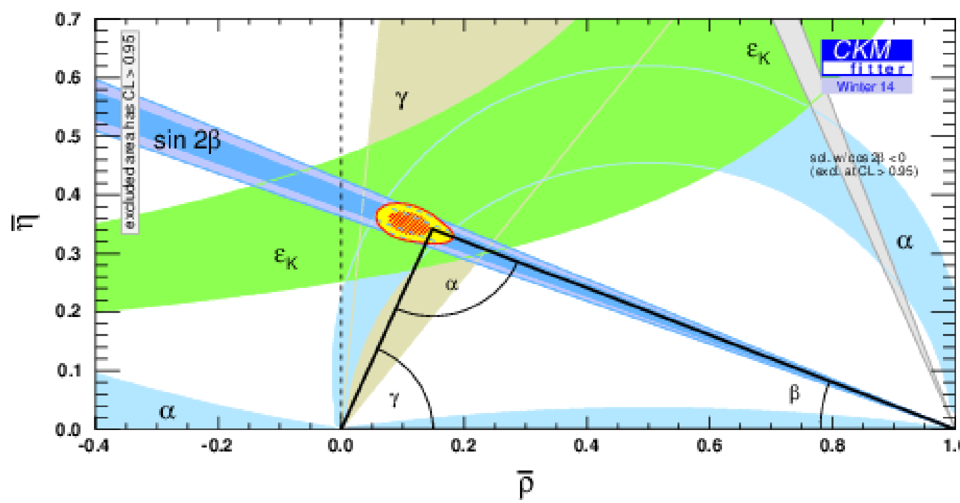


The Unitarity Triangle : CP vs ~~CP~~

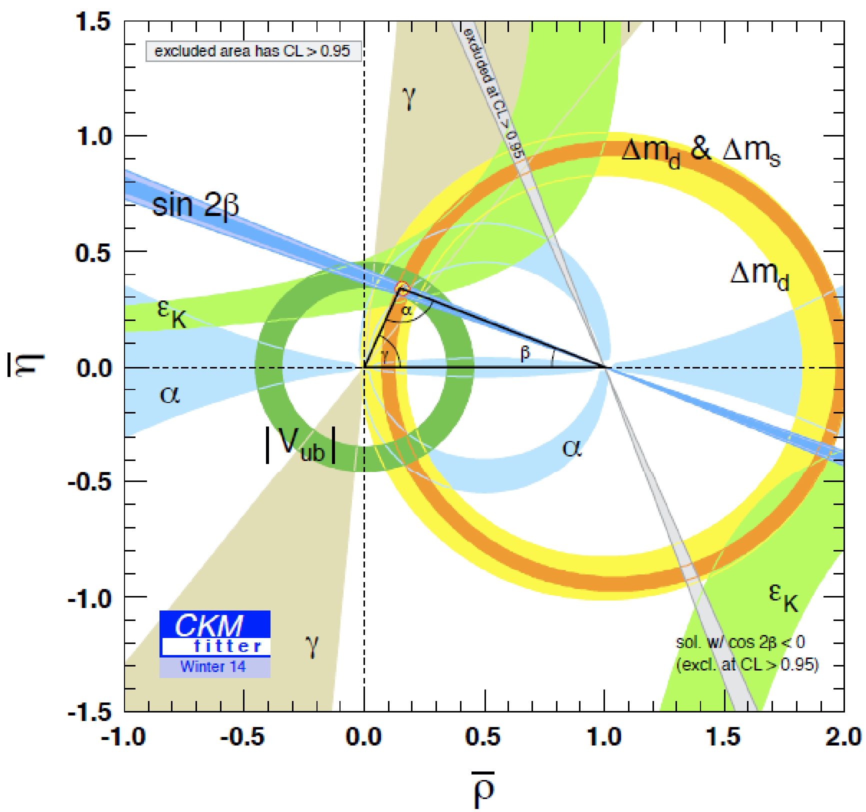
Unitarity triangle from "CP conserving observables"

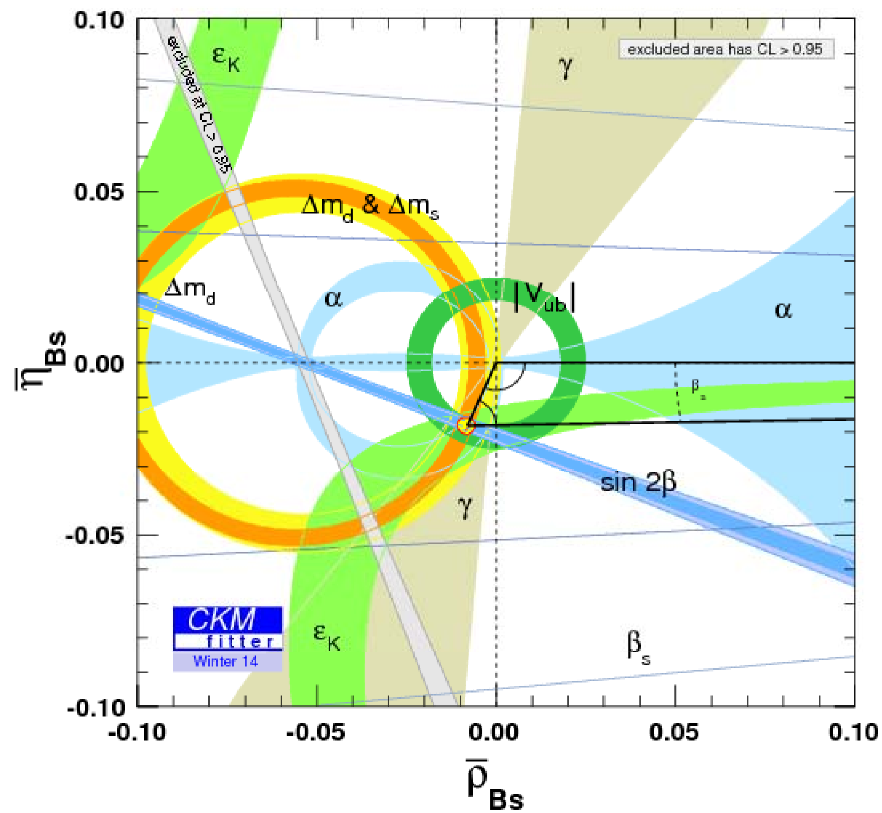


Unitarity triangle from "CP violating observables"



Unitarity Triangles

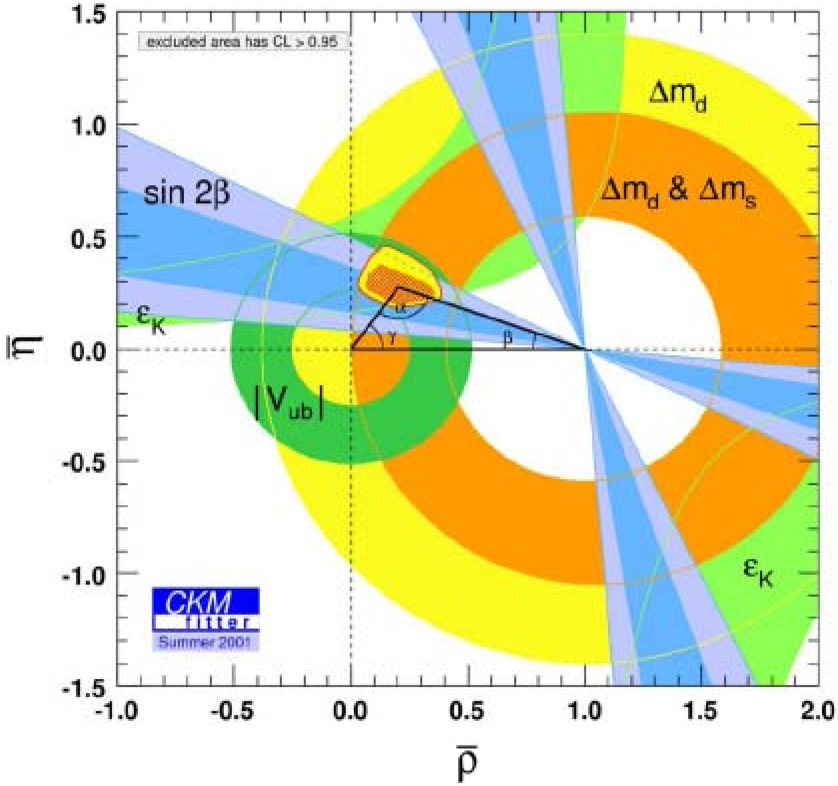




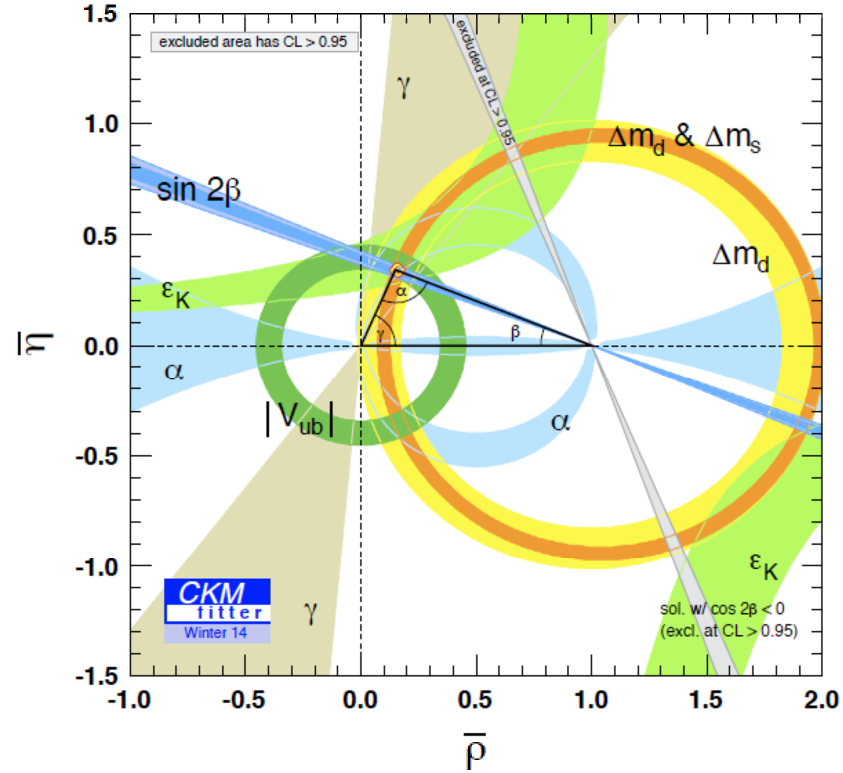
- so far a huge success for the Standard Model
- ~20% contribution from New Physics still possible within current precision

The Unitarity Triangle: 2001 vs 2014

Summer 2011



Winter 2014



$$\alpha + \beta + \gamma = (175.2 \pm 9.3)^o$$

	Direct	CKM fit
α	$= \left(88.8^{+4.5}_{-4.3}\right)^o$	$\left(93.6^{+3.2}_{-2.9}\right)^o$
β	$= \left(21.5^{+0.8}_{-0.7}\right)^o$	$\left(25.38^{+0.80}_{-1.57}\right)^o$
γ	$= \left(70^{+7.7}_{-9.0}\right)^o$	$\left(66.4^{+1.2}_{-3.3}\right)^o$
$-2\beta_s$	$= +0.00 \pm 0.07$	$-0.0363^{+0.0014}_{-0.0012}$

well with the CKM picture at O(10%) level

Cabibbo-Kobayashi-Maskawa (CKM)

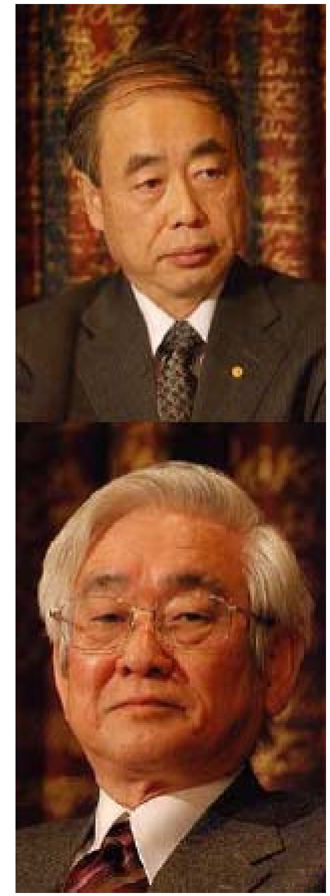
- The quark sector in the SM is well described by Cabibbo-Kobayashi-Maskawa (CKM) picture.

- The CKM description turns to be very successful.

Nobel Prize 2008

Cabibbo-Kobayashi-Maskawa (CKM)

- ❑ The quark sector in the SM is well described by Cabibbo-Kobayashi-Maskawa (CKM) picture.
- ❑ The CKM description turns to be very successful.
- ❑ But, it is just a **DESCRIPTION**.
- ❑ There should be something behind.
- ❑ Flavour physics beyond the SM….



Nobel Prize 2008