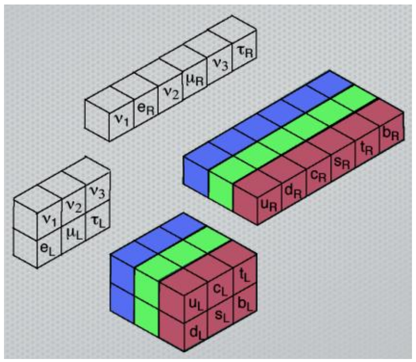
Theory of Electroweak Interactions



[Okun, Leptons and Quarks]
[Perkins, Introduction to HEP]
[Peskin, Perimeter, 2009]
[Altarelli, arXiv:1303.2842]
[Quigg, ICTP-SAIFR School, 2013]
[EW review, PDG 2012]
[Higgs review, PDG 2012]
...

fermions	SU(2)	$U(1)_Y$
$(\nu\ ,\ e^-)_L$	2	-1
e^-_R	1	-2
$(u\ ,\ d)_L$	2	$1/3$
u_R	1	$4/3$
d_R	1	$-2/3$

Theoretical structure
Discovery of W and Z
Quantum fluctuations

Open KIAS PCSI, 2013

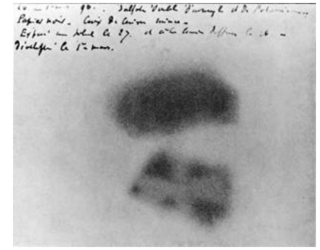
Milestones in Particle Physics

1897 Discovery of electron
1900 α, β and γ radioactivity
1905 Photon identified as quantum of electromagnetic field
1911 Discovery of atomic nucleus
1912 Discovery of cosmic rays
Invention of cloud chamber
1913 Bohr model of atom
1919 Discovery of proton
1923 de Broglie wave–particle duality
1925 Introduction of electron spin
1926 Wave mechanics
1927 Uncertainty Principle
1928 Dirac wave equation
1930 Neutrino hypothesis
1931 Operation of first cyclotron and of Van der Graaff accelerator
1932 Discovery of positron
Discovery of neutron
1933 Discovery of electromagnetic showers
1934 Theory of beta decay
Discovery of Čerenkov effect
1935 Yukawa theory of nuclear forces
1936 Breit–Wigner resonance formula
1937 First evidence for mesotron (= muon)
1939 Observation of mesotron (= muon) decay
1940 Spin-statistics theorem
1945 Phase stability in accelerators (synchrotron principle)
1946 First proposal of Big Bang model
Two-meson hypothesis
1947 Discovery of pion and $\pi \to \mu$ decay in cosmic rays
Prediction of muon-induced nuclear fusion
Two-meson hypothesis (again)
Discovery of V particles
1948 Quantum electrodynamics
Observation of $K \to 3\pi$ decay
Pion production at accelerators
1950 Spark chamber invented
Semiconductor detector invented
Discovery of neutral pion and $\pi^0 \to 2\gamma$ decay
1951 Observation of Λ hyperon and neutral kaon, K_S^0
1952 Evidence for $\Delta(1232)\pi p$ resonance
Strong focussing principle for synchrotron
Invention of bubble chamber
1953 Evidence for Σ and Ξ hyperons
First V events at accelerator: associated production
First hypernucleus event
τ–θ $(= K\pi 3/K\pi 2)$ paradox
1954 Prediction of long-lived K_L^0
Invention of strangeness quantum number and classification

[Perkins, Intro. to HEP]

1956 Observation of antiproton
Detection of (anti)neutrinos from reactor
Experimental evidence for K_L^0
Proposal for colliding-beam accelerators
1957 Observation of muon-induced nuclear fusion
Two-component neutrino, $V - A$ theory
Parity non-conservation in weak decays
Resolution of τ–θ paradox
1958 $(\pi \to e)/(\pi \to \mu)$ branching ratio
Neutrino helicity measurement
1959 Operation of CERN PS, Brookhaven AGS
1961 K_L–K_S regeneration
Discovery of ρ, ω, η pion resonances
1962 Pion β-decay $\pi^+ \to \pi^0 e^+ \nu$
First accelerator neutrino beams and interactions
ν_μ and ν_e as separate neutrino flavours
1963 Cabibbo theory of hadronic weak decays
1964 Streamer chamber invented
Introduction of quarks and quark model
First evidence for Ω^- hyperon
Discovery of CP violation in K^0 decay
Higgs mechanism of spontaneous symmetry breaking
1965 Observation of cosmic microwave background radiation
Introduction of colour quantum number and vector gluons
1967 Baryon asymmetry of universe (Sakharov criteria)
1968 Weinberg–Salam–Glashow electroweak model
Deep inelastic ep scattering. Bjorken scaling and partons
1970 Invention of multiwire proportional chamber
Proposal of fourth quark (charm)
1972 Solar neutrino deficit (^{37}Cl experiment)
Fermilab Tevatron operates
CKM matrix for weak quark decays
1973 QCD as field theory of interquark interactions
Neutrino scattering experiments confirm that partons are quarks
Discovery of neutral weak currents
1974 Discovery of J/ψ and ψ' $c\bar{c}$ resonances
1975 Charmed baryons and mesons
Discovery of τ lepton
$e^+e^- \to$ quark jets
1976 CERN SPS operates
1977 Discovery of $\Upsilon(= b\bar{b})$ states
Emergence of Standard Model
1978 Parity violation in polarised electron–deuterium scattering
1979 $e^+e^- \to$ three jets (PETRA)
1980 Evidence for Υ(3S) and Υ(4S) (CESR)
1981 Observation of mesons and baryons containing b quarks
1983 Discovery of Z^0 and $W^\pm$ bosons
1987 Observation of B^0–$\bar{B}^0$ mixing
SN 1987A Supernova neutrino burst
1990 Z^0 produced at e^+e^- colliders LEP and SLC
Number of neutrino flavours $N_\nu = 3$ from Z^0 width
1993 Solar neutrino deficit confirmed in gallium experiments
Atmospheric neutrino flavour anomaly
Precise measurements of Z^0 decay parameters confirm Standard Model
1995 Discovery of t quark at Fermilab collider
1997 $e^+e^- \to W^+W^-$ pair production at LEP 200 collider

Theoretical Structure

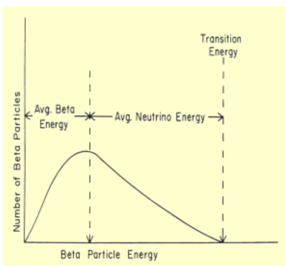


Antecedents :

- **1896** : Becquerel discovered "radioactivity"

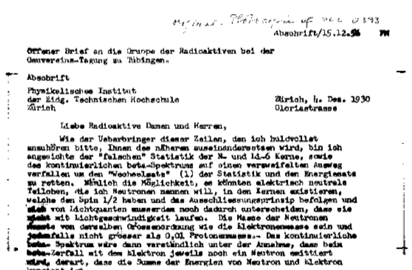
$$ {}^{A}Z \rightarrow {}^{A}(Z+1) + \beta^- \quad \text{such as} \quad {}^{3}H_1 \rightarrow {}^{3}He_2 + \beta^-, \quad n \rightarrow p + \beta^-, \quad {}^{214}Pb_{82} \rightarrow {}^{214}Bi_{83} + \beta^- $$

- **1914** : Chadwick saw a continuous β spectrum



Energy conservation in question

- **1930** : Pauli proposed an (almost) massless, neutral and penetrating particle (nuclear spin & statistics)



Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

⇨ neutrino (Fermi, 1933)

- **1932** : Chadwick discovered neutron

⇨ Isospin symmetry (Heisenberg, 1932)

$$ \begin{cases} m_n = 939.56563 \pm 0.00028 \ \mathrm{MeV}/c^2 \\ m_p = 938.27231 \pm 0.00028 \ \mathrm{MeV}/c^2 \end{cases} $$

$$ \Delta m / \bar{m} \simeq 1.4 \times 10^{-3} $$

$$ \begin{cases} {}^{3}H(pnn) : \ \text{B.E.} = 8.481855 \pm 0.000013 \ \mathrm{MeV} \\ {}^{3}He(ppn) : \ \text{B.E.} = 7.718109 \pm 0.000013 \ \mathrm{MeV} \end{cases} $$

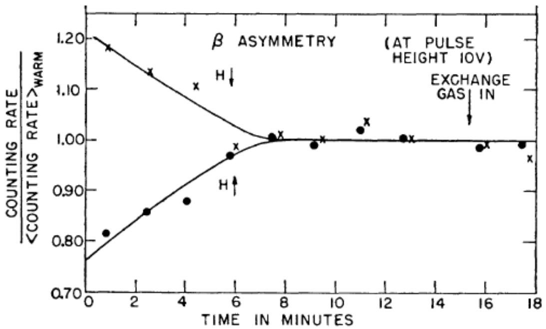
$$ \Delta(\text{B.E.}) = 0.76346 \ \mathrm{MeV} $$

⇨ $SU(2)_f$ on $\begin{bmatrix} p \\ n \end{bmatrix}$: 1st flavor symmetry

⊙ Parity violation in weak decays :

✓ 1956 : Wu et al. saw a strong correlation between the spin vector $\vec{J}$ of polarized ^{60}Co and the direction $\hat{p}_e$ of outgoing β particle

$$\langle \vec{J} \cdot \hat{p}_e \rangle < 0 \quad \text{in} \quad {}^{60}Co \rightarrow {}^{60}Ni^* + e^- + \bar{\nu}_e + 2\gamma$$



$$P : \vec{J} \rightarrow +\vec{J}$$
$$\hat{p}_e \rightarrow -\hat{p}_e$$

⇨ $\langle \vec{J} \cdot \hat{p}_e \rangle \neq 0$

Parity violation


Es ist uns eine traurige Pflicht, bekannt zu geben, daß unsere langjährige, liebe Freundin
PARITY
am 19. Januar 1957 nach kurzem Leiden bei weiteren experimentellen Eingriffen sanft entschlafen ist.
Für die Hinterbliebenen
e, μ, ν

It is our sad duty to announce that our royal friend of many years PARITY went peacefully to her eternal rest on the nineteenth of January, 1957, after a short period of suffering in the face of further experimental interventions. ...

✓ Experiments in late 1950s established that (charged-current) weak interactions left-handed

$$P : \nu_L \;\overset{\Leftarrow}{\longrightarrow} \;\longleftrightarrow\; \cancel{\nu_R} \;\overset{\Leftarrow}{\longleftarrow}$$

⇨ manifestly parity-violating theory with "only" ν_L

ex) $\pi^+ \rightarrow \mu^+ \nu_\mu$ ⇨ $h(\mu^+)$

$$\nu_\mu \overset{\Rightarrow}{\longleftarrow} (\pi^+) \overset{\Leftarrow}{\longrightarrow} \mu^+ \qquad h(\nu_\mu) = h(\mu^+)$$

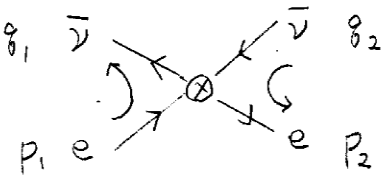
$J=0$

$$\Gamma(\pi^+ \to e^+ \nu_e) / \Gamma(\pi^+ \to \mu^+ \nu_\mu) = 1.23 \times 10^{-4}$$

⊙ Effective Lagrangian ⇨ late 1950s : current × current interaction

$$\mathcal{L}_{V-A} = \frac{G_F}{\sqrt{2}} [\bar{\nu} \gamma_\mu (1-\gamma_5) e] [\bar{e} \gamma^\mu (1-\gamma_5) \nu] + h.c. + \cdots \qquad G_F = 1.16632 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$\boxed{\bar{\nu} e \to \bar{\nu} e}$



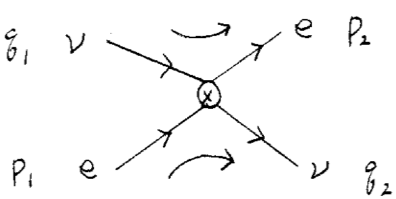
$$\mathcal{M} = \frac{G_F}{\sqrt{2}} [\bar{v}(\bar{\nu}, q_1) \gamma_\mu (1-\gamma_5) u(e^-, p_1)] [\bar{u}(e^-, p_2) \gamma^\mu (1-\gamma_5) v(\bar{\nu}, q_2)]$$

$$\frac{d\sigma(\bar{\nu}e)}{d\Omega_{cm}} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s} = \frac{G_F^2}{16\pi^2} 2m E_\nu (1-z)^2 \quad \text{with} \quad z = \cos\theta^*$$

$$\Rightarrow \sigma(\bar{\nu} e^- \to \bar{\nu} e^-) = \frac{G_F^2 \cdot 2m E_\nu}{3\pi} \simeq 0.57 \times 10^{-41}\ \mathrm{cm}^2 \left(\frac{E_\nu}{1\ \mathrm{GeV}}\right) : \text{extremely small}$$

$\boxed{\nu e \to \nu e}$

$$\bar{\nu} \gamma_\mu (1-\gamma_5) e = \bar{e} \gamma_\mu (1+\gamma_5) \nu$$



$$\mathcal{M} = \frac{G_F}{\sqrt{2}} [\bar{u}(e^-, p_2) \gamma_\mu (1+\gamma_5) u(\nu, q_1)] [\bar{u}(\nu, q_2) \gamma^\mu (1-\gamma_5) u(e^-, p_1)]$$

$$= \frac{G_F}{\sqrt{2}} 2 [\bar{u}(e^-, p_2) (1-\gamma_5) u(e, p_1)] [\bar{u}(\nu, q_2) (1+\gamma_5) u(\nu, q_1)]$$

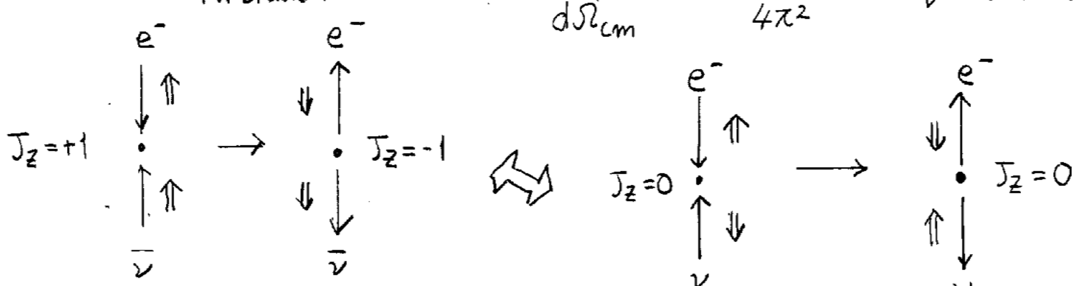
Fierzing

$$\frac{d\sigma(\nu e)}{d\Omega_{cm}} = \frac{G_F^2 \cdot 2m E_\nu}{4\pi^2} \Rightarrow \sigma(\nu e^- \to \nu e^-) = \frac{G_F^2 \cdot 2m E_\nu}{\pi} = 1.72 \times 10^{-41}\ \mathrm{cm}^2 \left(\frac{E_\nu}{1\ \mathrm{GeV}}\right)$$

$$= 3 \times \sigma(\bar{\nu} e^- \to \bar{\nu} e^-)$$

$$\int_0^1 (1-z)^2 dz = 1/3$$

Forbidden



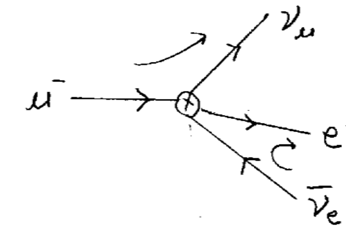
: allowed at all angles

⊙ <u>1962</u> : Lederman, Schwartz, Steinberger $\nu_\mu \neq \nu_e$

① Prepare HE $\pi \to \mu \nu$ beam $\to$ ② observe $\nu N \to \mu + X$

$\to$ ③ $\nu N \to e + X$: NOT observed

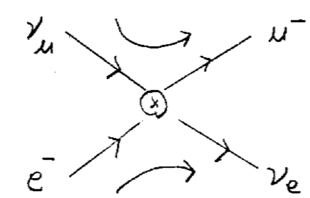
⇨ Extended "family" structure $\begin{bmatrix} \nu_e \\ e \end{bmatrix}_L \not\leftrightarrow \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix}_L$



⇨ Generalized effective current x current Lagrangian

$$\mathcal{L}^{(e\mu)}_{V-A} = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\, \mu \right]\left[\bar{e} \gamma^\mu (1-\gamma_5)\, \nu_e \right] + \text{h.c.}$$

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3} \to \tau_\mu = 2.2 \times 10^{-6}\,\text{s}$$



⇨ $$\sigma(\nu_\mu e^- \to \mu^- \nu_e) = \sigma(\nu_e e^- \to \nu_e e^-) \left[1 - \frac{(m_\mu^2 - m_e^2)}{2 m_e E_\nu} \right]^2$$

agrees with CHARM II and CCFR data ($E_\nu \lesssim 600$ GeV)

⇨ Partial-wave unitarity : $|\mathcal{T}^J| < 1$ on $\nu_\mu e^- \to \mu^- \nu_e$

$$\mathcal{T}^0 = \frac{G_F\, 2 m_e E_\nu}{\pi\sqrt{2}} \left[1 - \frac{(m_\mu^2 - m_e^2)}{2 m_e E_\nu} \right]^2 \longrightarrow E_\nu < \frac{\pi}{\sqrt{2}\, G_F m_e} \simeq 3.7 \times 10^8\ \text{GeV}$$

V-A theory cannot be "complete" $\to$ Physics must change before $\sqrt{s} \simeq 600$ GeV

A theory of leptons ← S. Weinberg, "A Model of Leptons", 1967

3 crucial clues

① Left-handed weak-isospin doublets + right-handed weak-isospin singlets $\begin{bmatrix} \nu_e \\ e^- \end{bmatrix}_L$, e_R^-

② Universal strength of the (charged-current) weak interactions $(\frac{\vec{\tau}}{2}, Y)$ $(\mathbb{1}, Y)$

③ Neutrinos are (almost) massless → no ν_R is required

generators

A model (based on a local gauge symmetry) ⇨ straightforward extension to (μ, ν_μ) ...

- Matter : $L_L = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L$, e_R
- Gauge group : $SU(2)_L \otimes U(1)_Y$

$$W_\mu^{1,2,3} \qquad B_\mu$$
$$g \qquad g'$$

$Q = I_3 + \frac{1}{2} Y$ — $Y(L) = -1 \equiv Y_L$

$I_3(\nu_e) = +\frac{1}{2}$, $I_3(e^-) = -\frac{1}{2}$ — $Y(e_R^-) = -2 \equiv Y_R$

⇨ Field-strength tensors

$U(1)_Y$: $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$SU(2)_L$: $W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g \epsilon^{ijk} W_\mu^j W_\nu^k$

- Lagrangian : $\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{leptons}$

$$\mathcal{L}_{gauge} = -\frac{1}{4} W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{L}_{leptons} = \bar{e}_R i\gamma^\mu \left(\partial_\mu + i g' \frac{Y_R}{2} B_\mu\right) e_R + \bar{L}_L i\gamma^\mu \left(\partial_\mu + i g' \frac{Y_L}{2} B_\mu + i g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu\right) L_L$$

*** Local gauge invariance $\not\to$ $\mathcal{L}_e = -m_e (\bar{e}_R e_L + \bar{e}_L e_R)$

→ forces $\vec{W}$ and B to be massless ↔ Nature has only one massless gauge boson (γ)

⊙ Solution to mass generation :

Higgs mechanism : "relativistic" generalization of Ginzburg-Landau superconducting phase transition

- Introduce a complex doublet of scalar fields

$$\Phi \equiv \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} \quad w/ \quad Y(\Phi) = +1$$

- Add the gauge-invariant terms for interaction and propagation of the scalars

$$\mathcal{L}_\Phi = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi^\dagger \Phi)$$

$$D_\mu = \partial_\mu + i\frac{g'}{2} B_\mu + i\frac{g'}{2} \vec{\tau}\cdot\vec{W}_\mu$$

$$V(\Phi^\dagger\Phi) = \mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2 \quad w/ \quad \mu^2 < 0$$

$$\Rightarrow \text{VEV} : \langle\Phi\rangle_0 = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} \quad w/ \quad v = \sqrt{-\mu^2/\lambda}$$

hides (or breaks) $SU(2)_L$ and $U(1)_Y$, while preserving $U(1)_{EM}$ invariance

$$SU(2)_L \otimes U(1)_Y \Rightarrow U(1)_{EM}$$

- Add a Yukawa-interaction term

$$\mathcal{L}_Y = -\lambda_e [(\bar{L}_L \cdot \phi) e_R + h.c.]$$

$$\langle\Phi^\dagger\Phi\rangle_0 = \frac{v^2}{2} \Rightarrow V(\langle\Phi^\dagger\Phi\rangle_0) = \frac{\mu^2}{2}v^2 + \frac{\lambda}{4}v^4$$

$$\frac{\partial V_0}{\partial v^2} = \frac{\mu^2}{2} + \frac{1}{2}\lambda v^2 = 0 \rightarrow v = \sqrt{-\mu^2/\lambda}$$

$$\tau_1 \langle\Phi\rangle_0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} = \begin{bmatrix} v/\sqrt{2} \\ 0 \end{bmatrix} \neq 0 \quad \text{broken}$$

$$\tau_2 \langle\Phi\rangle_0 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} = \begin{bmatrix} -i\,v/\sqrt{2} \\ 0 \end{bmatrix} \neq 0 \quad \text{broken}$$

$$\tau_3 \langle\Phi\rangle_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ -v/\sqrt{2} \end{bmatrix} \neq 0 \quad \text{broken}$$

$$Y\langle\Phi\rangle_0 = Y(\Phi)\langle\Phi\rangle_0 = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} \neq 0 \quad \text{broken}$$

$$\Rightarrow Q\langle\Phi\rangle_0 \equiv \frac{1}{2}(\tau_3 + Y)\langle\Phi\rangle_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \; !$$

"unbroken"

⇨ A massless gauge boson (= photon) + 3 massive gauge bosons (W^1, W^2, Z)

$Q = \frac{1}{2}(\tau_3 + Y)$ $\qquad\qquad$ τ_1, τ_2, $\frac{1}{2}(\tau_3 - Y)$

- Expand the scalar field about the vacuum state in the unitary gauge

$$\Phi = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+H) \end{bmatrix} \Rightarrow \mathcal{L}_\Phi = \frac{1}{2}(\partial_\mu H)\partial^\mu H) - \frac{1}{2}\mu^2 H^2 + \frac{v^2}{8}\left[g^2 |W_\mu^1 - i W_\mu^2|^2 + (g' B_\mu - g W_\mu^3)^2 \right] + \cdots$$

$$\vec{\tau}\cdot\vec{W}_\mu \, \Phi = \frac{1}{\sqrt{2}}(v+H)\begin{bmatrix} W_\mu^1 - i W_\mu^2 \\ -W_\mu^3 \end{bmatrix} \Rightarrow D_\mu \Phi = \frac{1}{\sqrt{2}}\partial_\mu H + i\frac{1}{2\sqrt{2}}(v+H)\begin{bmatrix} g(W_\mu^1 - i W_\mu^2) \\ g' B_\mu - g W_\mu^3 \end{bmatrix}$$

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \frac{1}{8}(v+H)^2\left[g^2 |W_\mu^1 - i W_\mu^2|^2 + (g' B_\mu - g W_\mu^3)^2 \right]$$

$$-V(\Phi^\dagger \Phi) = -\frac{\lambda}{4}(v+H)^4 - \frac{1}{2}\mu^2 (v+H)^2 = \cdots - \left(\frac{1}{2}\mu^2 + \frac{3}{2}\lambda v^2\right) H^2 - \lambda v H^3 - \frac{1}{4}\lambda H^4$$

$$\xrightarrow{\mu^2 = -\lambda v^2} = -\lambda v^2 H^2 - \lambda v H^3 - \frac{1}{4}\lambda H^4 = -\mu^2 H^2 - \lambda v H^3 - \frac{1}{4}\lambda H^4$$

$$\boxed{\begin{aligned} M_H^2 &= -2\mu^2 = 2\lambda v^2 \\ M_{W^\pm}^2 &= \frac{1}{4} g^2 v^2 \\ M_Z^2 &= \frac{1}{4}(g^2 + g'^2) v^2 \\ \cos\theta_W &= g / \sqrt{g^2 + g'^2} \\ \sin\theta_W &= g' / \sqrt{g^2 + g'^2} \end{aligned}}$$

$$g_Z \equiv \sqrt{g^2 + g'^2}$$

$$\boxed{\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}}(W_1^\mu \mp i W_2^\mu) \\ Z_\mu &= \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu \\ A_\mu &\equiv \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu \end{aligned}}$$

$$\boxed{\begin{aligned} \mathcal{L}_\Phi = & \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - \frac{1}{2} M_H^2 H^2 \\ & + M_W^2 \left(1 + \frac{H}{v}\right)^2 W_\mu^+ W^{-\mu} \\ & + \frac{1}{2} M_Z^2 \left(1 + \frac{H}{v}\right)^2 Z_\mu Z^\mu \\ & - \frac{M_H^2}{2v} H^3 - \frac{M_H^2}{8v^2} H^4 \end{aligned}}$$

* A_μ remains "massless" ⇒ photon ?!

$$\mathcal{L}_\gamma = -\lambda_e [(\bar{L}_L \cdot \Phi)\, e_R + h.c.] = -\frac{\lambda_e}{\sqrt{2}} (v+H)(\bar{e}_L e_R + \bar{e}_R e_L) = -m_e\left(1+\frac{H}{v}\right)\bar{e}e$$

$$a\cdot b = \begin{bmatrix} a^+ \\ a^- \end{bmatrix} \cdot \begin{bmatrix} b^+ \\ b^0 \end{bmatrix} \equiv a^0 b^0 - a^- b^+ \qquad \boxed{m_e \equiv \frac{1}{\sqrt{2}} \lambda_e v} \qquad P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$$

⊙ Interactions :

$$\mathcal{L}_{W-\ell} = \bar{L}_L i \not{D} L_L + \bar{e}_R i \not{D} e_R = (\bar{\nu}_e, \bar{e})\, i\gamma^\mu \left\{ \partial_\mu + i\frac{g'}{2}(-1) B_\mu + i \frac{g}{2} \underbrace{\begin{bmatrix} W_\mu^3 & \sqrt{2} W_\mu^+ \\ \sqrt{2} W_\mu^- & -W_\mu^3 \end{bmatrix} \right\} P_L \begin{bmatrix} \nu_e \\ e \end{bmatrix}}_{P_L \begin{bmatrix} W_\mu^3 \nu_e + \sqrt{2} W_\mu^+ e \\ \sqrt{2} W_\mu^- \nu_e - W_\mu^3 e \end{bmatrix}}$$

$$+ \bar{e}\, i\gamma^\mu [\partial_\mu + i\frac{g'}{2}(-2) B_\mu]\, P_R\, e$$

$$\vec{\tau}\cdot\vec{W}_\mu = \sqrt{2}\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix}$$

$$= i(\bar{\nu}_e \not{\partial} P_L \nu_e + \bar{e} \not{\partial} P_L e) + i(\bar{e} \not{\partial} P_R e)$$

$$+ \frac{1}{2}[\, g'(\bar{\nu}_e \gamma^\mu P_L \nu_e) B_\mu + g'(\bar{e}\gamma^\mu P_L e) B_\mu$$

$$- g(\bar{\nu}_e \gamma^\mu P_L \nu_e) W_\mu^3 + g(\bar{e}\gamma^\mu P_L e) W_\mu^3 + 2g'(\bar{e}\gamma^\mu P_R e) B_\mu\,]$$

$$- \frac{1}{\sqrt{2}} g [(\bar{\nu}_e \gamma^\mu P_L e) W_\mu^+ + (\bar{e}\gamma^\mu P_L \nu_e) W_\mu^-]$$

$$B_\mu = c_W A_\mu - s_W Z_\mu$$

$$W_\mu^3 = s_W A_\mu + c_W Z_\mu$$

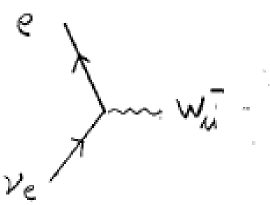
$$g W_\mu^3 - g' B_\mu = g_Z Z_\mu$$

$$g W_\mu^3 + g' B_\mu = g s_W A_\mu + g c_W Z_\mu + g' c_W A_\mu - g' s_W Z_\mu$$

$$2 g' B_\mu = 2 g' c_W A_\mu - 2 g s_W Z_\mu$$

• Charged-current interactions

$$\mathcal{L}_{W-\ell} = -\frac{g}{2\sqrt{2}} [(\bar{\nu}_e \gamma^\mu (1-\gamma_5) e) W_\mu^+ + (\bar{e}\gamma^\mu (1-\gamma_5)\nu_e) W_\mu^-]$$



$$= -i\frac{g}{2\sqrt{2}}\gamma_\mu(1-\gamma_5) \qquad W_\mu \underset{k}{\sim\!\sim\!\sim} W_\nu = \frac{-i(g_{\mu\nu} - k_\mu k_\nu / M_W^2)}{k^2 - M_W^2}$$ in the unitary gauge

$\boxed{\tan\theta_W = \frac{g'}{g}}$ $\qquad$ $\sin / \cos\theta_W = \frac{g'/g}{\sqrt{g^2+g'^2}}$

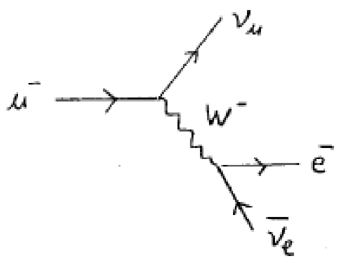
$$
\begin{tikzcd}
 & \boxed{g_Z = \sqrt{g^2+g'^2}} \arrow[dl, "\times\cos\theta_W"'] \arrow[dr, "\times\sin\theta_W"] & \\
SU(2)_L \ \boxed{g} \arrow[dr, "\times\sin\theta_W"'] & & \boxed{g'} \ U(1)_Y \arrow[dl, "\times\cos\theta_W"] \\
 & \boxed{e = \frac{gg'}{\sqrt{g^2+g'^2}}} &
\end{tikzcd}
$$

$U(1)_{EM}$

$$\mathcal{L}_{V-A}\,[\mu\nu_\mu \leftrightarrow e\nu_e] \simeq -\frac{1}{M_W^2}\cdot\underbrace{\left(\frac{g}{2\sqrt{2}}\right)^2}_{=\frac{G_F}{\sqrt{2}}} g^{\mu\nu}\,[\,\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu^-\,][\,\bar{e}\,\gamma_\nu (1-\gamma_5)\nu_e\,]$$

$M_W = \frac{1}{2} g v$

$M_W^2 \gg m_\mu, m_e$



$$\frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{1/2}$$

$$\Rightarrow \boxed{\begin{aligned} G_F &= \frac{g^2}{4\sqrt{2}\,M_W^2} = \frac{1}{\sqrt{2}\,v^2} \\ v &= (\sqrt{2}\,G_F)^{-1/2} \simeq 246\ \text{GeV} \end{aligned}}$$

$$\langle \phi^0 \rangle_0 = \frac{v}{\sqrt{2}} \simeq 174\ \text{GeV}$$

Revisiting $\nu_\mu + e^- \to \mu^- + \nu_e$

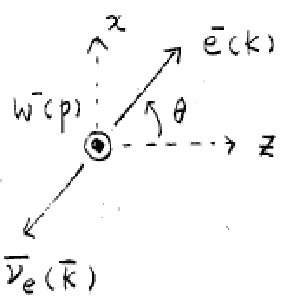
Independent of energy !

$$\sigma(\nu_\mu e^- \to \mu^+ \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4}\,\frac{[\,1-(m_\mu^2-m_e^2)/2m_e E_\nu\,]^2}{(1+2m_e E_\nu/M_W^2)} \xrightarrow{E_\nu\to\infty} \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\pi}$$

much better HE behavior

$\Rightarrow$ The $W^\pm$ propogator renders the HE behavior much more reliable !

- Leptonic W^- decay : $W^- \to e\bar{\nu}_e$ (m_e neglected)



$$\mathcal{M} \simeq -\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} [\,\bar{u}(e^-,k)\,\gamma_\mu (1-\gamma_5)\, v(\bar{\nu}_e, \bar{k})\,]\,\epsilon^\mu(p,\lambda)$$

$\epsilon(p,\pm) = \mp\frac{1}{\sqrt{2}}(0,1,\pm i,0)$: W^- pol. vector

$\epsilon(p,0) = (0,0,0,1)$ at rest

$\epsilon_{0123} = +1$
BD convention

$p = (M_W, 0, 0, 0)$

$k = \frac{M_W}{2}(1, \sin\theta, 0, \cos\theta) = \frac{M_W}{2}(1, \hat{k})$

$\bar{k} = \frac{M_W}{2}(1, -\sin\theta, 0, -\cos\theta) = \frac{M_W}{2}(1, -\hat{k})$

$$|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}}\,\mathrm{tr}\,[\,\not{\epsilon}\,(1-\gamma_5)\,\bar{\not{k}}\,(1-\gamma_5)\,\not{\epsilon}^*\,\not{k}\,]$$

$$= 4\sqrt{2}\,G_F M_W^2\,[\,\epsilon\cdot\bar{k}\;\epsilon^*\cdot k - \epsilon\cdot\epsilon^*\;k\cdot\bar{k} + \epsilon\cdot k\;\epsilon^*\cdot\bar{k} + i\langle\epsilon\,\bar{k}\,\epsilon^*\,k\rangle\,]$$

① Longitudinal pol. with $\lambda = 0$ $\rightarrow$ $\epsilon\cdot\bar{k} = \frac{M_W}{2}\cos\theta = -\epsilon^*\cdot k$, $\epsilon\cdot\epsilon^* = -1$, $k\cdot\bar{k} = \frac{1}{2}M_W^2$

$$|\mathcal{M}|_0^2 = 4\sqrt{2}\,G_F M_W^2\, M_W^2 \left[\frac{1}{2} - 2\times\frac{1}{4}\cos^2\theta \right] = \frac{4G_F M_W^4}{\sqrt{2}}\sin^2\theta$$

② Transverse pol. with $\lambda = \pm 1$ $\rightarrow$ $\epsilon\cdot\epsilon^* = -1$, $\epsilon\cdot\bar{k}\,\epsilon^*\cdot k = \epsilon\cdot k\,\epsilon^*\cdot\bar{k} = -\frac{1}{8}M_W^2\sin^2\theta$

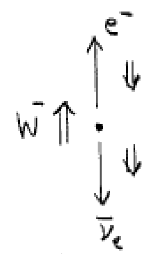
$$\langle \epsilon\bar{k}\epsilon^* k\rangle = -\langle \bar{k}k\epsilon\epsilon^*\rangle = -\frac{M_W^2}{4}\langle 0\hat{k}\epsilon\epsilon^*\rangle\times 2$$

$$= -\frac{1}{2}M_W^2\cos\theta\,\langle 0\,\epsilon\epsilon^* 3\rangle = -\frac{1}{2}M_W^2\cos\theta\,(\mp i)$$

$$|\mathcal{M}|_\pm^2 = 4\sqrt{2}\,G_F M_W^2\, M_W^2 \times\left[\frac{1}{2} - \frac{1}{4}\sin^2\theta \mp \frac{1}{2}\cos\theta\right]$$

$$= \sqrt{2}\,G_F M_W^4\,(1\mp\cos\theta)^2$$

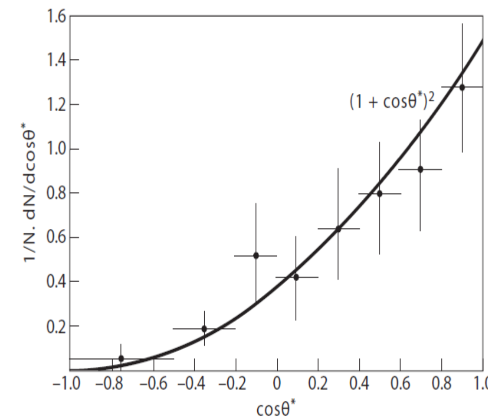
$\Rightarrow$
$$\boxed{\frac{d\Gamma_0}{d\cos\theta} = \frac{G_F M_W^3}{8\pi\sqrt{2}}\sin^2\theta \qquad \frac{d\Gamma_\pm}{d\cos\theta} = \frac{G_F M_W^2}{8\pi\sqrt{2}}\,\frac{(1\mp\cos\theta)^2}{2}}$$



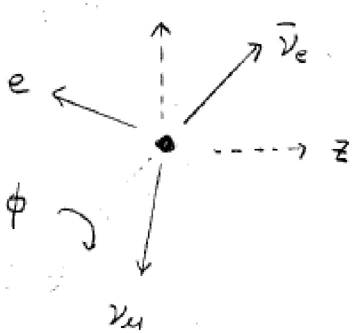
($\theta = 0$: forbidden)

(reversed for $W^+ \rightarrow e^+\nu_e$)

$$\bar{\nu}_e\gamma_\mu(1-\gamma_5)e^- = -\bar{e}^+\gamma_\mu(1+\gamma_5)\nu_e^c$$



Problem : Work out the process $\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu$ $\Rightarrow$ polarized muon decay



$x_e \equiv E_e/m_\mu$

$$\frac{d\Gamma}{dx_e} = \frac{G_F m_\mu^5}{16\pi^3}\left[\frac{x_e^2}{2} - \frac{x_e^3}{3}\right]$$

(unpolarized)

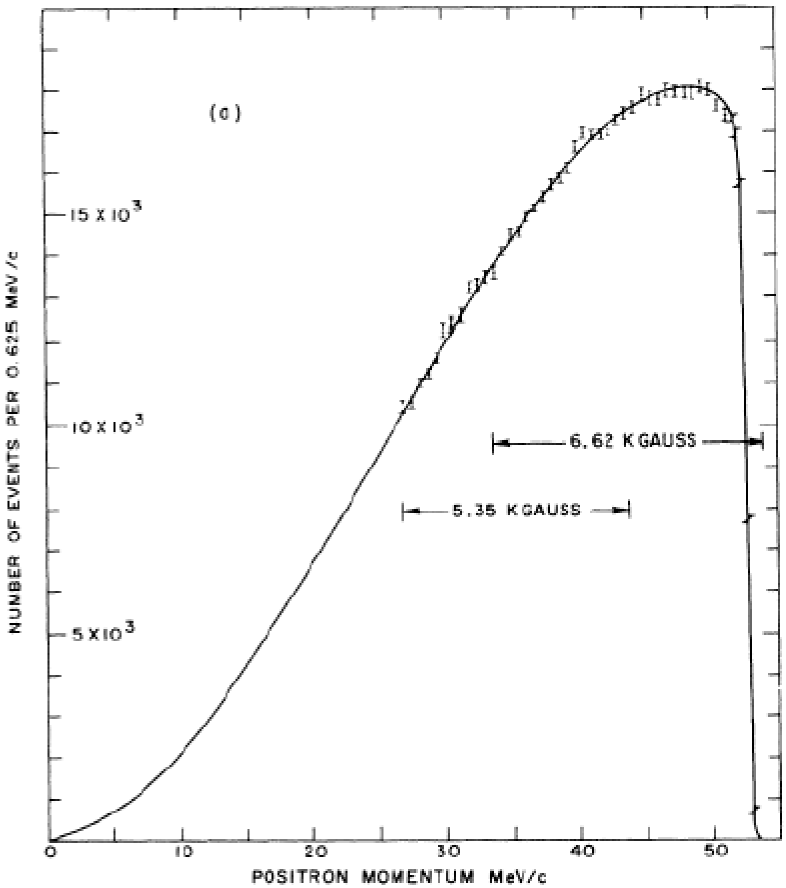
$$\frac{1}{\Gamma}\frac{d^2\Gamma}{dx_e\,d\cos\theta} = \frac{1}{2}(1-\hat{S}_e\cdot\hat{m})\left[3-2x_e + (\hat{S}_\mu\cdot\hat{m})(1-2x_e)\right]x_e^2$$

$\theta = \angle(\hat{p}_e, \hat{S}_\mu)$

$\hat{m} \equiv \hat{p}_e$

$\Rightarrow$ How about $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$?

$$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$$



- Neutral-current interactions $\qquad \boxed{g s_W = g' c_W \equiv e}$

3-8 $\Rightarrow$

$$\frac{1}{2}\left[(g' B_\mu - g W^3_\mu)(\bar\nu_e \gamma^\mu P_L \nu_e) + (g' B_\mu + g W^3_\mu)(\bar e \gamma^\mu P_L e) + 2 g' B_\mu (\bar e \gamma^\mu P_R e)\right]$$

$$= -\frac{g_Z}{4}(\bar\nu_e \gamma^\mu (1-\gamma_5)\nu_e) Z_\mu + e(\bar e \gamma^\mu P_L e + \bar e \gamma^\mu P_R e) A_\mu$$

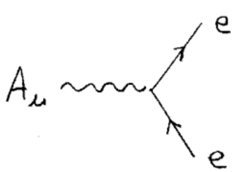
$$+ \underbrace{\frac{1}{2}(g c_W - g' s_W)(\bar e \gamma^\mu P_L e) Z_\mu + g' s_W (\bar e \gamma^\mu P_R e) Z_\mu}_{g_Z (\frac{1}{2} - s_W^2) P_L \;-\; g_Z s_W^2 P_R \;=\; g_Z [(\frac{1}{2} - s_W^2) P_L - s_W^2 P_R]}$$

$e \to -e Q_f$

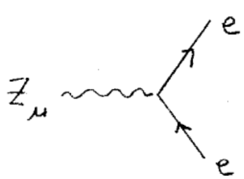
$\frac{1}{2} - s_W^2 \to Q_f s_W^2 - I_3$

$-s_W^2 \to Q_f s_W^2$

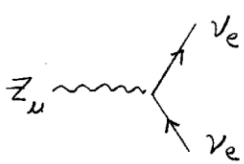
$$\boxed{\mathcal{L}_{A/Z-\ell} = -\frac{g_Z}{4}[\bar\nu_e \gamma^\mu (1-\gamma_5)\nu_e] Z_\mu - \frac{g_Z}{4}\left[\bar e \gamma^\mu \left((2s_W^2-1)(1-\gamma_5) + 2s_W^2(1+\gamma_5)\right) e\right] Z_\mu + e[\bar e \gamma^\mu e] A_\mu}$$



$= i e \gamma_\mu$



$= -i \frac{g_Z}{4} \gamma^\mu [(2s_W^2 - 1)(1-\gamma_5) + 2s_W^2(1+\gamma_5)]$



$= -i \frac{g_Z}{4} \gamma^\mu (1-\gamma_5)$

- Z properties

$$\Gamma(Z \to \bar\nu_e \nu_e) = \frac{G_F M_Z^2}{12\pi\sqrt{2}}$$

$$\Gamma(Z \to e^+ e^-) = \Gamma(Z \to \bar\nu_e \nu_e)[L_e^2 + R_e^2]$$

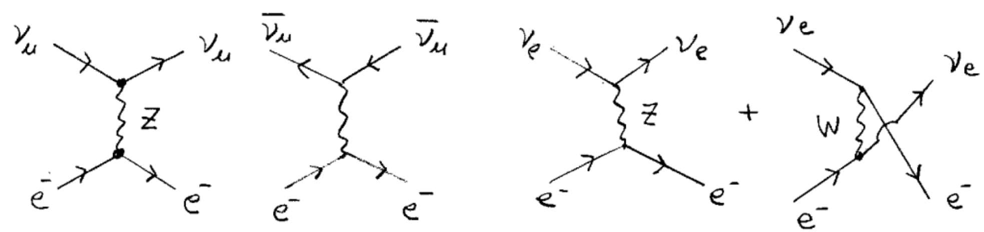
$L_e = 2 s_W^2 - 1$

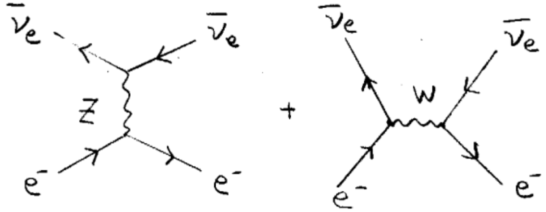
$R_e = 2 s_W^2$

$x_W \equiv s_W^2$

- Neutral-current interactions

"New νe reaction NOT present in the old V-A theory"





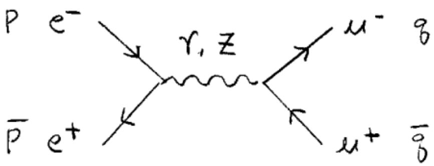
$$\boxed{\begin{aligned}
\sigma(\nu_\mu e^- \to \nu_\mu e^-) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2 + R_e^2/3 \right] \\
\sigma(\bar\nu_\mu e^- \to \bar\nu_\mu e^-) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2/3 + R_e^2 \right] \\
\sigma(\nu_e e^- \to \nu_e e^-) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e+2)^2 + R_e^2/3 \right] \\
\sigma(\bar\nu_e e^- \to \bar\nu_e e^-) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e+2)^2/3 + R_e^2 \right]
\end{aligned}}$$

- Model-independent analysis

$$v = g_V = \tfrac{1}{2}(L_e + R_e) = 2s_W^2 - 1/2$$
$$a = g_A = \tfrac{1}{2}(L_e - R_e) = -1/2$$

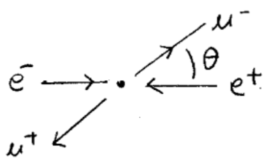
$\Rightarrow$ $\exists$ two-fold ambiguity ($R_e \leftrightarrow -R_e$ or $g_V \leftrightarrow g_A$)

- To resolve the 2-fold ambiguity, consider $e^+e^- \to \mu^+\mu^-$



$$\mathcal{M} = -e^2 \left[\bar u(\mu^-, q)\gamma^\mu Q_\mu v(\mu^+, \bar q)\right] \frac{1}{s} \left[\bar v(e^+, \bar p)\gamma_\mu u(e^-, p)\right] \qquad \boxed{Q_\mu = -1}$$
$$+ \frac{1}{2}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)\left[\bar u(\mu^-, q)\gamma^\mu \{R_\mu(1+\gamma_5) + L_\mu(1-\gamma_5)\} v(\mu^+, \bar q)\right]$$
$$\times \frac{1}{s - M_Z^2 + iM_Z\Gamma_Z} \times \left[\bar v(e^+, \bar p)\gamma_\mu \{R_e(1+\gamma_5) + L_e(1-\gamma_5)\} u(e^-, p)\right]$$

$z = \cos\theta$



$$\Rightarrow \frac{d\sigma}{dz} = \frac{\pi\alpha^2}{2s} Q_\mu^2 (1+z^2) - \frac{\alpha Q_\mu G_F M_Z^2}{8\sqrt{2}} \frac{(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\left[\overbrace{(R_e+L_e)(R_\mu+L_\mu)(1+z^2) + 2(R_e-L_e)(R_\mu-L_\mu)z}^{(R_eR_\mu + L_eL_\mu)(1+z)^2 + (R_eL_\mu + L_eR_\mu)(1-z)^2}\right]$$
$$+ \frac{G_F^2 M_Z^4}{64\pi} \frac{s}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\left[(R_e^2+L_e^2)(R_\mu^2+L_\mu^2)(1+z^2) + 2(R_e^2-L_e^2)(R_\mu^2-L_e^2)z\right]$$

$$A_{FB} \equiv \frac{\int_0^1 \frac{d\sigma}{dz}dz - \int_{-1}^0 \frac{d\sigma}{dz}dz}{\int_0^1 \frac{d\sigma}{dz}dz + \int_{-1}^0 \frac{d\sigma}{dz}dz} \Rightarrow \lim_{s/M_Z^2 \ll 1} A_{FB} = \frac{3 G_F s}{16\sqrt{2}\pi\alpha Q_\mu}(R_e - L_e)(R_\mu - L_\mu)$$

$$\simeq -6.7\times 10^{-5}\left(\frac{s}{1\,\mathrm{GeV}^2}\right)(R_e - L_e)(R_\mu - L_\mu)$$

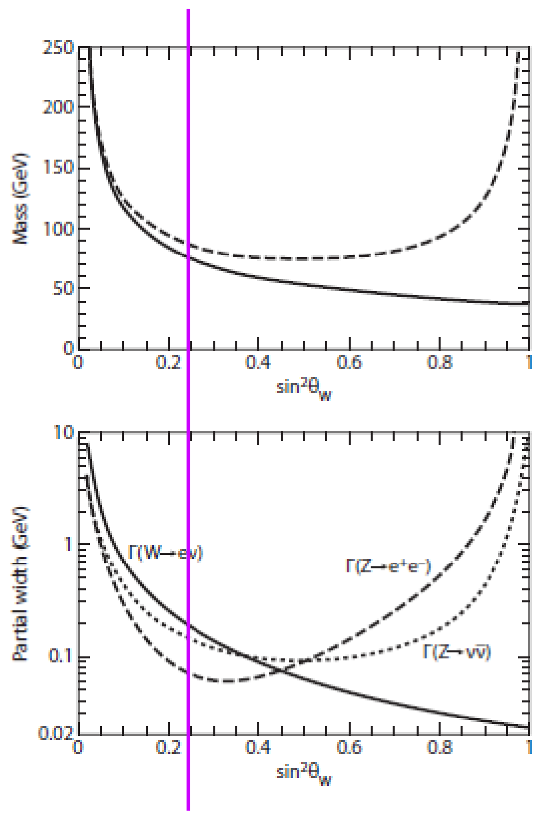
$R - L = 2a$

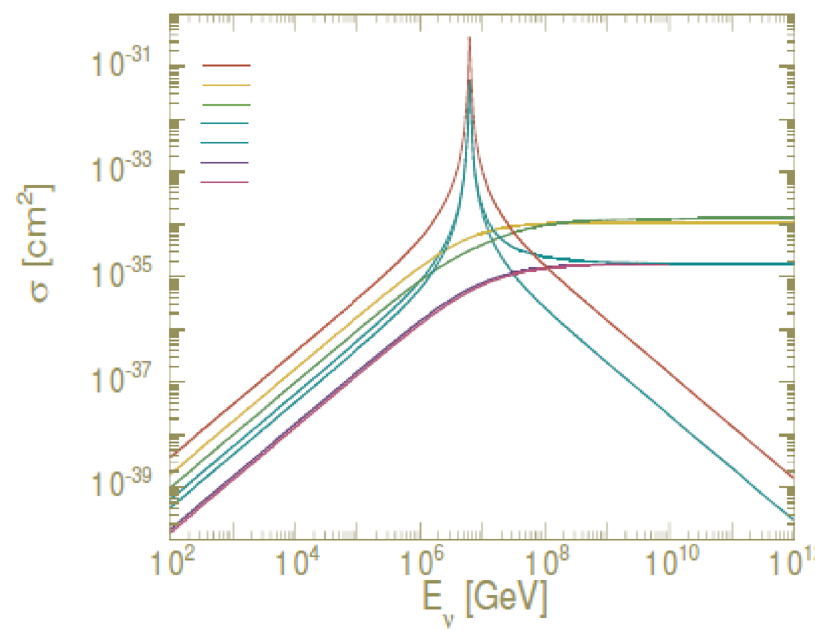
$Q_\mu = -1$.

$$= -\frac{3 G_F}{4\sqrt{2}\pi\alpha} a^2 s \longrightarrow |a| \text{ determined !}$$

With a measurement of $\sin^2\theta_W$, it is possible to predict

$$\boxed{M_W^2 = \frac{1}{4}g^2 v^2 = \frac{e^2}{4\sqrt{2}G_F s_W^2} \simeq (37.3\,\mathrm{GeV})^2/\sin^2\theta_W \quad \oplus \quad M_Z^2 = M_W^2/\cos^2\theta_W}$$

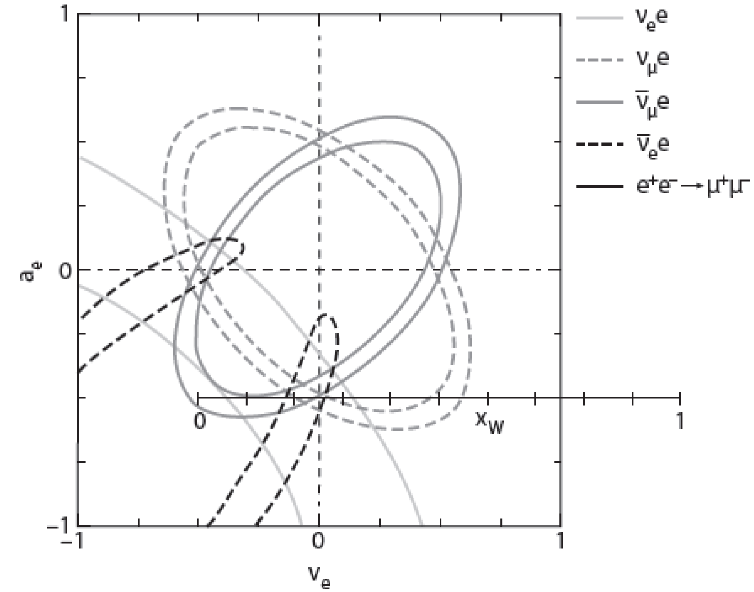




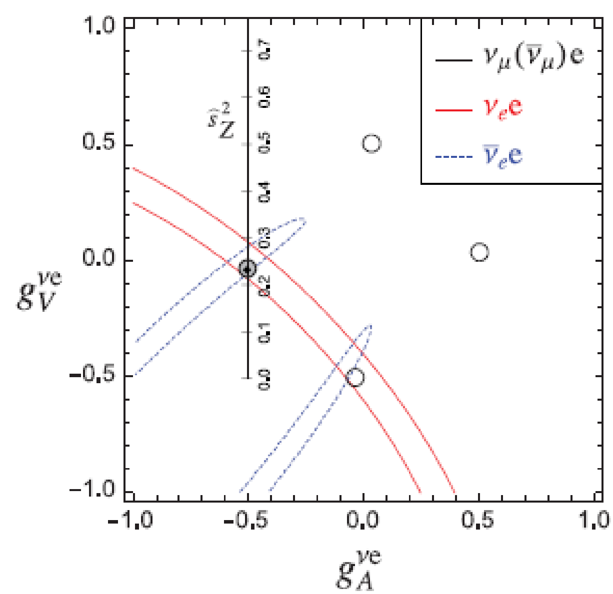
$\sigma(\bar\nu_e e^- \to \text{hadrons}) > \sigma(\nu_\mu e^- \to \mu^- \nu_e) > \sigma(\nu_e e^- \to \nu_e e^-) > \sigma(\bar\nu_e e^- \to \bar\nu_\mu \mu^-)$

$> \sigma(\bar\nu_e e^- \to \bar\nu_e e^-) > \sigma(\nu_\mu e^- \to \nu_\mu e^-) > \sigma(\bar\nu_\mu e^- \to \bar\nu_\mu e^-)$

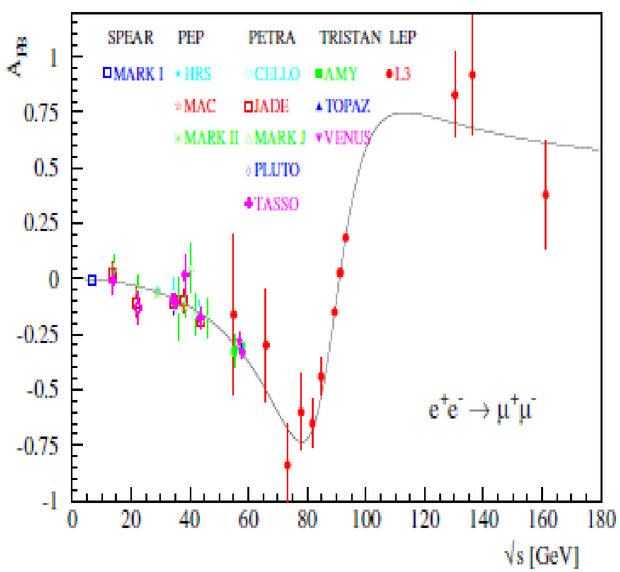
Gargamelle $\nu_\mu e$ Event 1973



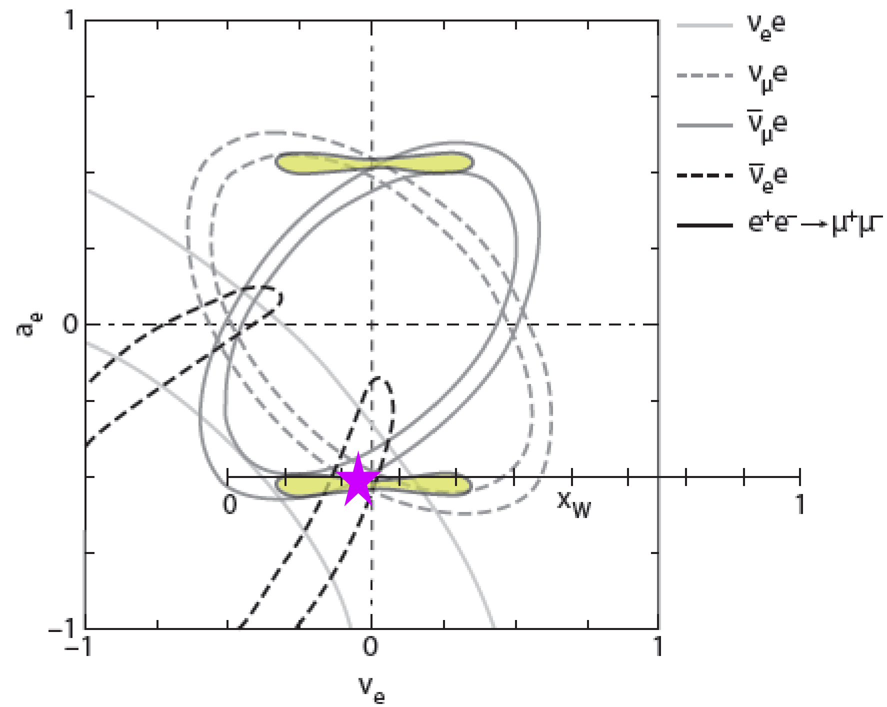
X ⇔ Y



FB Asymmetry

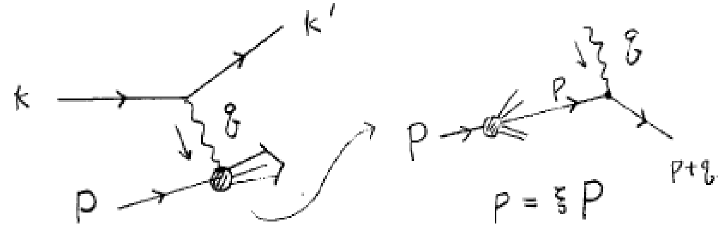


Two-fold ambiguity disappeared!



Neutrino-Nucleon Scattering

Kinematics: (Masses neglected)



$$p = \xi P \qquad p^2 \simeq 0$$

$$\boxed{\begin{aligned} Q^2 &= -q^2 > 0 \\ x &\equiv \frac{Q^2}{2P\cdot q} \\ y &\equiv \frac{2P\cdot q}{2P\cdot k} \end{aligned}}$$

$$x = \xi \quad \left(\because\ (p+q)^2 \simeq 0 = 2p\cdot q - Q^2 = 2\xi P\cdot q - Q^2\right)$$

DIS for neutrino-nucleon scattering: (for simplicity, consider a nucleus with $\# n = \# p$)

$\sum_f \int dx\, f_f(x)$: parton distribution functions before including Q^2-dependent corrections

$$\boxed{\begin{aligned} \frac{d^2\sigma}{dx\,dy}(\nu_\mu A \to \mu^- X) &= \frac{G_F s}{\pi}\left[x f_q(x) + x f_{\bar q}(x)(1-y)^2\right] \\ \frac{d^2\sigma}{dx\,dy}(\bar\nu_\mu A \to \mu^+ X) &= \frac{G_F s}{\pi}\left[x f_q(x)(1-y)^2 + x f_{\bar q}(x)\right] \end{aligned}}$$

$f_{q/\bar q}(x)$: summed quark/anti-quark distributions

$$\frac{d^2\sigma}{dx\,dy}(\nu A \to \nu X) = \frac{G_F s}{\pi}\left[x f_q(x)\left\{\left(\tfrac{1}{2} - s_W^2\right) + \tfrac{5}{9} s_W^4\left(1+(1-y)^2\right)\right\} + x f_{\bar q}(x)\left\{\left(\tfrac{1}{2} - s_W^2\right)(1-y)^2 + \tfrac{5}{9} s_W^4\left(1+(1-y)^2\right)\right\}\right]$$

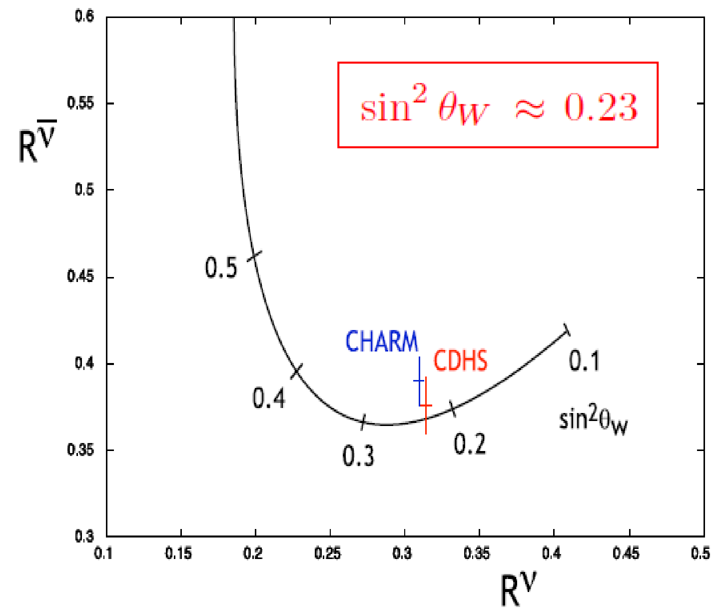
$$\frac{d^2\sigma}{dx\,dy}(\bar\nu A \to \bar\nu X) = \frac{G_F s}{\pi}\left[x f_q(x)\left\{\left(\tfrac{1}{2} - s_W^2\right)(1-y)^2 + \tfrac{5}{9} s_W^4\left(1+(1-y)^2\right)\right\} + x f_{\bar q}(x)\left\{\left(\tfrac{1}{2} - s_W^2\right) + \tfrac{5}{9} s_W^4\left(1+(1-y)^2\right)\right\}\right]$$

$$r \equiv \frac{\sigma(\bar{\nu}, CC)}{\sigma(\nu, CC)} = \frac{f_{\bar{q}}(x) + f_q(x)(1-y)^2}{f_q(x) + f_{\bar{q}}(x)(1-y)^2}$$: directly measurable ~ 0.4

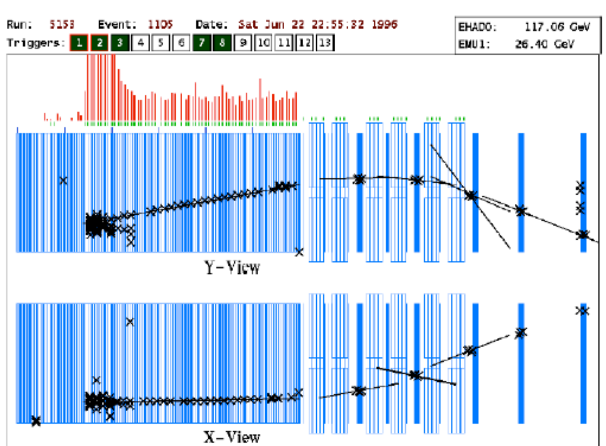
$$\boxed{\begin{aligned} R^{\nu} &\equiv \frac{\sigma(\nu, NC)}{\sigma(\nu, CC)} = \frac{1}{2} - s_W^2 + \frac{5}{9} s_W^4 (1 + r) \\ R^{\bar{\nu}} &\equiv \frac{\sigma(\bar{\nu}, NC)}{\sigma(\bar{\nu}, CC)} = \frac{1}{2} - s_W^2 + \frac{5}{9} s_W^4 \left(1 + \frac{1}{r}\right) \end{aligned}}$$

$(R^{\nu}, R^{\bar{\nu}})$ lies on a specific curve in the plane of possible values, swept by the values of $\sin^2\theta_W$

[Weinberg's nose]

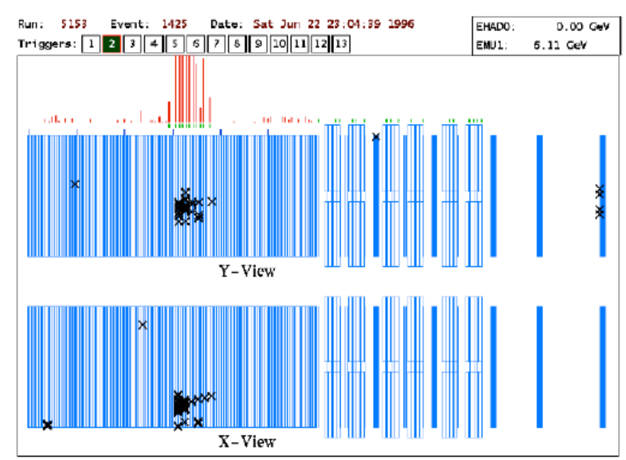


CC



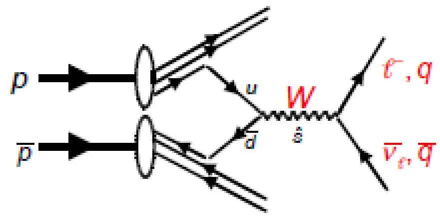
CCFR experiment

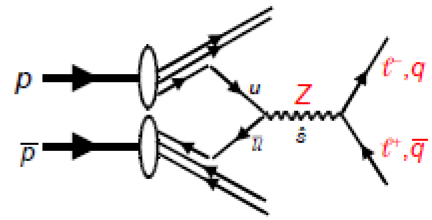
NC



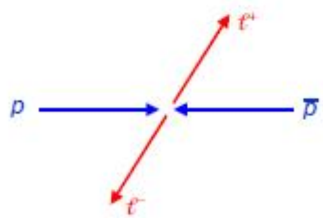
CCFR experiment

Discovery of Z and W 1983



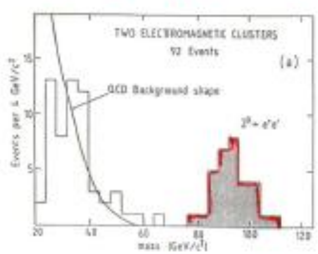


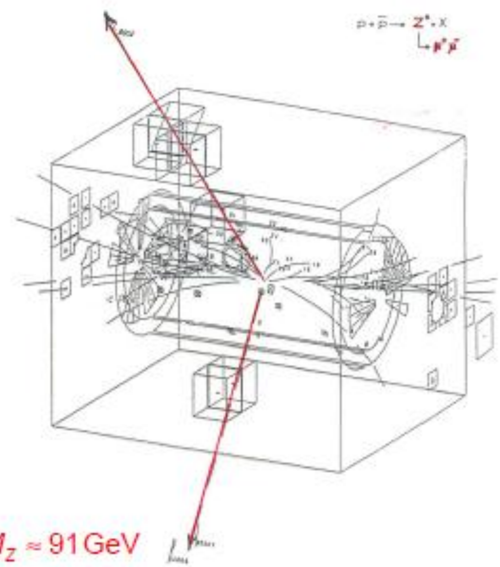
Z boson



High-energy lepton pair:

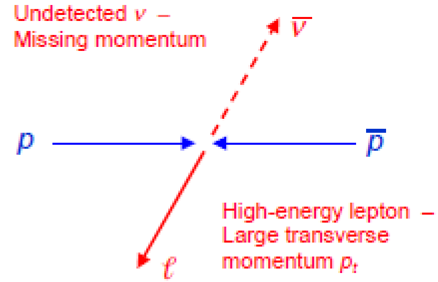
$$m_{\ell\ell}^2 = (p_{\ell^+} + p_{\ell^-})^2 = M_Z^2$$





$M_Z \approx 91\,\text{GeV}$

W boson



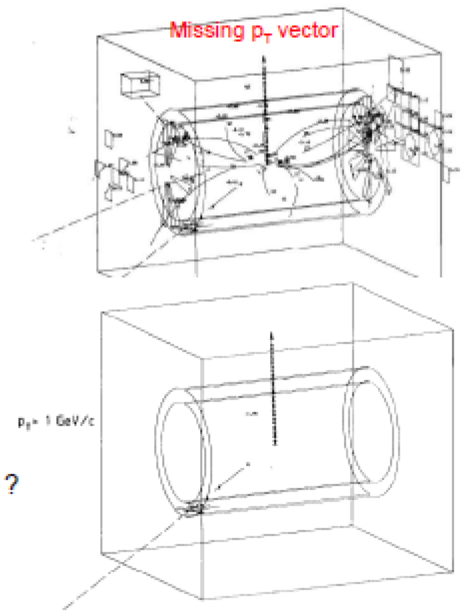
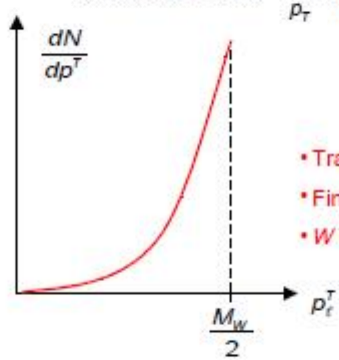


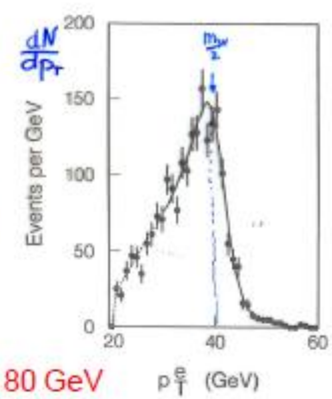
Fig. [illegible]. The same as picture (a), except that now only particles with p_T>1 GeV/c and calorimeters with E_T>1 GeV are shown.

How can the W mass be reconstructed ?

Jacobian Peak: $$\frac{dN}{p_T} \sim \frac{2p_T}{M_W} \cdot \left(\frac{M_W^2}{4} - p_T^2 \right)^{-1/2}$$



- Trans. Movement of the W
- Finite W decay width
- W decay is not isotropic



$M_W \approx 80$ GeV

EW interactions of quarks :

- Left-handed doublet

$$Q_L = \begin{bmatrix} u \\ d \end{bmatrix}_L \quad \begin{array}{ccc} 1/2 & +2/3 & \\ -1/2 & -1/3 & 1/3 \\ I_3 & Q & Y \end{array}$$

- 2 right-handed singlets

$$\begin{array}{cccc} u_R & 0 & +2/3 & +4/3 \\ d_R & 0 & -1/3 & -2/3 \\ & I_3 & Q & Y \end{array}$$

- CC interaction

$$\mathcal{L}_{W-q} = -\frac{g}{2\sqrt{2}} \left[\bar{u} \gamma^\mu (1-\gamma_5) d \, W_\mu^+ + \bar{d} \gamma^\mu (1-\gamma_5) u \, W_\mu^- \right]$$

identical to $\mathcal{L}_{W-\ell}$ in form reflecting "universality"

- NC interaction

$$\mathcal{L}_{Z-q} = -\frac{g_Z}{4} \sum_{q=u,d} \left[\bar{q} \gamma^\mu \{ L_q (1-\gamma_5) + R_q (1+\gamma_5) \} q \right] Z_\mu$$

$$L_q = 2 (I_3 - Q_q s_W^2), \quad R_q = -2 Q_q s_W^2$$

equivalent in form to $\mathcal{L}_{Z-\ell}$

Universality between $u \leftrightarrow d$ and $\nu_e \leftrightarrow e$ is NOT quite right.

$\exists$ Strangeness quark (s) ⇨ $\Delta S = 1$ transitions, although rather suppressed, occur !

Cabbibo (1963) : $\begin{bmatrix} u \\ d \end{bmatrix}_L \longrightarrow \begin{bmatrix} u \\ d_\theta \end{bmatrix}$ with $d_\theta \equiv d \cos\theta_c + s \sin\theta_c$, $\cos\theta_c = 0.9736 \pm 0.0010$

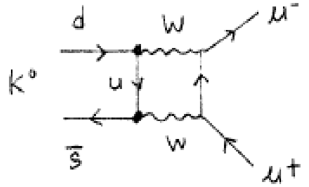
$\theta_c \simeq 12^\circ$

⇨ The "Cabbibo-rotated" doublet perfects CC interactions up to small third-generation effects, but causes a SERIOUS trouble for NC interactions.

$\boxed{d_\theta \to d' \text{ later}}$

$$\mathcal{L}_{Z\text{-}q} = -\frac{g_Z}{4} Z_\mu \left[J^\mu_{\bar u u} + J^\mu_{\bar d d}\cos^2\theta_c + J^\mu_{\bar s s}\sin^2\theta_c + \underline{(J^\mu_{\bar d s} + J^\mu_{\bar s d})\cos\theta_c \sin\theta_c} \right]$$

Eg. $\dfrac{B_r(K_L \to \mu^+\mu^-)}{B_r(K_L \to \text{all})} \sim 7\times 10^{-9}$ $\not\Leftrightarrow$ Not so much suppressed $\Delta S = 1$ NC



$\longrightarrow$ $M \sim \cos\theta_c \sin\theta_c$ $\longrightarrow$ Resolution : GIM (1970)

☀ GIM (Glashow, Iliopoulos, Maiami) $\Rightarrow$ 2nd doublet $\begin{bmatrix} c \\ s_\theta \end{bmatrix}$ with $s_\theta = -\sin\theta_c\, d + \cos\theta_c\, s$

∇ $(\bar u, \bar d_\theta) \otimes \begin{bmatrix} u \\ d_\theta \end{bmatrix} + (\bar c, \bar s_\theta) \otimes \begin{bmatrix} c \\ s_\theta \end{bmatrix}$ No tree-level NC !

GIM

$= \bar u \otimes u + \bar d \otimes d \cos^2\theta_c + \bar s \otimes s \sin^2\theta_c + (\bar d \otimes s + \bar s \otimes d)\cos\theta_c \sin\theta_c$ $\Rightarrow$ $\bar u \otimes u + \bar c \otimes c$

$+ \bar c \otimes c + \bar d \otimes d \sin^2\theta_c + \bar s \otimes s \cos^2\theta_c - (\bar d \otimes s + \bar s \otimes d)\cos\theta_c \sin\theta_c$ $+ \bar d \otimes d + \bar s \otimes s$

- Another historical retrospect

1964 : Discovery of CP violation $\longrightarrow$ 1973 : 3-generation quark mixing for $\not{CP}$ — Kobayashi and Maskawa

$B_r(K_L^0 \to \pi^+\pi^-) \neq 0$

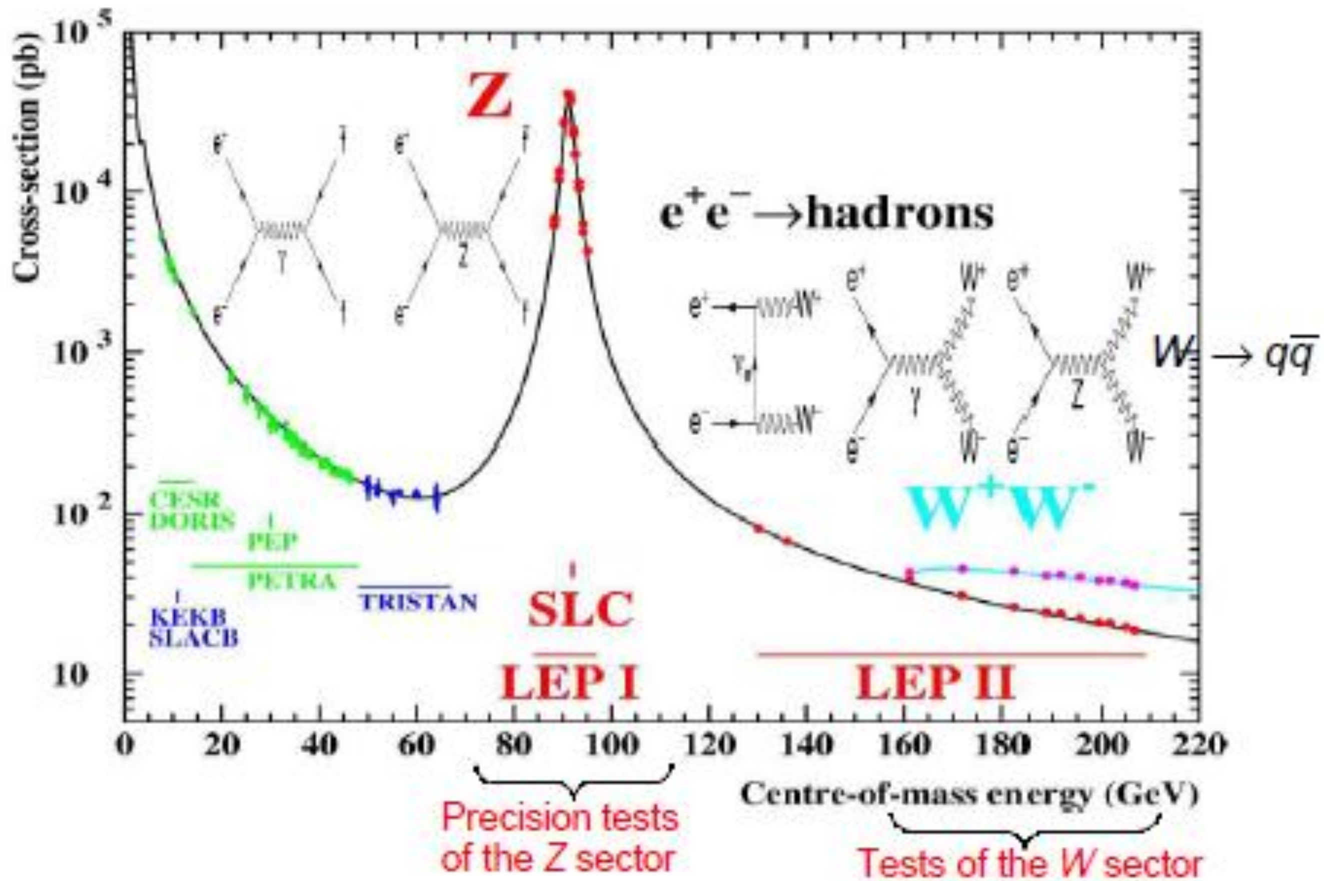
1974 : $c\bar c$ bound states discovered — 1995 : Discovery of t quark

1977 : $b\bar b$ 〃 $\Rightarrow$ CKM : $\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$ Valid ! (almost)

$N^2 - (2N-1) = (N-1)^2 = \underbrace{\tfrac{1}{2}(N-1)N}_{\text{angles}} \oplus \underbrace{\tfrac{1}{2}(N-1)(N-2)}_{\text{phases}} \xrightarrow[N=3]{} 3 \oplus 1$

$\hookrightarrow$ CP phase

Production of Z at LEP I & SLC and W pair at LEP II



Precision tests of the Z sector

(LEP and SLC)

~4.5M Z decays / experiment

Cross section for $e^+e^- \to \gamma / Z \to f\bar{f}$

$|M|^2 = \Big|$  $+$  $\Big|^2$

$$\boxed{e^+e^- \to \mu^+\mu^-}$$

$$M_\gamma = -e^2(\bar{\mu}\gamma_\mu \mu)\frac{1}{q^2}(\bar{e}\gamma^\mu e) \quad + \quad M_Z = -\frac{g^2}{\cos^2\theta_W}\left[\bar{\mu}\gamma^\nu \frac{1}{2}(g_V^\mu - g_A^\mu\gamma^5)\mu\right]\underbrace{\frac{g_{\nu\rho} - q_\nu q_\rho / M_Z^2}{(q^2 - M_Z^2) + iM_Z\Gamma_Z}}_{\text{Z propagator considering a finite Z width}}\left[\bar{e}\gamma^\rho \frac{1}{2}(g_V^e - g_A^e\gamma^5)e\right]$$

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[F_\gamma(\cos\theta) + F_{\gamma Z}(\cos\theta)\frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} + F_Z(\cos\theta)\frac{s^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]$$

γ $\qquad$ γ/Z interference $\qquad$ Z

$$F_{\gamma}(\cos\theta) = Q_e^2 Q_\mu^2 (1+\cos^2\theta) = (1+\cos^2\theta)$$

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W \cos^2\theta_W}\left[2 g_V^e g_V^\mu (1+\cos^2\theta) + 4 g_A^e g_A^\mu \cos\theta\right]$$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W \cos^4\theta_W}\left[({g_V^e}^2 + {g_A^e}^2)({g_V^\mu}^2 + {g_A^\mu}^2)(1+\cos^2\theta) + 8 g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta\right]$$

Z-boson pole

$$\sigma_{tot} = \sigma_Z = \frac{4\pi}{3s}\frac{\alpha^2}{16\sin^4\theta_w \cos^4\theta_w}\cdot\left[(g_V^e)^2 + (g_A^e)^2\right]\left[(g_V^\mu)^2 + (g_A^\mu)^2\right]\cdot\frac{s^2}{(s-M_Z^2)^2 + (M_Z\Gamma_Z)^2}$$

$$\sigma(s) = 12\pi\frac{\Gamma_e\Gamma_\mu}{M_Z^2}\cdot\frac{s}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$

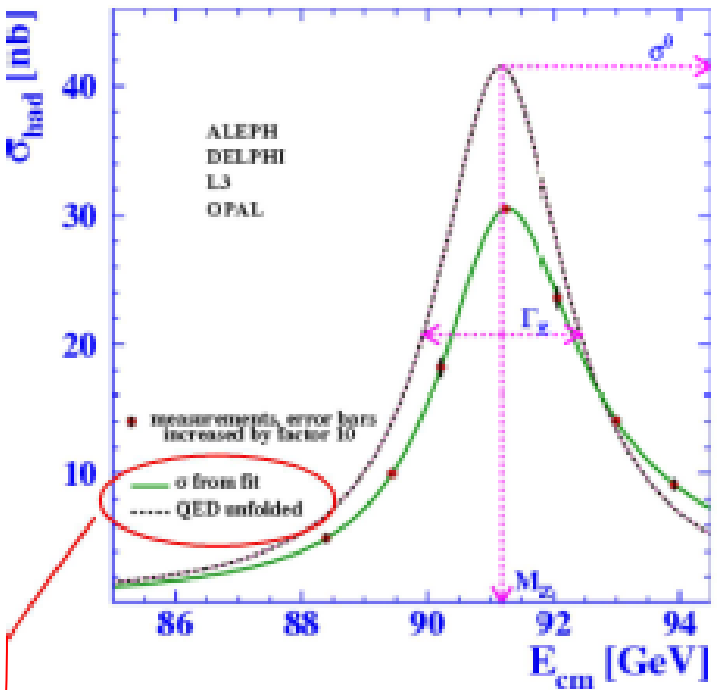
$$\sigma_Z\left(\sqrt{s} = M_Z\right) = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_\mu}{\Gamma_Z^2}$$

With partial and total widths:

$$\Gamma_f = \frac{\alpha M_Z}{12\sin^2\theta_w \cos^2\theta_w}\cdot\left[(g_V^f)^2 + (g_A^f)^2\right]$$

$$\Gamma_Z = \sum_i \Gamma_i$$

Z lineshape



Z resonance curve:

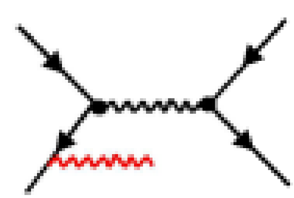
$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Peak: $$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

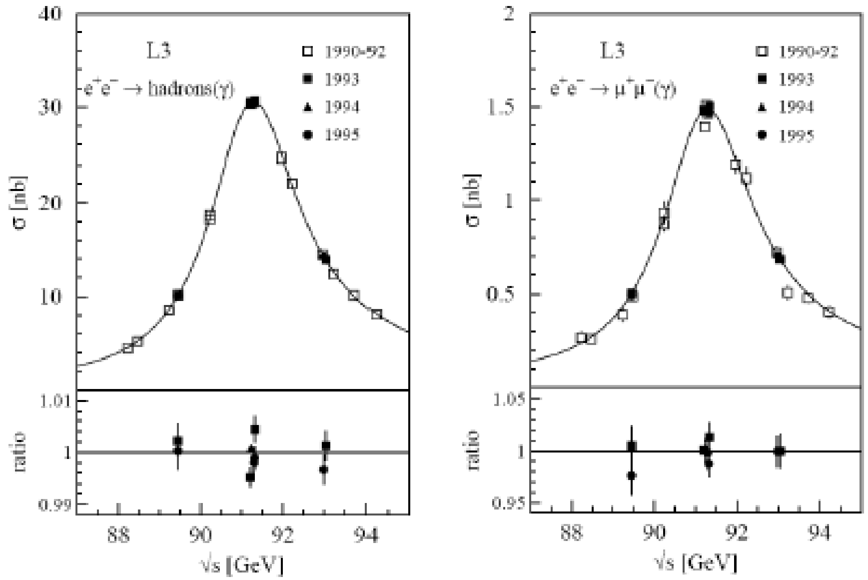
- Resonance position → M_Z
- Height → $\Gamma_e \Gamma_\mu$
- Width → Γ_Z

Initial state Bremsstrahlung corrections

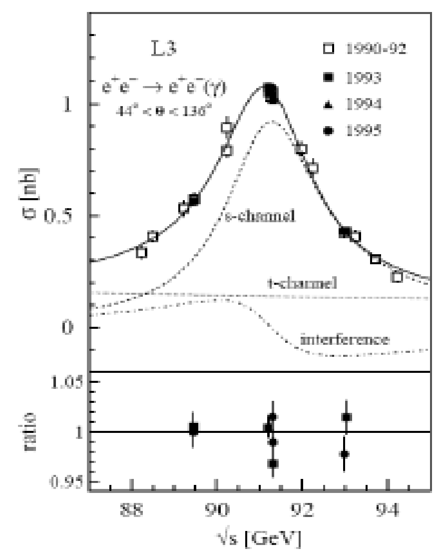
$$\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^{1} G(z)\sigma_{ff}^{0}(zs)\,dz \qquad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$



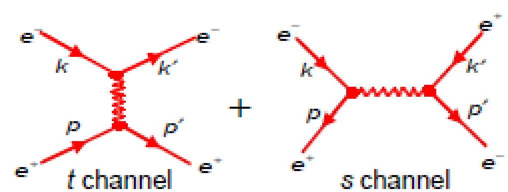
$e^+e^- \to hadrons$ $\qquad$ $e^+e^- \to \mu^+\mu^-$



$e^+e^- \to e^+e^-$



Identical resonance, independent of final states $\Rightarrow$ same propagator



Principal Z pole observables

$$R_\ell \equiv \Gamma(\text{had})/\Gamma(\ell^+\ell^-)$$

$$R_b \equiv \Gamma(b\bar{b})/\Gamma(\text{had})$$

$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$A_f \equiv \frac{2\bar{g}_V^f\,\bar{g}_A^f}{\bar{g}_V^{f2} + \bar{g}_A^{f2}}$$

Quantity	Value	Standard Model	Pull	Dev.
M_Z [GeV]	91.1876 ± 0.0021	91.1874 ± 0.0021	0.1	0.0
Γ_Z [GeV]	2.4952 ± 0.0023	2.4961 ± 0.0010	−0.4	−0.2
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7426 ± 0.0010	—	—
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.69 ± 0.06	—	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	84.005 ± 0.015	—	—
σ_{had}[nb]	41.541 ± 0.037	41.477 ± 0.009	1.7	1.7
R_e	20.804 ± 0.050	20.744 ± 0.011	1.2	1.3
R_μ	20.785 ± 0.033	20.744 ± 0.011	1.2	1.3
R_τ	20.764 ± 0.045	20.789 ± 0.011	−0.6	−0.5
R_b	0.21629 ± 0.00066	0.21576 ± 0.00004	0.8	0.8
R_c	0.1721 ± 0.0030	0.17227 ± 0.00004	−0.1	−0.1
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01633 ± 0.00021	−0.7	−0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.4	0.6
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5	1.6
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1034 ± 0.0007	−2.6	−2.3
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0739 ± 0.0005	−0.9	−0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1035 ± 0.0007	−0.5	−0.5
$\bar{s}_\ell^2(A_{FB}^{(0,q)})$	0.2324 ± 0.0012	0.23146 ± 0.00012	0.8	0.7
	0.23200 ± 0.00076		0.7	0.6
	0.2287 ± 0.0032		−0.9	−0.9
A_e	0.15138 ± 0.00216	0.1475 ± 0.0010	1.8	2.1
	0.1544 ± 0.0060		1.1	1.3
	0.1498 ± 0.0049		0.5	0.6
A_μ	0.142 ± 0.015		−0.4	−0.3
A_τ	0.136 ± 0.015		−0.8	−0.7
	0.1439 ± 0.0043		−0.8	−0.7
A_b	0.923 ± 0.020	0.9348 ± 0.0001	−0.6	−0.6
A_c	0.670 ± 0.027	0.6680 ± 0.0004	0.1	0.1
A_s	0.895 ± 0.091	0.9357 ± 0.0001	−0.4	−0.4

Number of light neutrino families

In the Standard Model:

$$\Gamma_Z = \Gamma_{\mathrm{had}} + 3\cdot\Gamma_\ell + \underbrace{N_\nu\cdot\Gamma_\nu}_{\text{invisible}\,:\,\Gamma_{inv}} \rightarrow \begin{cases} e^+e^- \to Z \to \nu_e\bar{\nu}_e \\ e^+e^- \to Z \to \nu_\mu\bar{\nu}_\mu \\ e^+e^- \to Z \to \nu_\tau\bar{\nu}_\tau \end{cases}$$

$$\Gamma_{inv} = 0.4990 \pm 0.0015\ \mathrm{GeV}$$

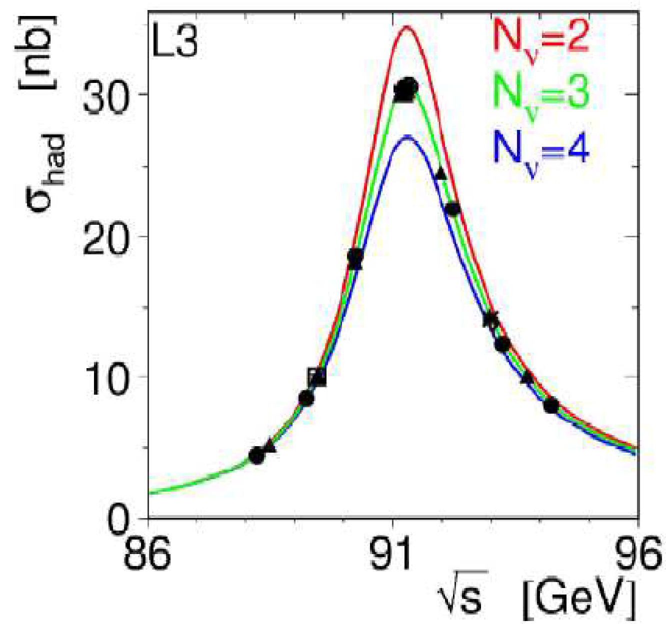
To determine the number of light neutrino generations:

$$N_\nu = \underbrace{\left(\frac{\Gamma_{inv}}{\Gamma_\ell}\right)_{exp}}_{5.9431\pm0.0163} \cdot \underbrace{\left(\frac{\Gamma_\ell}{\Gamma_\nu}\right)_{SM}}_{=1.991\pm0.001}$$

5.9431±0.0163 =1.991±0.001 (small theo. uncertainties from m_{top} M_{H})

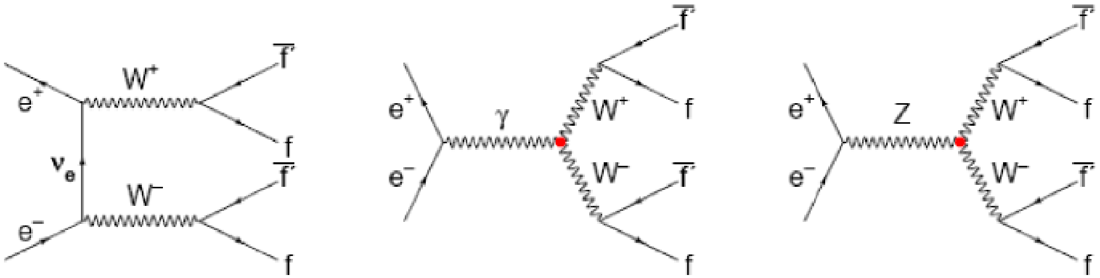
$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

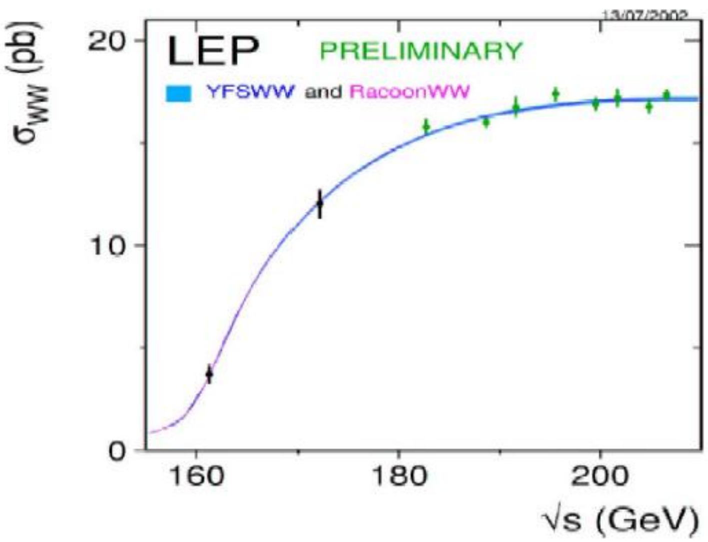
No room for new physics: $Z \to$ new

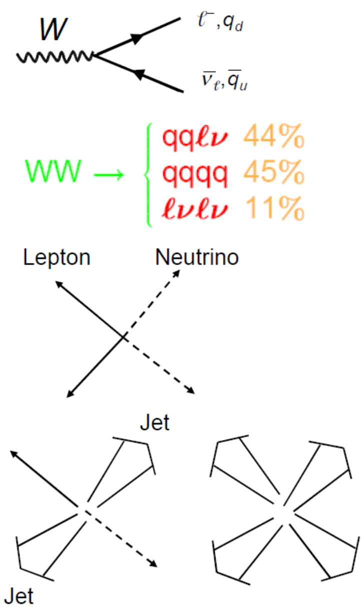


Precision tests of the W sector

(LEP2 and Tevatron)
~10K WW events / experiment

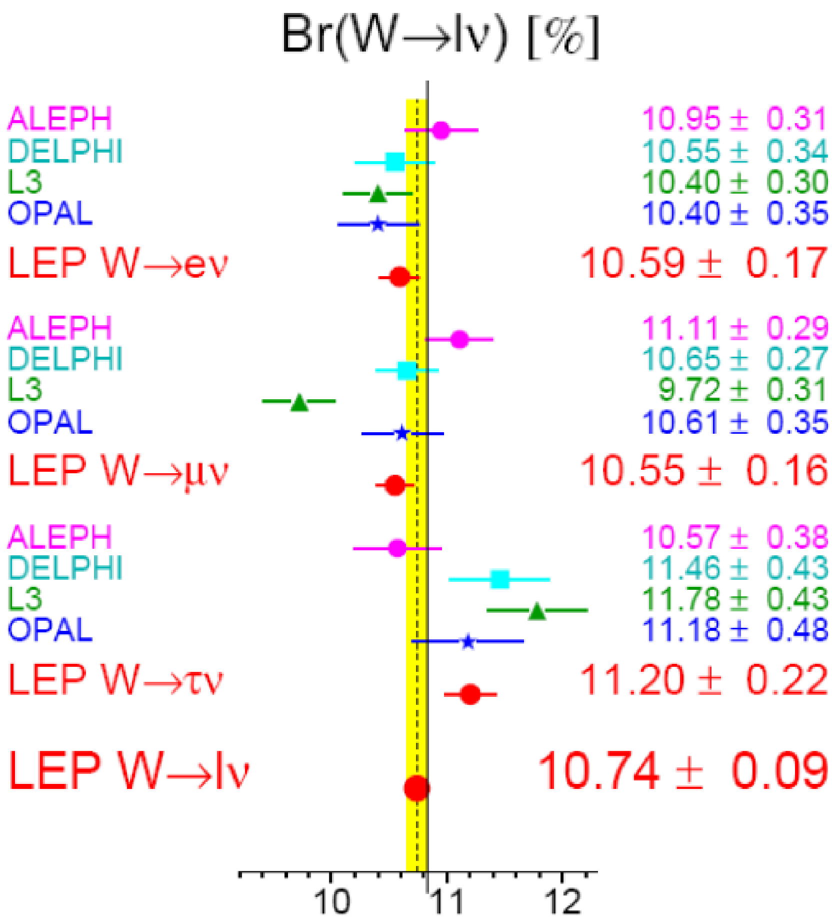






Allows determination of M_W

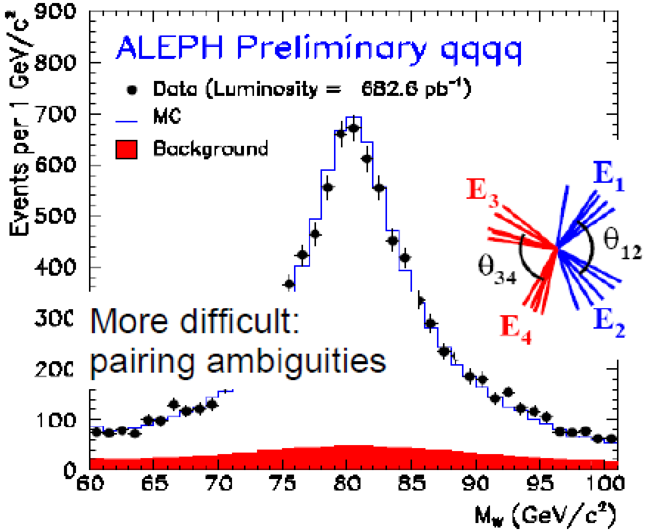
W branching ratios

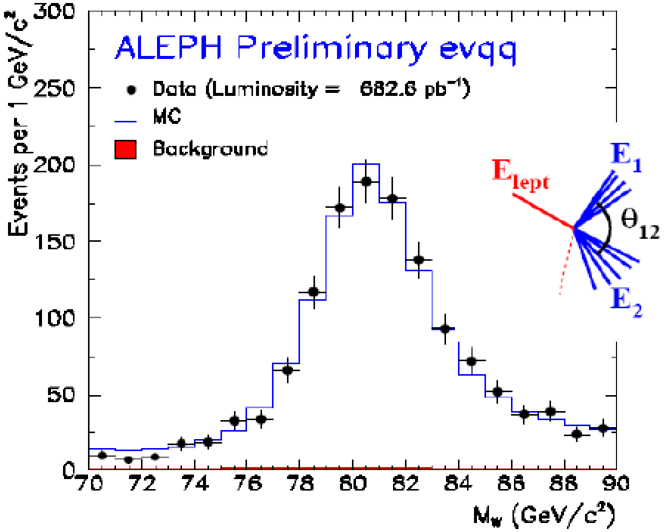


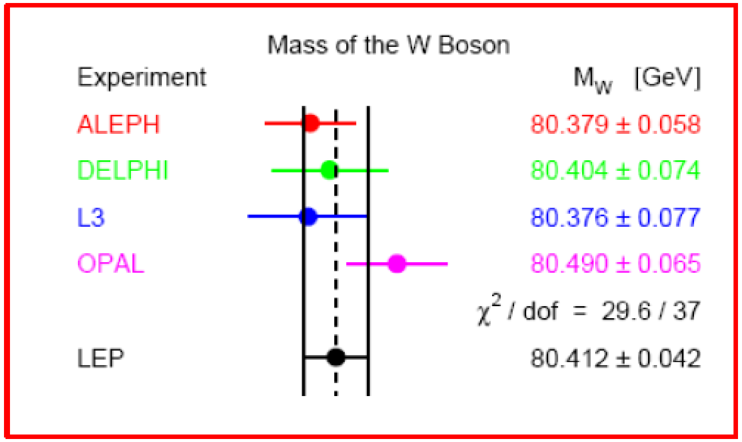
$$Br(W \to q\overline{q}) = (67.77 \pm 0.28)\%$$

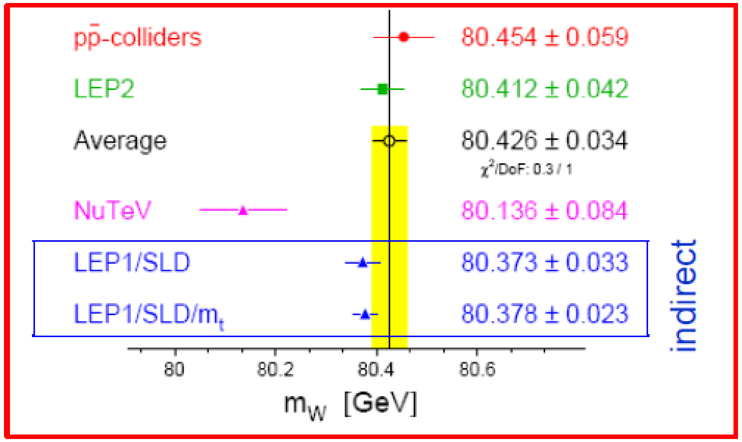
Lepton universality tested to 2%

W mass measurements

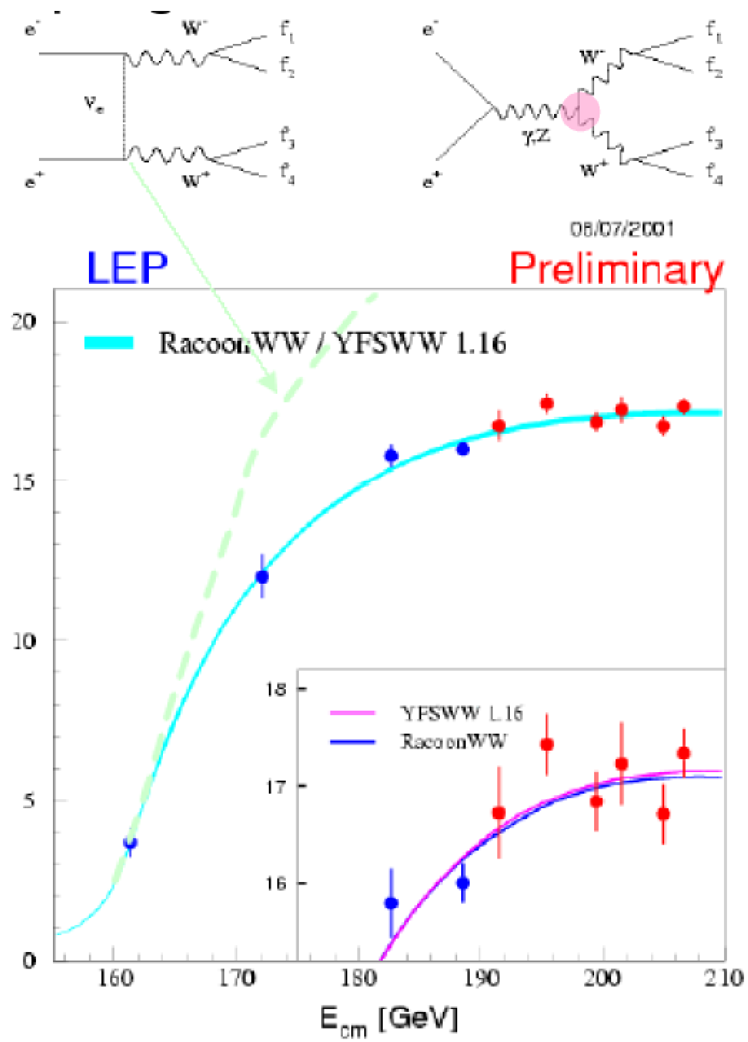




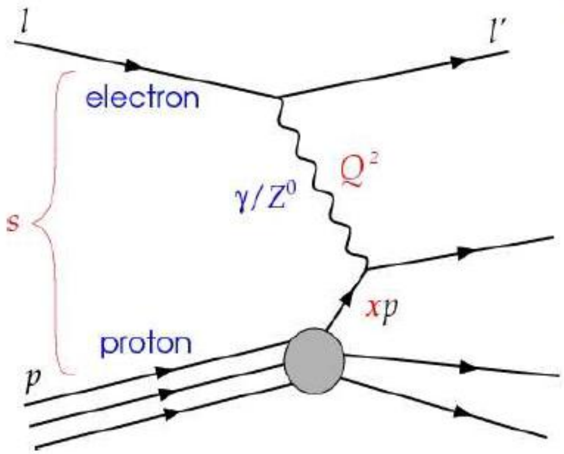




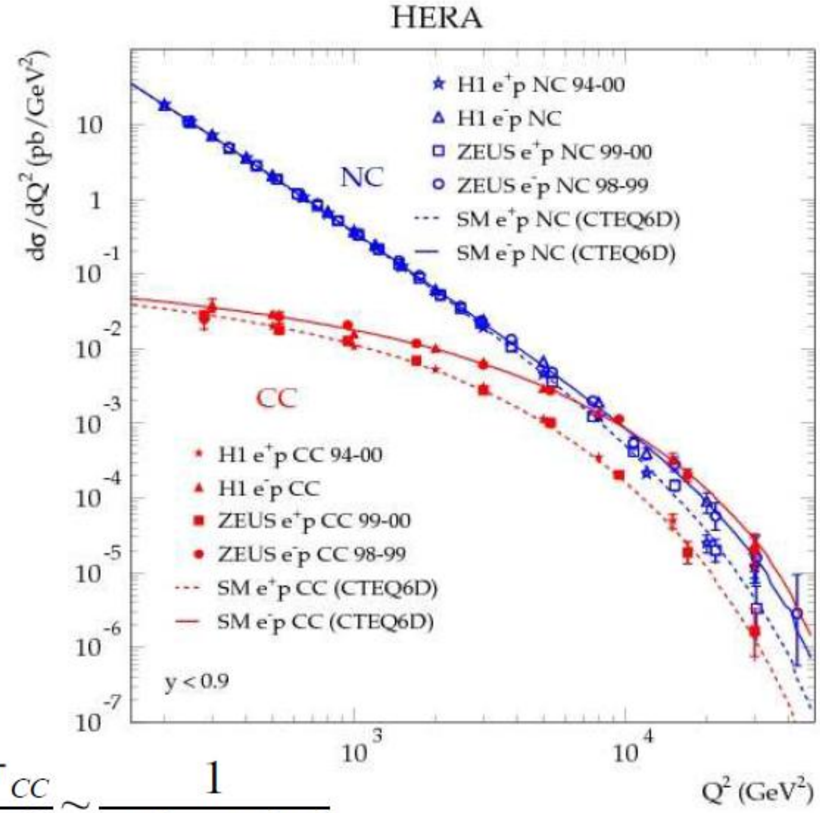
Triple gauge couplings

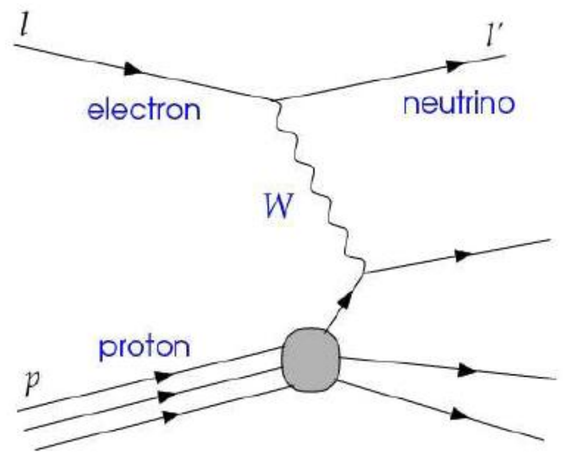


EW unification seen at HERA



$$\frac{d\sigma_{NC}}{dQ^2} \sim \frac{1}{(Q^2)^2} + \dots \frac{1}{Q^2(Q^2+M_Z^2)} + \dots \frac{1}{(Q^2+M_Z^2)^2}$$





$$\frac{d\sigma_{CC}}{dQ^2} \sim \frac{1}{(Q^2+M_W^2)^2}$$

Top quest

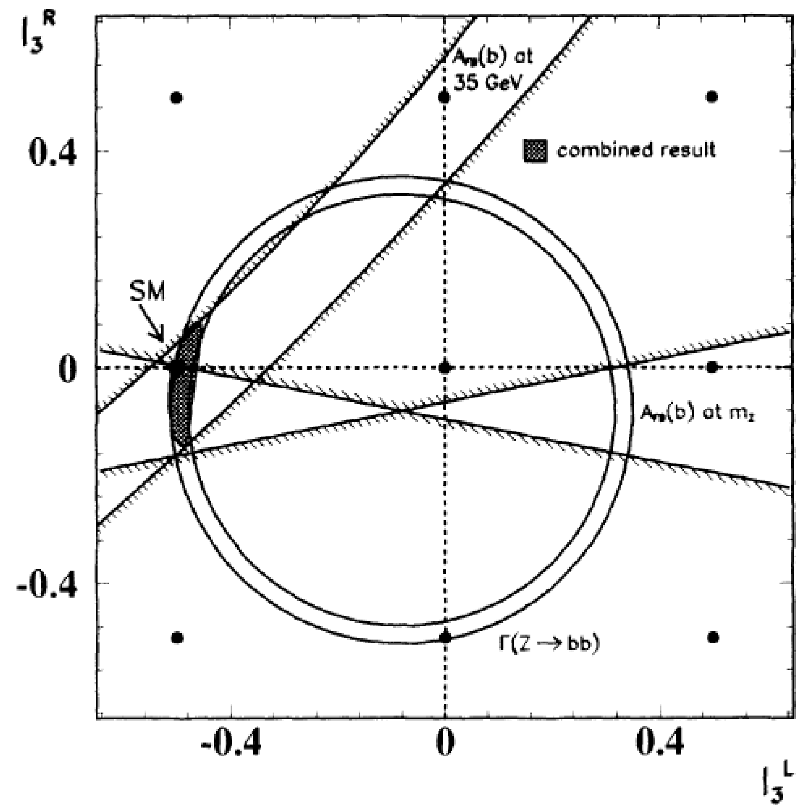
Raison d'être

Two families cannot account for CP violation in the quark sector

Given the existence of b and τ, the top quark is requested for anomaly cancellation

Absence of FCNC in (tree-level) b decays

b quark has a weak isospin of -1/2, demanding a partner



[Schaile and Zerwas, 1992]

Higher-order corrections ⇔ Top (and H) mass

Including radiative corrections

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \quad \Longrightarrow \quad \bar{\rho} = 1 + \Delta\rho$$

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \quad \Longrightarrow \quad \sin^2\theta_{\mathrm{eff}} = (1+\Delta\kappa)\sin^2\theta_W$$

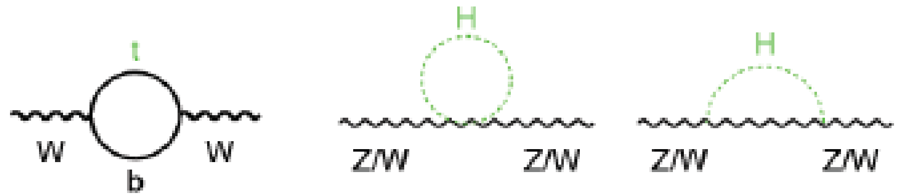
$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_W G_F} \quad \Longrightarrow \quad m_W^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_W G_F}(1+\Delta r)$$

$$\alpha(0) \quad \Longrightarrow \quad \alpha(m_Z^2) = \frac{\alpha(0)}{1-\Delta\alpha}$$

Lowest order SM predictions

$$\text{with :}\ \Delta\alpha = \Delta\alpha_{\mathrm{lept}} + \Delta\alpha_{\mathrm{top}} + \Delta\alpha_{\mathrm{had}}^{(5)}$$

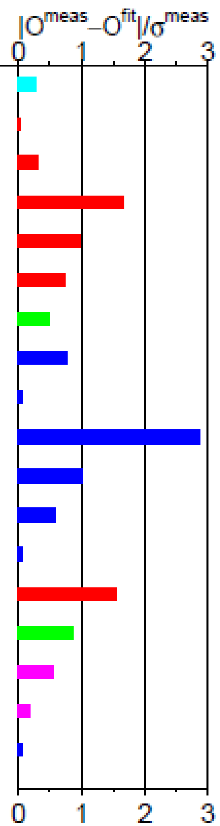
$$\Delta\rho, \Delta\kappa, \Delta r = f\left(m_t^2, \log(m_H), \ldots\right)$$



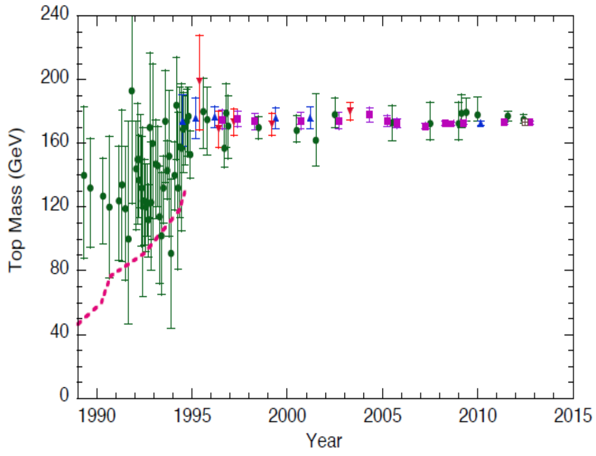
$$\Delta\rho = \frac{3G_F}{8\pi^2\sqrt{2}}\left[m_b^2 + m_t^2 + \frac{2m_t^2 m_b^2}{(m_t^2 - m_b^2)}\ln\left(\frac{m_b^2}{m_t^2}\right)\right] \quad \Rightarrow \quad \Delta\rho \to \frac{3G_F m_t^2}{8\pi^2\sqrt{2}}$$

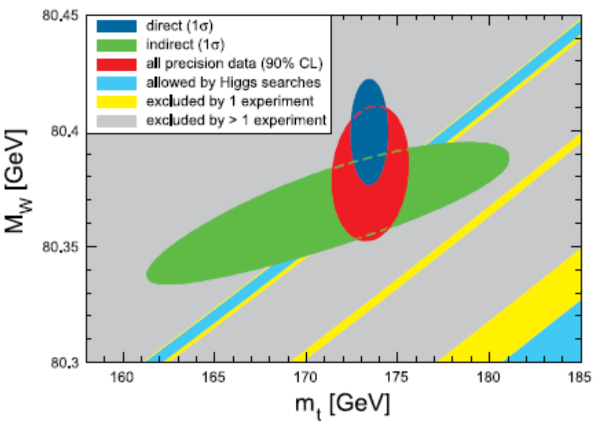
Global fits to precision EW measurements

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



March 2012

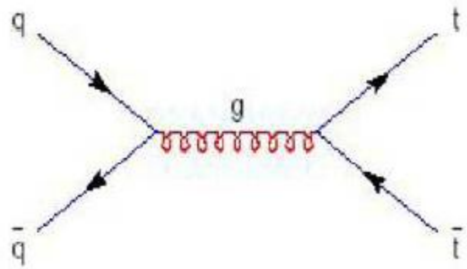




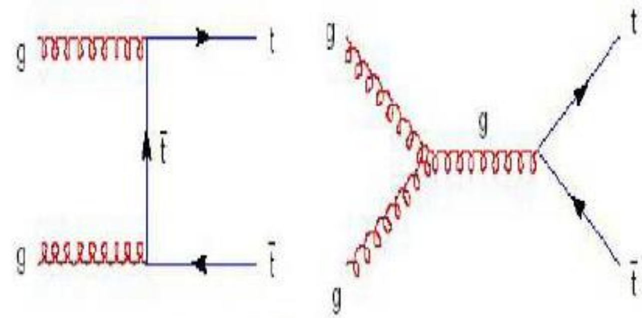
Precision and calculations improve with time!

Observation of the top quark at Tevatron (1995)

$p\bar{p}$ @ 2 TeV

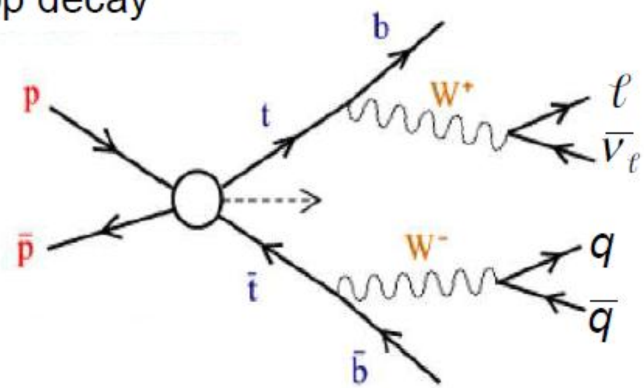


qq annihilation (85%)



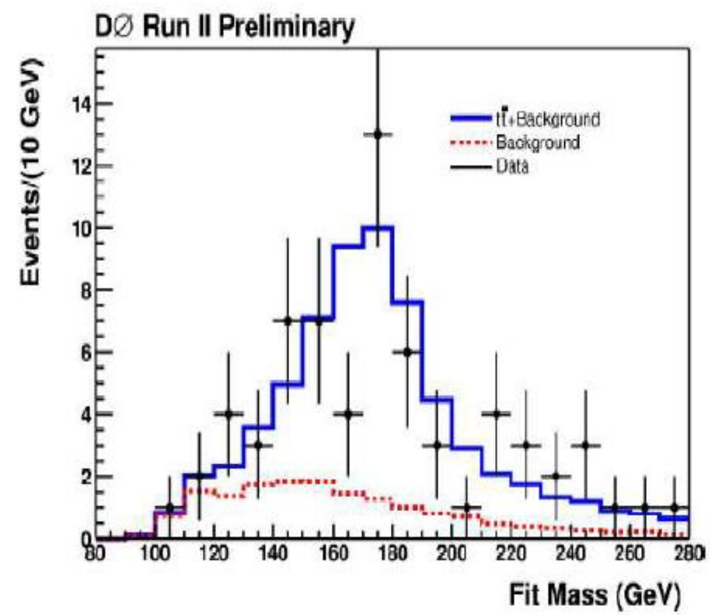
gluon fusion (15%)

Top decay



Channel used for mass reconstruction:

$$m_t = m_{inv}(b - jet, W \rightarrow jet + jet)$$



Successful SU(2) x U(1) EW structure

NC
Charm
W & Z

+

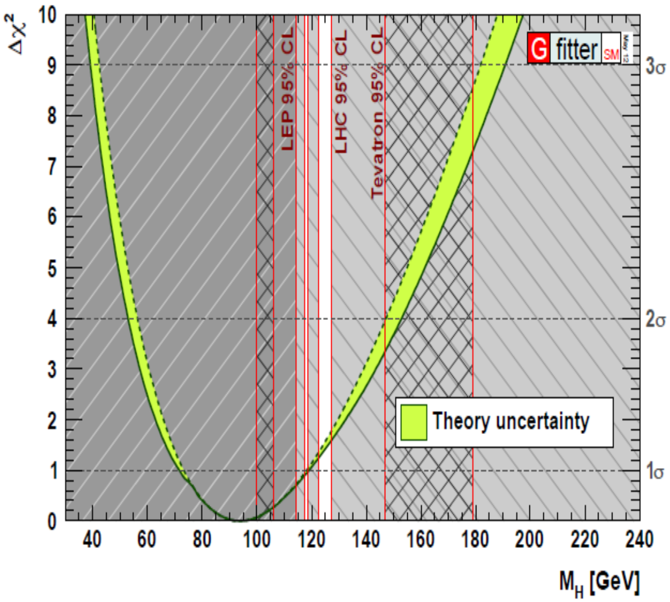
Precision
EW tests
(0.1%)

M_Z	$91\,187.5 \pm 2.1$ MeV
Γ_Z	2495.2 ± 2.3 MeV
$\sigma^0_{\text{hadronic}}$	41.540 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

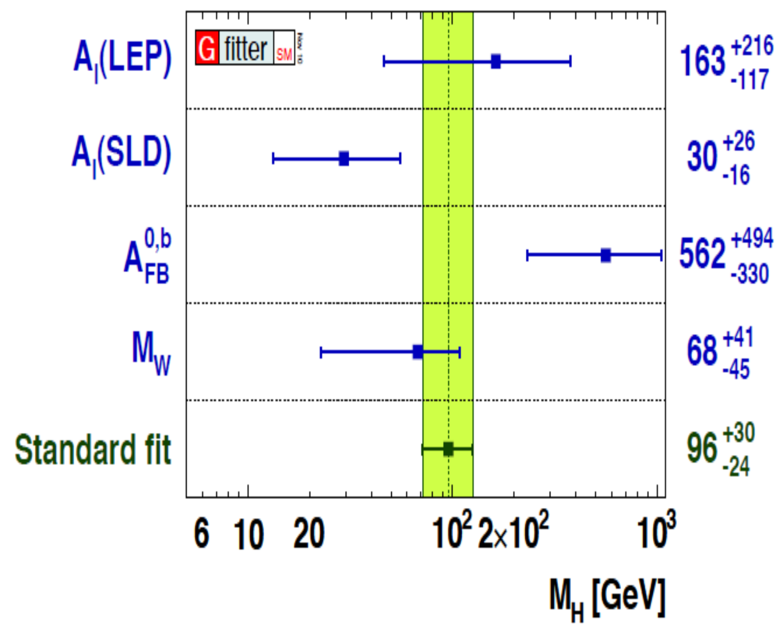
$$\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$$

$$N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i \bar{\nu}_i) = 2.984 \pm 0.008$$

H influence



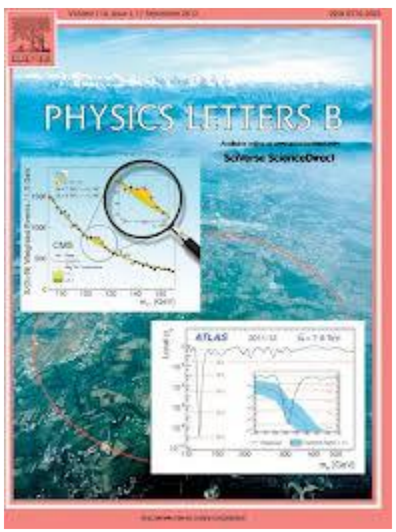
$\Delta\chi^2$
LEP 95% CL
LHC 95% CL
Tevatron 95% CL
Theory uncertainty
3σ
2σ
1σ
M_H [GeV]



A_l(LEP)
163^{+216}_{-117}
A_l(SLD)
30^{+26}_{-16}
$A_{FB}^{0,b}$
562^{+494}_{-330}
M_W
68^{+41}_{-45}
Standard fit
96^{+30}_{-24}
M_H [GeV]

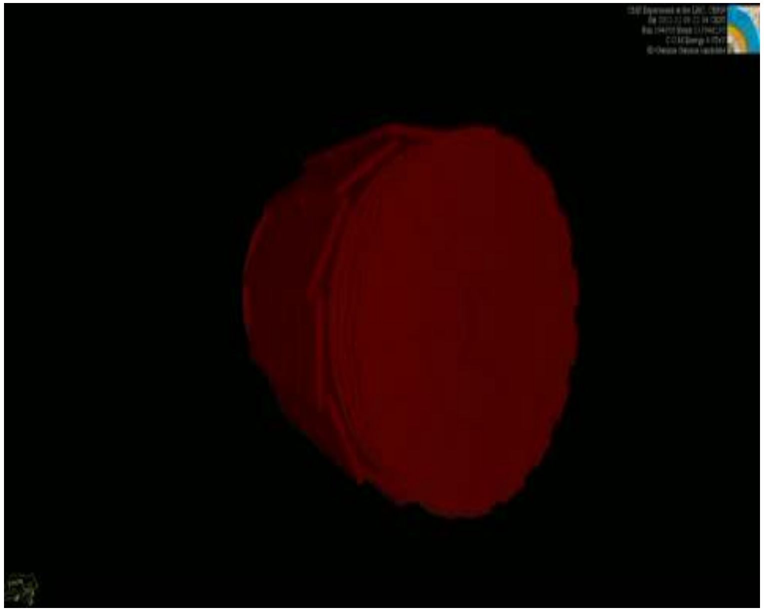
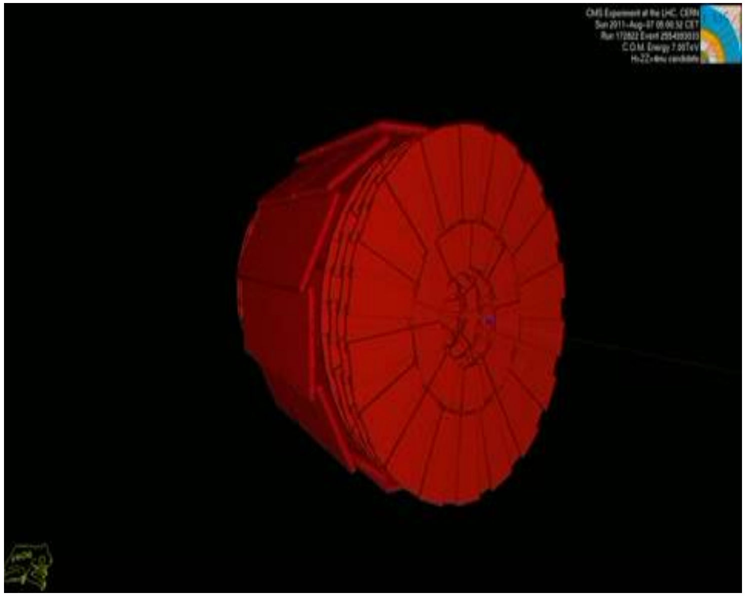
Higgs Odyssey

Discovery of a new boson on July 4, 2012



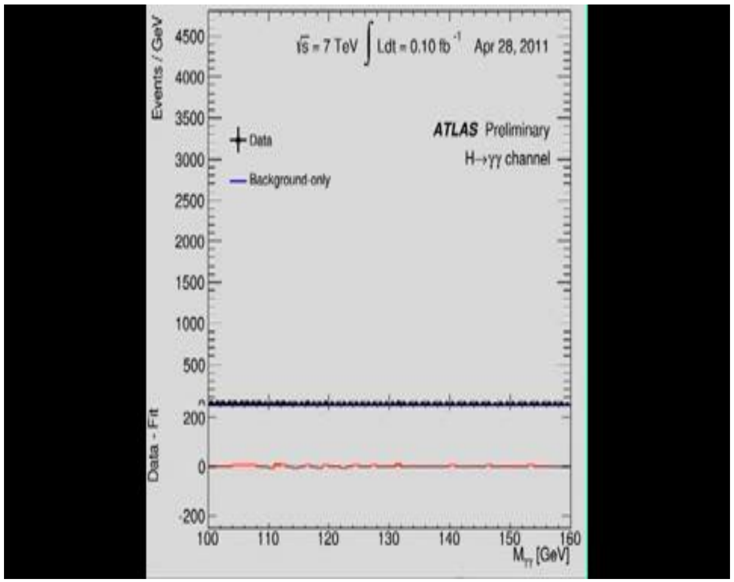
A Higgs boson
SM Higgs or not?
Implications
Prospects

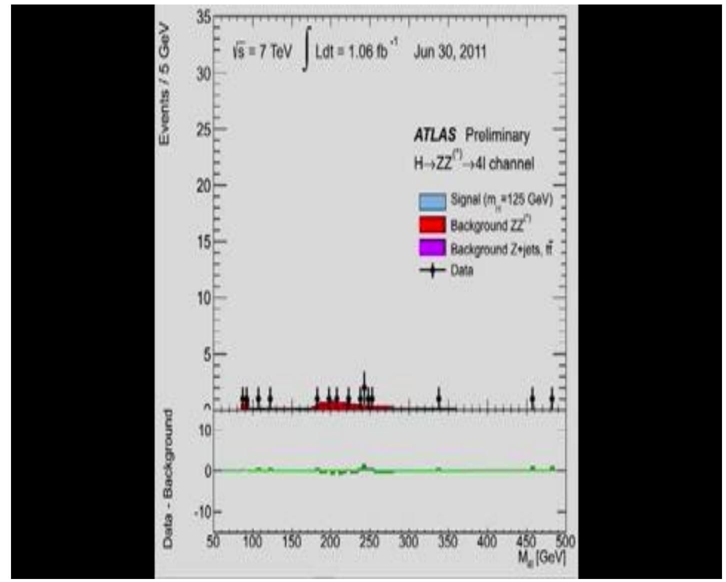
Characteristic 2 γ and 4 μ events

[CMS]

Accumulated 2-photon and **4-lepton** events



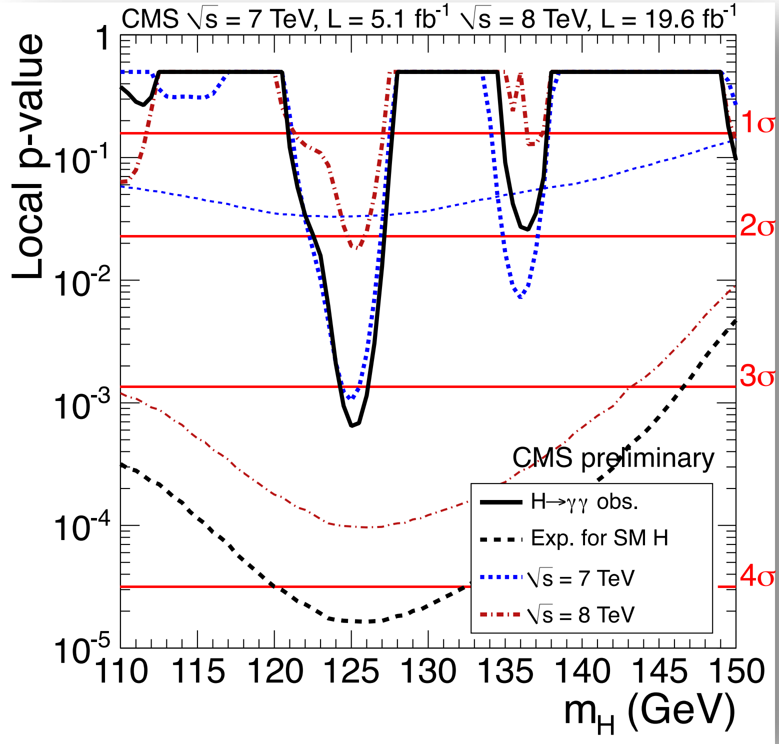


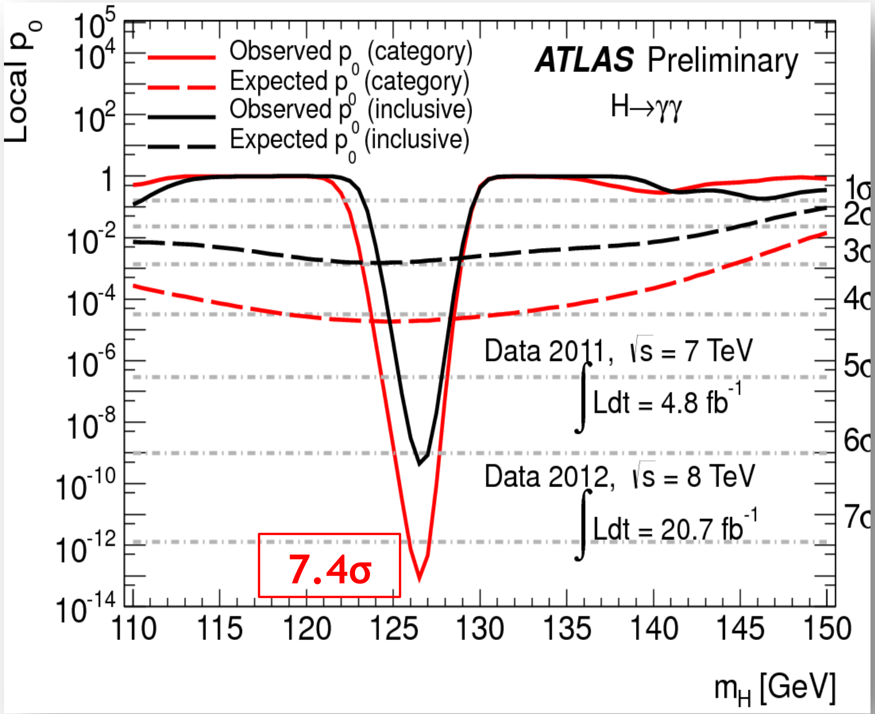
[ATLAS]

H→γγ

Moriond 2013

[Incandela @ Princeton]





With additional data, new analysis, significance decreased relative to July/4!!

7.4σ (obs) ⇔ 4.1σ (exp)

m_H = 126.8 ± 0.2(stat) ± 0.7(syst) GeV

m_H = 125.4 ± 0.5 (stat.) ± 0.6 (syst.)

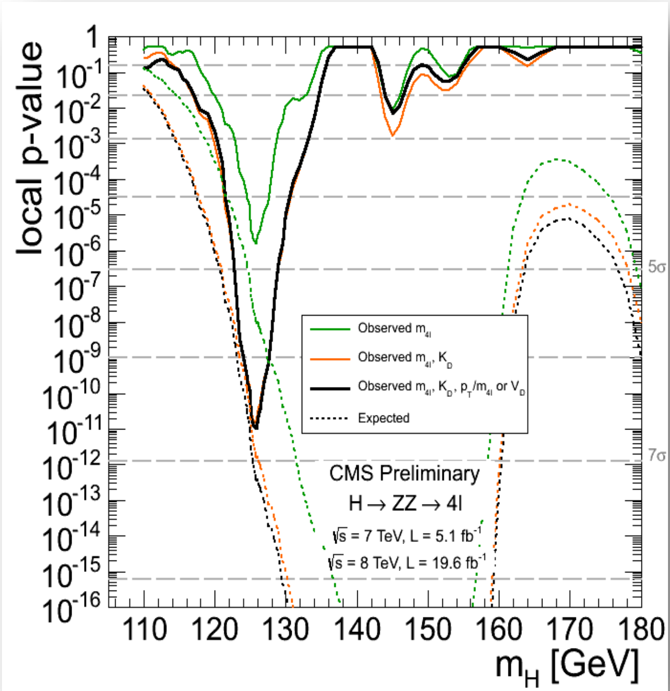
CMS HIG-13-001

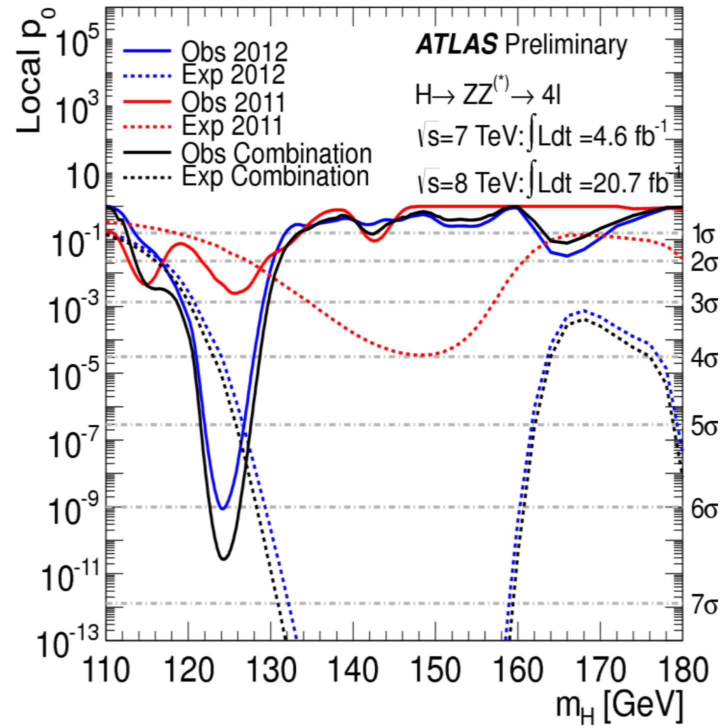
ATLAS-CONF-2013-021

$H \to Z^*Z \to 4\ l$

Moriond 2013

[Incandela @ Princeton]

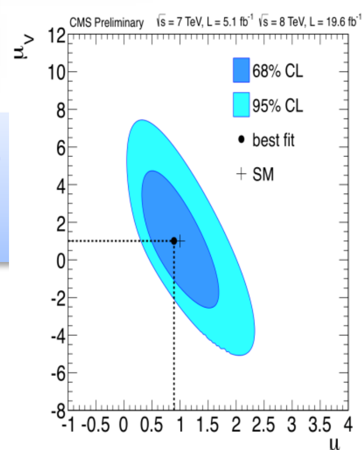




$125.8 \pm 0.5(stat.) \pm 0.2(syst.)$

6.7 σ (obs) ⇔ 7.1 σ (exp)

$\mu = \sigma/\sigma_{SM} = 0.92 \pm 0.28$



6.6σ (obs) ⇔ 4.4σ (exp)

$m_H = 124.3 \pm^{0.6}_{0.5} \pm^{0.5}_{0.3}$ GeV

$\mu = 1.7 \pm^{0.5}_{0.4}$

[CMS HIG-13-002](CMS HIG-13-002)

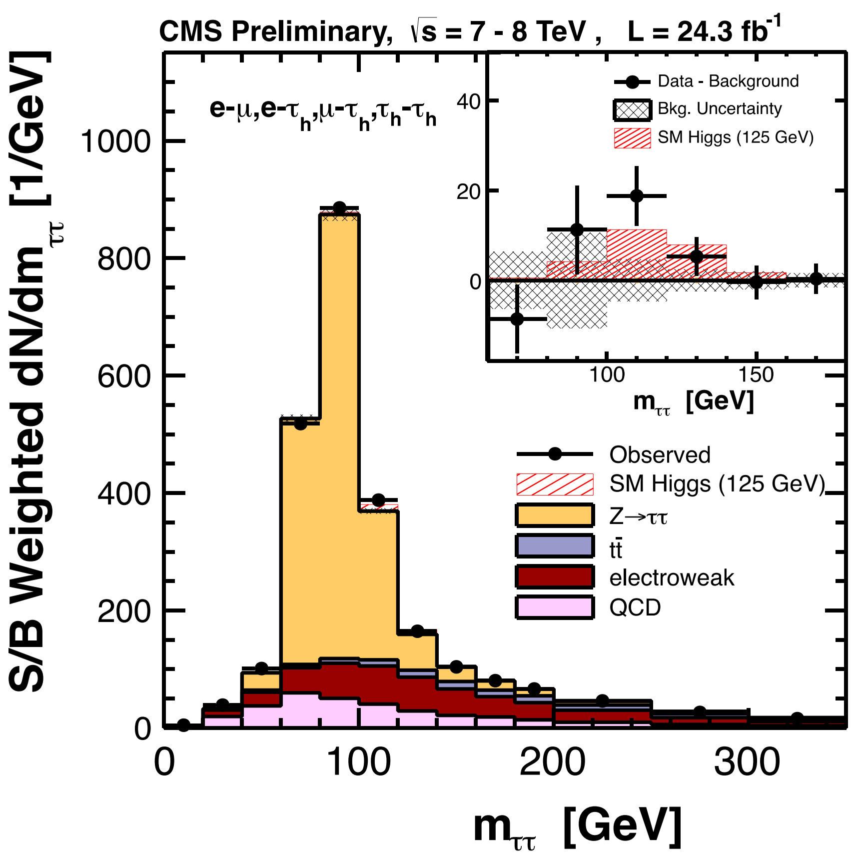
ATLAS-CONF-2013-013

H→ ττ

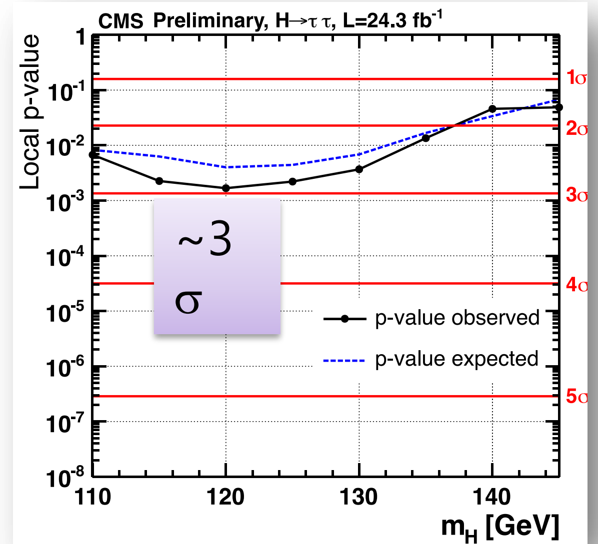
[Incandela @ Princeton]

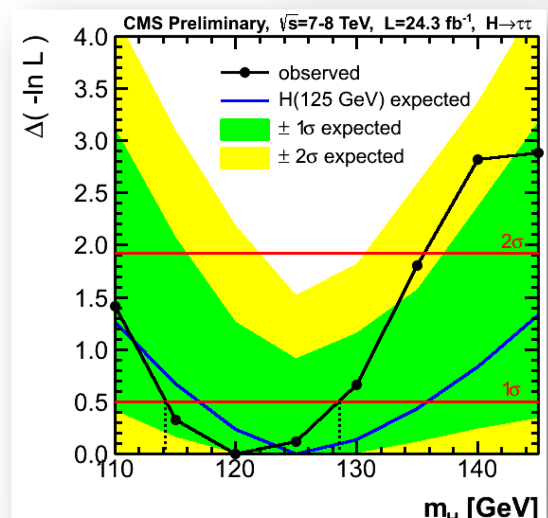
Moriond 2013

$\mu\tau_h$, $e\tau_h$, $e\mu$, $\tau_h\tau_h$, $\mu\mu$



First strong indication of decay to spin ½ particles



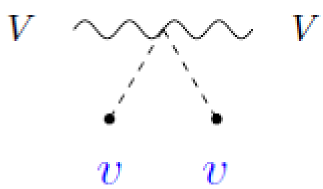


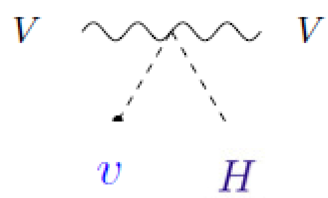
CMS HIG-13-004

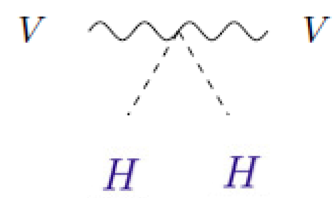
$m = 120^{+9}_{-7}$ (stat+syst) GeV

SM Higgs Couplings

$$\boxed{m \to m\left(1+\frac{H}{v}\right)}$$

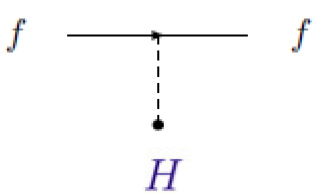


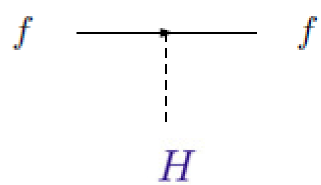




$$\boxed{2m_V^2/v}$$

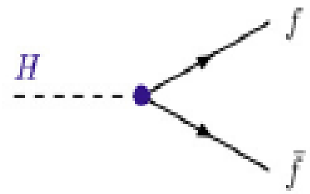
$$\boxed{2m_V^2/v^2}$$



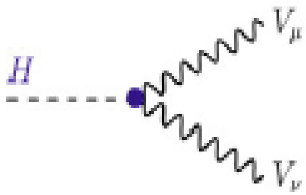


$$\boxed{m_f/v}$$

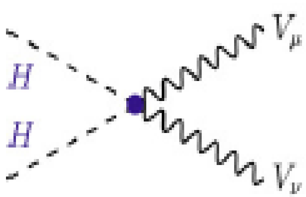
Basic Diagrams for the SM Higgs Boson



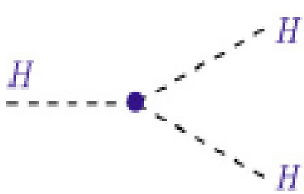
$$g_{Hff} = m_f/v = (\sqrt{2}G_\mu)^{1/2} m_f \qquad \times (-i)$$



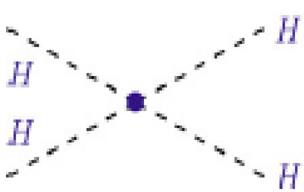
$$g_{HVV} = 2M_V^2/v = 2(\sqrt{2}G_\mu)^{1/2} M_V^2 \qquad \times (ig_{\mu\nu})$$



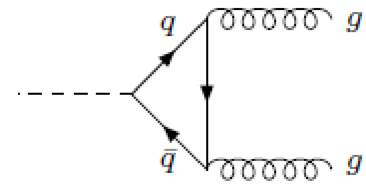
$$g_{HHVV} = 2M_V^2/v^2 = 2\sqrt{2}G_\mu M_V^2 \qquad \times (ig_{\mu\nu})$$

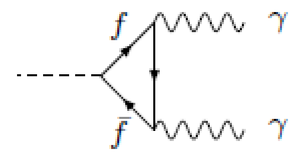


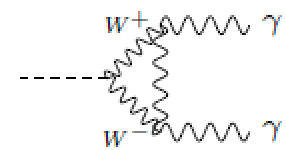
$$g_{HHH} = 3M_H^2/v = 3(\sqrt{2}G_\mu)^{1/2} M_H^2 \qquad \times (-i)$$

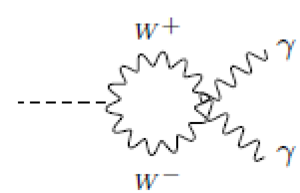


$$g_{HHHH} = 3M_H^2/v^2 = 3\sqrt{2}G_\mu M_H^2 \qquad \times (-i)$$







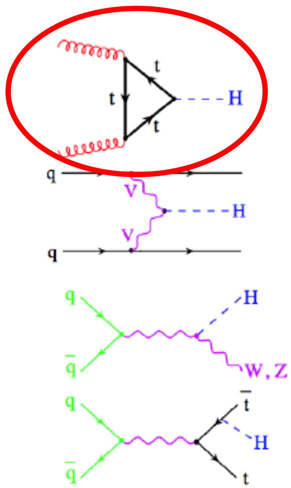


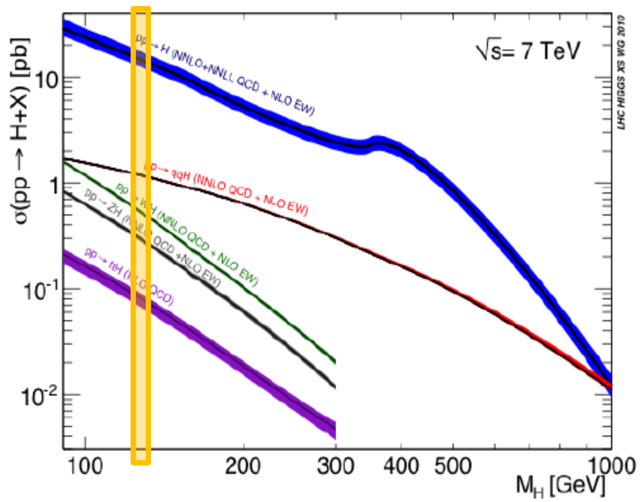
$[gg \rightarrow H]$

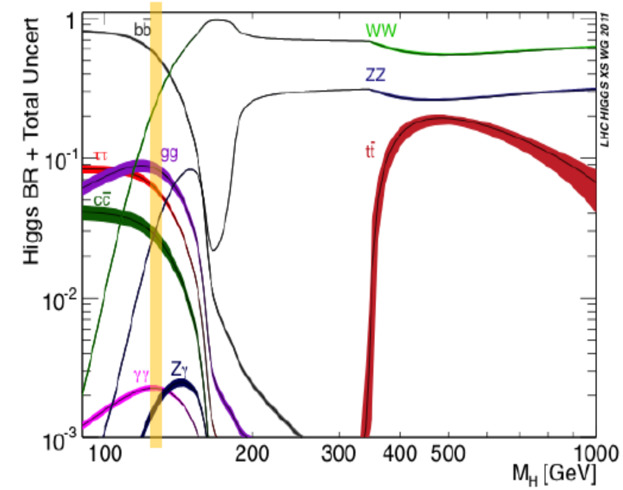
SM Higgs

Production

Decay

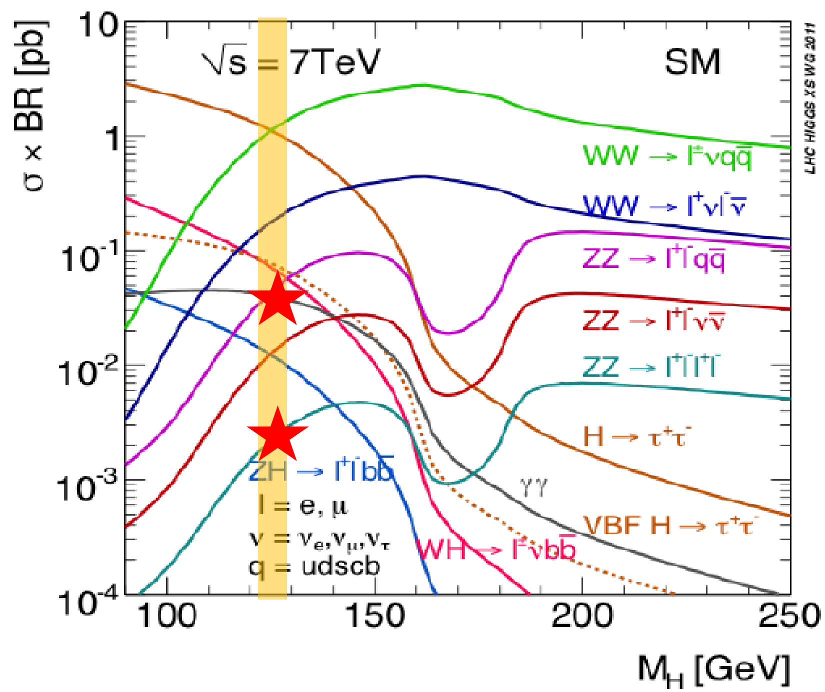






$W \rightarrow l\nu$ (11% each)
$W \rightarrow qq$ (68%)
$W \rightarrow invisible$ (1.4%)

$Z \rightarrow ll$ (3.4% each)
$Z \rightarrow invisible$ (20%)
$Z \rightarrow qq$ (70%)



Clean signature

$[H \rightarrow Z^{*}Z \rightarrow 4\ell]$

$\oplus$

$[H \rightarrow \gamma\gamma]$

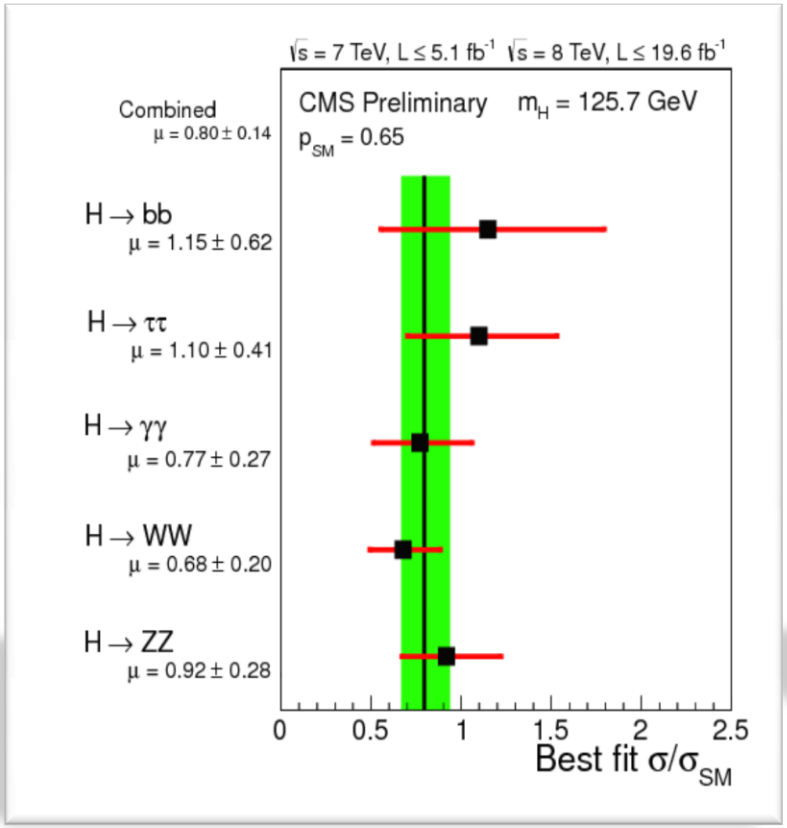
[Incandela @ Princeton]

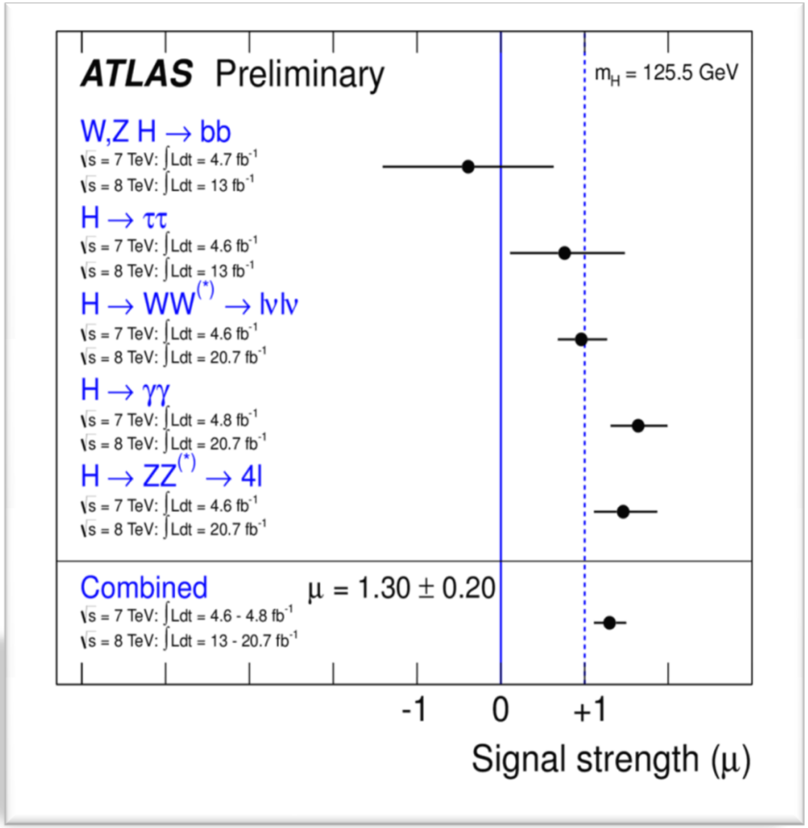
Signal strength and mass (combining $\gamma\gamma$ and ZZ*)

$\mu=0.80\pm0.14$

CERN 4/15, 2013

$\mu=1.30\pm0.20$





$m = 125.7 \pm 0.3 \pm 0.3$ GeV

$m = 125.5 \pm 0.2\,^{+0.5}_{-0.6}$ GeV

A Higgs Boson !

The SM Higgs Boson ?

$\boxed{\text{A Formal Proof of Goldstone Boson Equivalence Theorem}}$

BW Lee, Quigg, Thacker
PRD 16 (1977) 1519

We consider the generating functional of Green's functions

$$\boxed{Z[J_L] = -i\,\ln\left[\int (dV_\mu\, d\phi \cdots)\; \exp\left\{ i\left(S_{eff}(V_\mu, \phi, \cdots] + \int d^4x\, J_L V_L\right)\right\} \prod \delta(\partial^\mu V_\mu + iM\phi)\right]} \quad -①$$

from which connected Green's functions with external longitudinally polarized vector bosons are obtained by functional differentiations with respect to the source J_L. Here we suppress the group index, so that V_μ and ϕ stand collectively for $W^\pm_\mu$ and Z_μ, and for $w^\pm$ and z, respectively, with appropriate mass M. The constraints $\boxed{\partial^\mu V_\mu + iM\phi = 0}$ ② define the 't Hooft-Feynman gauge, and the effective action $S_{eff}[V_\mu, \phi, \cdots]$ includes the Faddeev-Popov term.

The longitudinal vector field V_L is defined as $\boxed{\tilde{V}_L(k) = \epsilon_L^\mu \tilde{V}_\mu(k) \text{ with } \epsilon_L^\mu = (|\vec{k}|, E\hat{k})/M}$ ③ where where k_μ is the four-momentum carried by the vector boson, and $\tilde{V}_\mu(k)$ is the Fourier transform of $V_\mu(x)$.

The equation ② states that $\boxed{k^\mu \tilde{V}_\mu(k)/M = \tilde{\phi}(k)}$ ④ while Eq. ③ implies

$$\boxed{\tilde{V}_L(k) = k^\mu \tilde{V}_\mu(k)/M + O(M/E) = \tilde{\phi}(k) + O(M/E)} \quad -⑤$$

$$V_\mu(x) = \frac{1}{(2\pi)^2}\int d^4k\; \tilde{V}_\mu(k)\, e^{-ik\cdot x}$$

$$\phi(x) = \frac{1}{(2\pi)^2}\int d^4k\; \tilde{\phi}(k)\, e^{-ik\cdot x}$$

$$J_L(x) = \frac{1}{(2\pi)^2}\int d^4k\; \tilde{J}_L(k)\, e^{-ik\cdot x}$$

$$\int d^4x\, J_L V_L = \int d^4k\; \tilde{J}_L(-k)\, \tilde{V}_L(k)$$

Thus, the equation ① may be cast in the form

$$\boxed{\begin{aligned} Z[J_L] = -i\,\ln\Big[\int (dV_\mu\, d\phi \cdots)\; &\exp\Big\{ i\Big(S_{eff}[V_\mu, \phi, \cdots] + \int d^4k\, \tilde{J}_L(-k)\left[\tilde{\phi}(k) + O(M/E)\right]\Big)\Big\} \\ &\times \prod \delta(\partial^\mu V_\mu + iM\phi)\Big] \end{aligned}} \quad -⑥$$

In vector-vector scattering, all E's are of order $\sqrt{s}$, and we obtain the so-called Goldstone boson equivalence theorem that

$$\boxed{T(V_L's) = T(\phi's) + O(M/\sqrt{s}) \quad \text{for large } s \gg M^2} \quad \text{⑦}$$

Inclusion of the physical Higgs bosons as external lines in the transition matrix T does not alter the above statement.

Partial-Wave Unitarity

◎ Unitarity : The unitarity relation for the S matrix, $\boxed{S^\dagger S = SS^\dagger = 1}$ ①, can be written in terms of the transition amplitude T defined by

$$\boxed{S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4(P_f - P_i) T_{fi} \longrightarrow -i[T_{fi} - T^\dagger_{fi}] = \sum_n (2\pi)^4 \delta^4(P_n - P_i) T^\dagger_{fn} T_{ni}} \quad \text{②}$$

$$1 = \sum_n |n\rangle\langle n| \rightarrow \delta_{fi} = \langle f|S^\dagger S|i\rangle = \sum_n \langle f|S^\dagger|n\rangle\langle n|S|i\rangle = \delta_{fi} + i\sum_n \delta_{fn}(2\pi)^4\delta^4(P_n - P_i)T_{ni}$$

$$- i\sum_n (2\pi)^4 \delta^4(P_f - P_n) T^\dagger_{fn}\delta_{ni}$$

$$+ \sum_n (2\pi)^4(2\pi)^4 \underbrace{\delta^4(P_f - P_n)\delta^4(P_n - P_i)}_{\delta^4(P_f - P_i)\delta^4(P_n - P_i)} T^\dagger_{fn}T_{ni}$$

$$\Rightarrow -i[T_{fi} - T^\dagger_{fi}] = \sum_n (2\pi)^4\delta^4(P_n - P_i)T^\dagger_{fn}T_{ni} \;!$$

◎ Optical theorem : Taking $f = i$ gives

$$\boxed{2\,\mathrm{Im}(T_{ii}) = \sum_n (2\pi)^4\delta^4(P_n - P_i)T^\dagger_{in}T_{ni} = \sum_n (2\pi)^4\delta^4(P_n - P_i)|T_{ni}|^2} \quad \text{③}$$

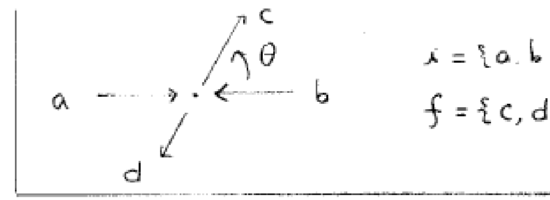
$$= \sum_f \int d\Phi_f\, |T_{fi}|^2 \quad \text{for all possible final states}$$

For any 2-body collision processes, $\boxed{\sigma_{i,tot}(s) = \frac{1}{2s\beta_i} \sum_f \int d\bar{\Phi}_f \, |T_{fi}|^2}$, leading to the following optical theorem

(4)

$$\boxed{Im(T_{ii}) = s\beta_i \cdot \sigma_{i,tot}(s)} \quad (5)$$

◎ partial-wave unitarity : For simplicity, we consider a 2 to 2 scattering process, in which case the transition amplitude can be written as



$i = \{a, b\}$

$f = \{c, d\}$

$$(6) \quad \boxed{T = \frac{16\pi}{\sqrt{\beta_i \beta_f}} \sum_{J=0}^{\infty} (2J+1)\, a_J\, P_J(\cos\theta)}$$

where a_J is a function of $\sqrt{s}$ only. In the partial-wave basis, the S matrix decouples into submatrices for each J because of angular momentum conservation

$$\int d\bar{\Phi}_{i'} |T_{i'i}|^2 \le \sum_f \int d\bar{\Phi}_f |T_{fi}|^2 = Im(T_{ii}) = \frac{16\pi}{\beta_i} \sum_{J=0}^{\infty} (2J+1)\, Im(a_J)_{ii}\, \underbrace{P_J(1)}_{=1}$$

$\theta = 0$

$$= \int_{-1}^{1} \frac{\beta_i}{16\pi} d\cos\theta \, \frac{(16\pi)^2}{\beta_i^2} \sum_J \sum_{J'} (2J+1)(2J'+1)\, |(a_J)_{i'i}|^2\, P_J(\cos\theta)\, P_{J'}(\cos\theta)$$

$$= \frac{16\pi}{\beta_i} \sum_J (2J+1)\, |(a_J)_{i'i}|^2 \Rightarrow \text{For each } J, \; |a_J|^2 \le Im(a_J)$$

$d\bar{\Phi}_i \overset{\text{eff}}{=} \frac{\beta_i}{16\pi} d\cos\theta$

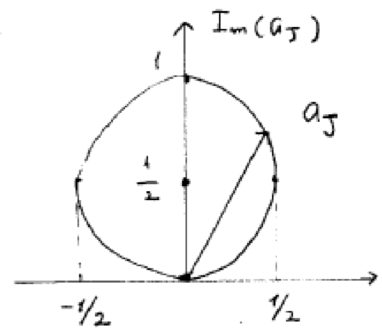
$\int_{-1}^{1} dx\, P_J(x) P_{J'}(x) = \frac{2}{2J+1} \delta_{J'J}$

$P_J(1) = 1$

i' : final state with the same particle content with the initial state but with an arbitrary scattering angle

$$\boxed{Re(a_J)^2 + [Im(a_J) - 1/2]^2 \le 1/4}$$

forms a closed disc of radius 1/2 for any $2 \to 2$ elastic process (7)



Argand diagram

For an inelastic channel $f \neq i$, we have $\boxed{|(a_J)_{fi}| \leq 1/2}$ — ⑧

∇ $|(a_J)_{fi}|^2 \leq \sum_{m \neq i} |(a_J)_{mi}|^2 \overset{<}{=} \mathrm{Im}\,(a_J)_{ii} - |(a_J)_{ii}|^2 \leq |(a_J)_{ii}| - |(a_J)_{ii}|^2 \leq 1/4$ △

To summarize,

$$\boxed{\begin{array}{l} |(a_J)_{ii}|^2 \leq \mathrm{Im}\,(a_J)_{ii} \quad \longrightarrow \quad \mathrm{Re}\,(a_J)_{ii} \leq 1/2 \\ |(a_J)_{fi}| \leq 1/2 \,,\; f \neq i \end{array}}$$

Goldstone - Boson Scattering Amplitudes

According to the Goldstone boson equivalence theorem, the longitudinal massive vector bosons can be replaced by the Goldstone bosons at very high energies. In addition, in many processes such as vector boson scattering, the amplitudes involving the longitudinal vector bosons are more dominant than those involving transverse vector bosons. Thus, in this high-energy limit, we can simply replace the $W^\pm$ and Z bosons by their corresponding Goldstone bosons $w^\pm, z$ in the SM scalar potential, leading to

∇ $V = \lambda \left[|\Phi|^2 - v^2/2 \right]^2 = \frac{M_H^2}{2v^2} \left[|\Phi|^2 - v^2/2 \right]^2 = \frac{M_H^2}{2v^2} \left[vH + \frac{1}{2}(H^2 + z^2 + 2w^+w^-) \right]^2$ △

$$\Phi = \begin{pmatrix} i w^+ \\ \frac{1}{\sqrt{2}}(v + H + i z) \end{pmatrix}, \quad |\Phi|^2 = w^+w^- + \frac{1}{2}v^2 + vH + \frac{1}{2}H^2 + \frac{1}{2}z^2$$

$$\boxed{V_{Int} = \frac{M_H^2}{2v}(H^2 + z^2 + 2w^+w^-)H + \frac{M_H^2}{8v^2}(H^2 + z^2 + 2w^+w^-)^2} \quad \text{— ①}$$

and use this potential to calculate the amplitudes for the processes involving weak vector bosons. The calculations are then extremely simple, since we have to deal only with interactions among scalar particles.

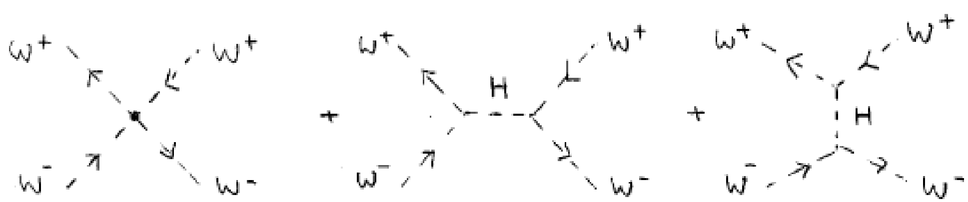
$HHH = -i3\frac{M_H^2}{v}$ $\quad HHHH = -i3\frac{M_H^2}{v^2}$ $\quad HHW^+W^- = -i\frac{M_H^2}{v^2} = ZZW^+W^-$

$ZZH = -i\frac{M_H^2}{v}$ $\quad HHZZ = -i\frac{M_H^2}{v^2}$ $\quad ZZZZ = -i3\frac{M_H^2}{v^2}$ $\quad W^+W^-W^+W^- = -i2\frac{M_H^2}{v^2}$

$W^+W^-H = -i\frac{M_H^2}{v}$

$$\boxed{\frac{1}{v^2} = \sqrt{2}\,G_F}$$

<u>$A(W^+W^- \to W^+W^-)$ at the tree level</u>



$$= (-i)\left[-i\,2\frac{M_H^2}{v^2} + (-i)^2\left(\frac{M_H^2}{v}\right)^2\frac{i}{s-M_H^2} + (-i)^2\left(\frac{M_H^2}{v}\right)^2\frac{i}{t-M_H^2}\right]$$

$$= \boxed{-\left[2\frac{M_H^2}{v^2} + \left(\frac{M_H^2}{v}\right)^2\frac{1}{s-M_H^2} + \left(\frac{M_H^2}{v}\right)^2\frac{1}{t-M_H^2}\right] = A_{tree}(W^+W^- \to W^+W^-)}$$

$$\Rightarrow \quad a_0(W^+W^- \to W^+W^-) \xrightarrow[s \gg M_H^2]{} -\frac{G_F M_H^2}{4\sqrt{2}\pi} \quad \longleftarrow \quad a_0 = \frac{1}{16\pi}\int_{-1}^{1} d\cos\theta\, A(s,t)$$

Why is the TeV scale special?

Conditional upper bound on M_H from unitarity:

- Gauge-boson & H scattering : $\{ W_L^+ W_L^-,\ Z_L Z_L/\sqrt{2},\ HH/\sqrt{2},\ W_L^+ H,\ Z_L H,\ W_L^+ Z_L \}$
  $\rightarrow \{ W_L^+ W_L^-,\ Z_L Z_L/\sqrt{2},\ HH/\sqrt{2},\ W_L^+ H,\ Z_L H,\ W_L^+ Z_L \}$

- Compute the scattering amplitudes for the 6×6 processes and make a partial-wave decomposition. In the high-energy limit, the amplitudes can be derived easily by use of the Goldstone boson equivalence theorem

$$\lim_{s \gg M_H^2} (a_0) = -\frac{G_F M_H^2}{4\sqrt{2}\pi} \left[\begin{array}{ccc|ccc} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 & 0 & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & & & \\ 1/\sqrt{8} & 1/4 & 3/4 & & & \\ \hline 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 \end{array} \right]$$

Require that the largest eigenvalue respect partial-wave unitarity condition $|a_0| \leq 1$

$$\Rightarrow \boxed{M_H \leq \left(\frac{8\sqrt{2}\pi}{3 G_F} \right)^{1/2} \simeq 710\ \text{GeV}} \quad - (*)$$

Implication : ① If the bound $(*)$ is respected, weak interactions remain weak at all energies and perturbation theory is everywhere reliable.

② If the bound $(*)$ is violated, perturbation theory breaks down and weak interactions among $W^{\pm}$, Z, H become strong around the TeV scale.

The unitarity argument is more rigorous and robust in the opposite limit with $M_H^2 \gg s \gg M_W^2$.
In this case, the unitarity constraint gives, if one takes into account only the $W_L^+W_L^- \to W_L^+W_L^-$

$$\boxed{a_0 \xrightarrow[s \ll M_H^2]{} -\frac{G_F}{16\sqrt{2}\pi}\, s}$$

and the result is valid to all orders in the Higgs self-coupling and receives only small corrections from the gauge couplings.

① with the unitarity condition $|\mathrm{Re}(a_0)| \leq 1/2$, we then obtain ②

$$\boxed{|\mathrm{Re}(a_0)| \longrightarrow \sqrt{s} \lesssim \left(\frac{8\sqrt{2}\pi}{G_F}\right)^{1/2} \simeq 1.7\ \mathrm{TeV} \implies \text{More stringent bound } \sqrt{s} \lesssim 1.2\ \mathrm{TeV}}$$

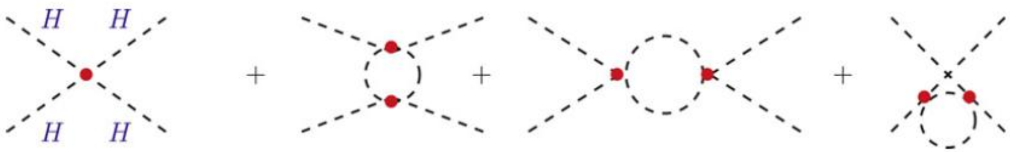
Implication: If the Higgs boson is too heavy (or, equivalently, not existing at all), some New Physics beyond the SM should manifest itself at energies in the TeV range to restore unitarity in the scattering amplitudes of longitudinal gauge bosons.

To conclude, (i) Some New Physics, which plays a role similar to that of the Higgs particle, should appear in the TeV range or

(ii) The unitarity breakdown is concealed by high-order terms which signal the failure of perturbation theory and the loss of the predictive power of the SM.

RGE running of the quartic Higgs coupling

QFT vacuum is a dielectric medium that screens charge $\Rightarrow$ the effective charge is a function of the distance or the energy.



$$\frac{d\lambda}{d\log Q^2} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

Triviality : M_H from above

$$\frac{\mathrm{d}}{\mathrm{d}Q^2}\lambda(Q^2) = \frac{3}{4\pi^2}\lambda^2(Q^2) + \text{higher orders}$$

$$\lambda(Q^2) = \lambda(v^2)\left[1 - \frac{3}{4\pi^2}\lambda(v^2)\log\frac{Q^2}{v^2}\right]^{-1}$$

$$\Lambda_C = v\exp\left(\frac{4\pi^2}{3\lambda}\right) = v\exp\left(\frac{4\pi^2 v^2}{M_H^2}\right)$$

If the SM is valid to infinite energy, then $\lambda\,(v^2)$ =0, i.e. non-interacting.

Since the Higgs mass is non-zero, then the theory has a cutoff Λ_C

Stability : M_H from below

$$\lambda \ll \lambda_t, g_1, g_2$$

$$\lambda(Q^2) = \lambda(v^2) + \frac{1}{16\pi^2}\left[-12\frac{m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]\log\frac{Q^2}{v^2}$$

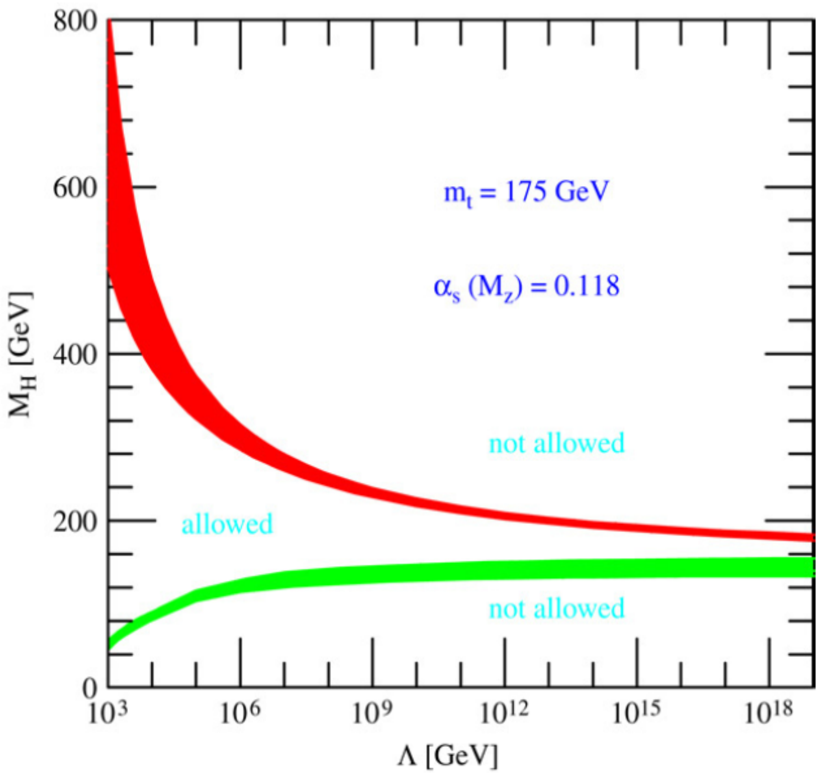
$$\lambda(Q^2) > 0$$

$$M_H^2 > \frac{v^2}{8\pi^2}\left[-12\frac{m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]\log\frac{Q^2}{v^2}$$

$$\Lambda_C \sim 10^3 \text{ GeV} \Rightarrow M_H \gtrsim 70 \text{ GeV}$$

$$\Lambda_C \sim 10^{16} \text{ GeV} \Rightarrow M_H \gtrsim 130 \text{ GeV}$$

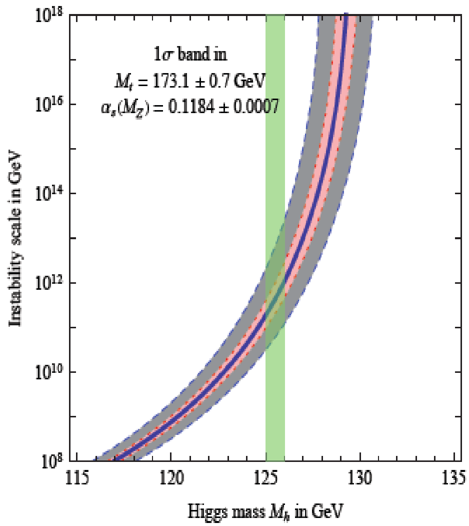
Message

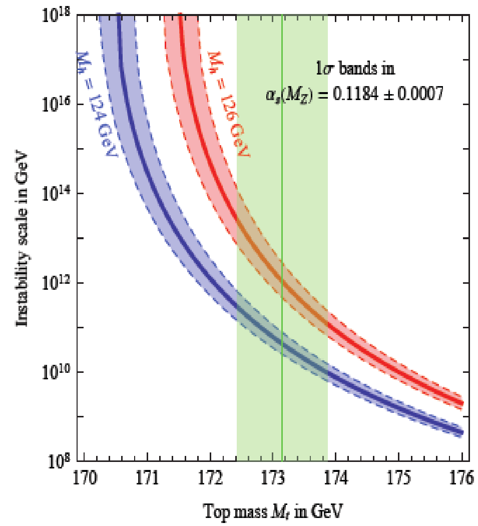


For any Higgs mass, there is a maximum energy scale beyond which the theory ceases to make sense.

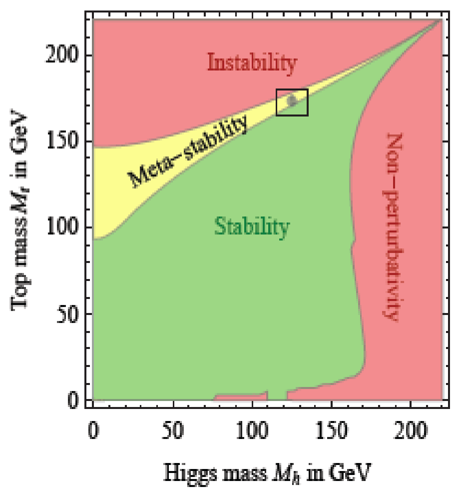
The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies.

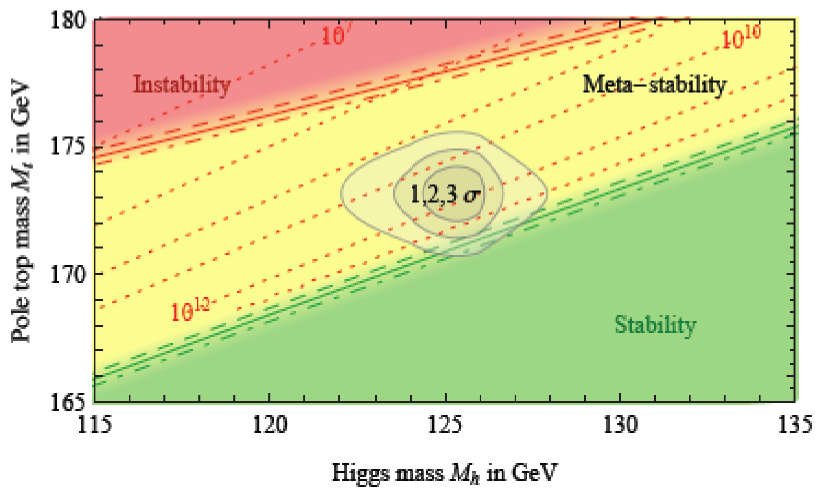
Extremely sensitive to the top quark mass





Meta-stable vacuum





New Endless Issues?!

The key questions looking forward

[Haber]

- Does the new boson discovered at the LHC exhibit the expected properties of the SM Higgs boson (spin? parity? couplings?)
- Will further study of the properties of this new state yield significant deviations from the SM Higgs boson expectations?
- How accurately can one measure the Higgs properties at the LHC? Do we need a dedicated precision Higgs factory?
- Will new BSM physics be discovered at the LHC that will shed light on the origin of EWSB?

These are exciting times. The July 4 discovery is not the end of the Standard Model but the beginning of an exploration that will yield profound insights into the theory of the fundamental particles and their interactions.