Perturbative QCD

Part IV : NLO computation in DY

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Matching

- Factorization Theorem for DY

$$F_{DY}(\tau) = \int_\tau^1 \frac{dz}{z} H_{DY}(z, Q^2, \mu) L_{q\bar{q}}(\frac{\tau}{z}, \mu)$$

- At NLO at the parton level

$$\begin{aligned} F_{DY}^{(1)}(\tau) &= H_{DY}^{(1)} \otimes f_{q/q}^{(0)} \otimes f_{\bar{q}/\bar{q}}^{(0)} + H_{DY}^{(0)} \otimes f_{q/q}^{(1)} \otimes f_{\bar{q}/\bar{q}}^{(0)} + H_{DY}^{(0)} \otimes f_{q/q}^{(0)} \otimes f_{\bar{q}/\bar{q}}^{(1)} \\ &= f_{q/q}^{(1)}(\tau) + f_{\bar{q}/\bar{q}}^{(1)}(\tau) + H^{(1)}(\tau) \end{aligned}$$

$$\therefore H_{DY}^{(1)}(\tau) = F_{DY}^{(1)}(\tau) - f_{q/q}^{(1)}(\tau) - f_{\bar{q}/\bar{q}}^{(1)}(\tau) = F_{DY}^{(1)}(\tau) - 2 f_{q/q}^{(1)}(\tau)$$

- Hard function should be infrared finite
- RG behaviors of STF and PDF are each different

NLO Computation

- Structure function in D dimension

$$F_{DY}(\tau, Q^2) = -\frac{N_c}{1-\varepsilon}\sum_X \int d^D q\,\delta(q^2 - Q^2)\delta^{(D)}(p_1 + p_2 - q - p_X)\langle q(p_1)\bar{q}(p_2)|J^\dagger_\mu|X\rangle\langle X|J^\mu|q(p_1)\bar{q}(p_2)\rangle$$

$$= -\frac{N_c}{1-\varepsilon}\sum_X \delta(q^2 - Q^2)\Big|_{q=p_1+p_2-p_X}\langle q(p_1)\bar{q}(p_2)|J^\dagger_\mu|X\rangle\langle X|J^\mu|q(p_1)\bar{q}(p_2)\rangle$$

⊗ LO computation in full QCD

$$F^{(0)}(\tau) = -\frac{N_c}{1-\varepsilon}\delta(q^2-Q^2)\langle p_1 p_2|\bar{q}\gamma^\mu q\cdot\bar{q}\gamma_\mu q|p_1 p_2\rangle$$

※ $q^2 = (p_1+p_2)^2 = 2p_1\cdot p_2 = S$

$\tau = \frac{Q^2}{S}$

$$= \langle p_1 p_2|\bar{u}_\alpha(p_1)\gamma^\mu v_\alpha(p_2)\cdot\bar{v}_\beta(p_2)\gamma_\mu u(p_1)_\beta|p_1 p_2\rangle$$

$$= \frac{1}{4N_c^2}\delta_{\alpha\gamma}\delta_{\beta\gamma}\delta_{\alpha\delta}\delta_{\beta\delta}\,\mathrm{Tr}\,\not{p}_1\gamma^\mu\not{p}_2\gamma_\mu, \quad p_1^\mu = \bar{n}\cdot p_1\frac{n^\mu}{2}$$

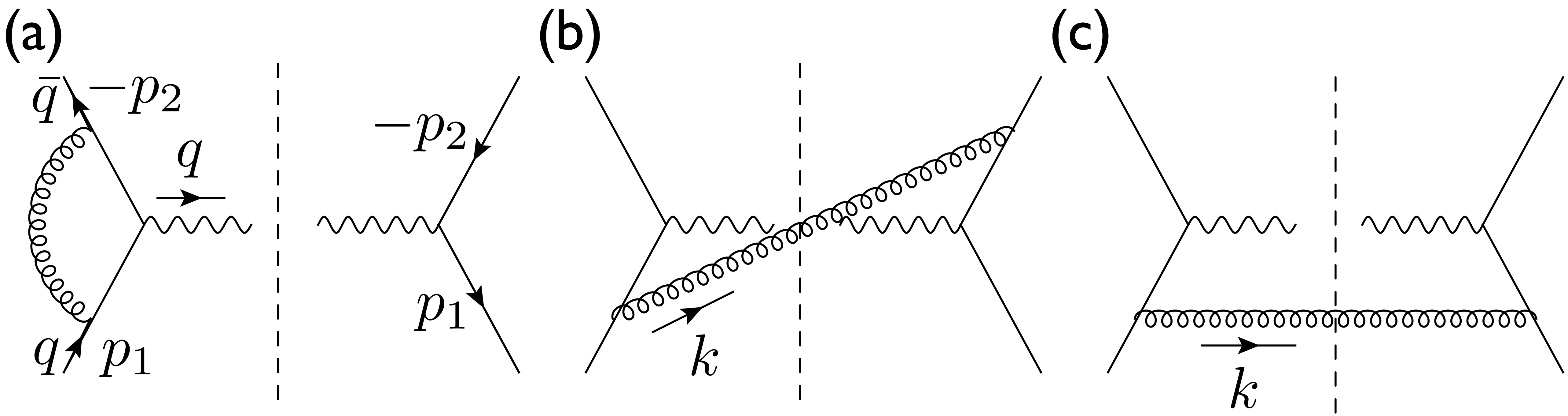
$= N_c$, $\quad p_2^\mu = n\cdot p_2\frac{\bar{n}^\mu}{2}$

$$= \frac{1}{4N_c}(2-D)\cdot 4p_1\cdot p_2$$

$$= \frac{-(1-\varepsilon)}{N_c}2p_1\cdot p_2 = -\frac{1-\varepsilon}{N_c}\bar{n}\cdot p_1\, n\cdot p_2$$

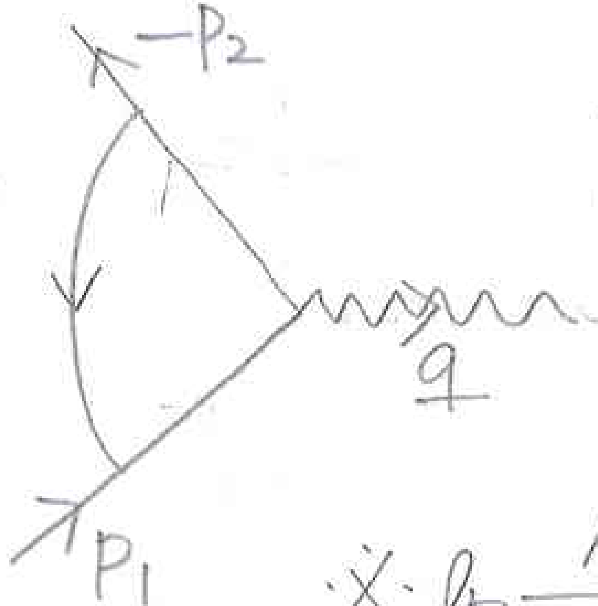
$$\therefore F^{(0)}(\tau) = \delta(S-Q^2)\cdot S = \delta(1-\tau).$$

- Feynman diagrams



Feynman diagrams to contribute to F_{DY} at NLO in α_s. Diagram (a) has a mirror image as a Hermitian conjugate. Diagrams (b) and (c) have cross diagrams with $q \leftrightarrow \bar{q}$, $p_1 \leftrightarrow -p_2$. Diagrams (b) and (c) emit real gluons, which cross unitary cuts.

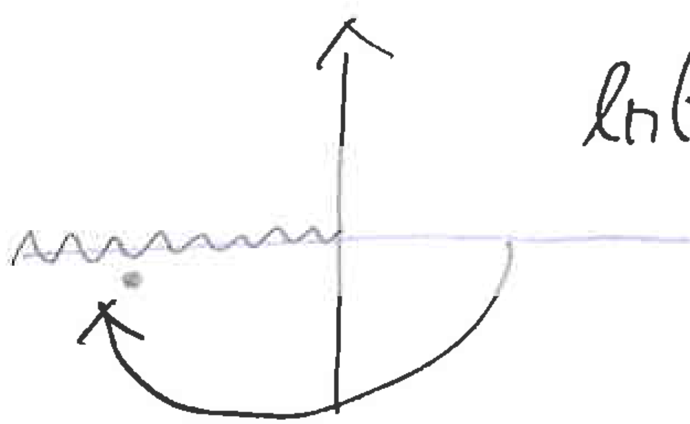
- Diagram (a)



$$\overline{M}_V = \frac{\alpha_s C_F}{2\pi}\left\{\frac{1}{2\varepsilon_{UV}} - \frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\left(2+\ln\frac{\mu^2}{M^2}\right) - 4 + \frac{\pi^2}{12} - \frac{3}{2}\ln\frac{\mu^2}{M^2} - \frac{1}{2}\ln^2\frac{\mu^2}{M^2}\right\},\quad M^2 = -2P_1\cdot P_2 = -q^2 = -Q^2$$

$$M^2 - i\varepsilon = -(Q^2 + i\varepsilon)$$

※ $\ln\frac{\mu^2}{M^2 - i\varepsilon} = \ln\frac{\mu^2}{-Q^2 - i\varepsilon}$



$$\ln(-Q^2 - i\varepsilon) = \ln Q^2 e^{-i\pi} = \ln Q^2 - i\pi.$$

$$\therefore \ln\frac{\mu^2}{-Q^2 - i\varepsilon} = \ln\frac{\mu^2}{Q^2} + i\pi.$$

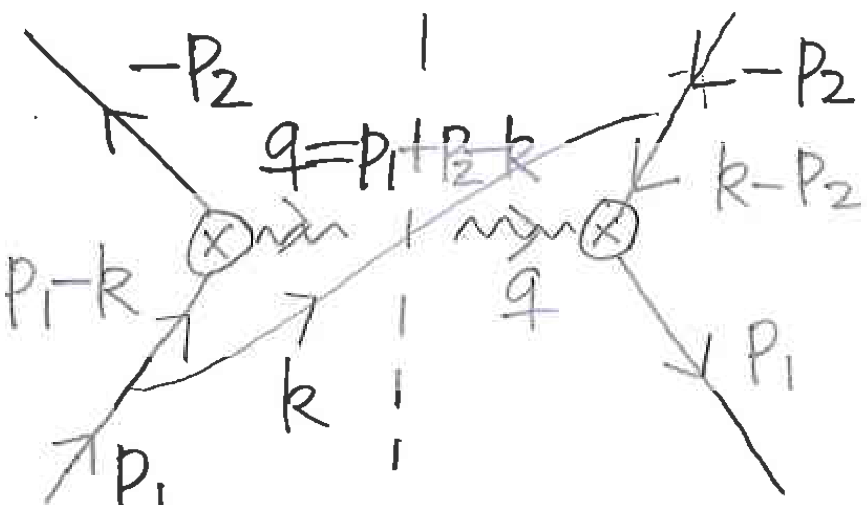
$$-\frac{1}{2}\left(\ln^2\frac{\mu^2}{Q^2} + 2i\pi\ln\frac{\mu^2}{Q^2} - \pi^2\right)$$

$$\therefore \overline{M}_V = \frac{\alpha_s C_F}{2\pi}\left\{\frac{1}{2\varepsilon_{UV}} - \frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\left(2+\ln\frac{\mu^2}{Q^2} + i\pi\right) - 4 + \frac{7\pi^2}{12} - \left(\frac{3}{2} + i\pi\right)\ln\frac{\mu^2}{Q^2} - \frac{3}{2}i\pi - \frac{1}{2}\ln^2\frac{\mu^2}{Q^2}\right\}.$$

Diagram (b)

- Real gluon emission

(A)



$$M_{RA} = -\frac{N_c}{1-\varepsilon}\mu_{\overline{MS}}^{2\varepsilon}\int\frac{d^Dk}{(2\pi)^D}\delta(q^2-Q^2)$$

$$\otimes \langle p_1 p_2|\bar q_1\gamma^\mu \frac{i(\not k-\not p_2)}{(k-p_2)^2}(+ig\gamma^\alpha T^a)\, q_2$$

$$\cdot\, \bar q_2\gamma_\mu\frac{i(\not p_1-\not k)}{(p_1-k)^2}(+ig\gamma_\alpha T^a)\, q_1\, (-2\pi\delta(k^2))$$

$$= -\frac{2\pi g^2 C_F}{1-\varepsilon}\mu^{2\varepsilon}_{\overline{MS}}\int \frac{d^Dk}{(2\pi)^D}\frac{\delta(k^2)\delta(q^2-Q^2)}{(k-p_1)^2(k-p_2)^2},\qquad p_1^\mu = \sqrt{s}\,\frac{n^\mu}{2},\quad p_2=\sqrt{s}\,\frac{\bar n^\mu}{2}$$

$$\otimes \frac{1}{4}\mathrm{Tr}\,\not p_1\gamma^\mu(\not k-\not p_2)\underline{\gamma^\alpha\not p_2\gamma_\mu(\not k-\not p_1)\gamma_\alpha}$$

$$= -2(\not k-\not p_1)\gamma_\mu\not p_2 + (4-D)\not p_2\gamma_\mu(\not k-\not p_1)$$

$$\frac{1}{4}\mathrm{Tr}\,\not p_1\gamma^\mu(\not k-\not p_2)\gamma^\nu\not p_2\gamma_\mu(\not k-\not p_1)\gamma_\nu = -2(1-\varepsilon)[s^2-2sk\cdot(p_1+p_2)-4\varepsilon k\cdot p_1 k\cdot p_2]$$

$$\therefore \frac{1}{4}\mathrm{Tr}[\ ] = -2(1-\varepsilon)\left[s^2 - s(2k\cdot p_1 + 2k\cdot p_2) - \varepsilon\, 2k\cdot p_1\, 2k\cdot p_2\right]$$

$$\because\ 2k\cdot p_1 = \sqrt{s}\, n\cdot k,\quad 2k\cdot p_2 = \sqrt{s}\, \bar{n}\cdot k$$

$$= -2(1-\varepsilon)\left[s^2 - s^{3/2}(n\cdot k + \bar{n}\cdot k) - \varepsilon s\, \bar{n}\cdot k\, n\cdot k\right].$$

$$\because\ q^2 - Q^2 = (p_1 + p_2 - k)^2 - Q^2 = s - \sqrt{s}(\bar{n}\cdot k + n\cdot k) - Q^2.$$

$$= s\left[1 - \frac{1}{\sqrt{s}}(\bar{n}\cdot k + n\cdot k) - \tau\right] = 0.$$

$$\therefore\ \bar{n}\cdot k = \sqrt{s}(1-\tau) - n\cdot k \geq 0.\quad 0 \leq n\cdot k \leq \sqrt{s}(1-\tau).$$

$$\therefore \frac{1}{4}\mathrm{Tr}[\ ] = -2(1-\varepsilon)\left[s^2\tau - \varepsilon s\left(\sqrt{s}(1-\tau) - n\cdot k\right) n\cdot k\right]$$

$$\therefore\ M_{RA} = -\frac{2\pi g^2 C_F}{1-\varepsilon}\frac{(\mu^2 e^{\gamma})^{\varepsilon}}{32\pi^3}\frac{1}{\Gamma(1-\varepsilon)}\int d\bar{n}\cdot k\, dn\cdot k\, d\vec{k}_\perp^2 (\vec{k}_\perp^2)^{-\varepsilon}\frac{\delta(k^2)\,\delta(q^2-Q^2)}{2k\cdot p_1\, 2k\cdot p_2}$$

$$\otimes \frac{1}{4}\mathrm{Tr}[\ ].$$

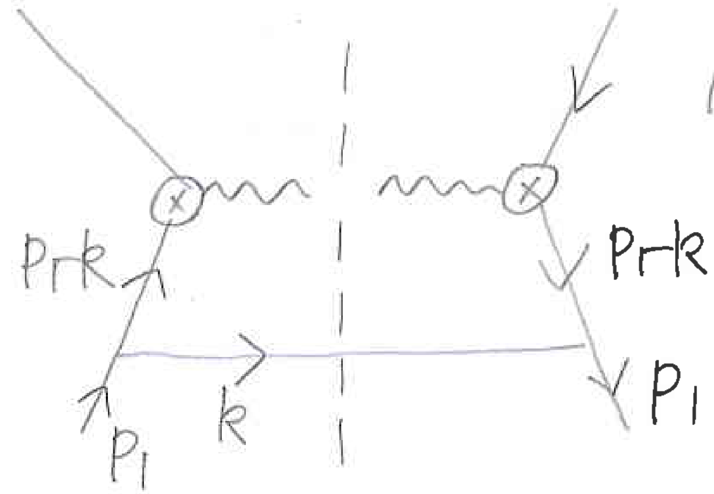
$$
\begin{aligned}
M_b &= \frac{\alpha_s C_F}{2\pi}\frac{(\mu^2 e^{\gamma})^{\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^{\sqrt{s}(1-\tau)} dn\cdot k\,\frac{[(\sqrt{s}(1-\tau)-n\cdot k)n\cdot k]^{-1-\varepsilon}}{s^{3/2}} \\
&\quad\times\left[s^2\tau-\varepsilon(\sqrt{s}(1-\tau)-n\cdot k)n\cdot k)\right] \\
&= \frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2 e^{\gamma}}{s}\right)^{\varepsilon}\frac{\tau(1-\tau)^{-1-2\varepsilon}}{\Gamma(1-\varepsilon)}\frac{\Gamma^2(-\varepsilon)}{\Gamma(-2\varepsilon)} \\
&= \frac{\alpha_s C_F}{2\pi}\Big[\delta(1-\tau)\Big(\frac{1}{\varepsilon_{\rm IR}^2}+\frac{1}{\varepsilon_{\rm IR}}\ln\frac{\mu^2}{Q^2}-\frac{\pi^2}{4}+\frac{1}{2}\ln\frac{\mu^2}{Q^2}\Big) \\
&\quad -\frac{2\tau}{(1-\tau)_+}\Big(\frac{1}{\varepsilon_{\rm IR}}+\ln\frac{\mu^2}{s}\Big)+4\tau\Big(\frac{\ln(1-\tau)}{1-\tau}\Big)_+\Big]
\end{aligned}
$$

Cancelled by the virtual contribution

- In full theory, on-shell gluon emission contribution gives only infrared divergence

▪ Diagram (c)

(B),



$$M_{RB} = -\frac{N_c}{1-\varepsilon}\mu_{\overline{MS}}^{2\varepsilon}\int\frac{d^Dk}{(2\pi)^D}\delta(q^2-Q^2)\left[-2\pi\delta(k^2)\right]$$

$$\otimes \langle P_1P_2|\bar q_1(+ig\gamma^\alpha T^a)\frac{i(\not p_1-\not k)}{(p_1-k)^2}\gamma^\mu q_2$$

$$\otimes \bar q_2\gamma_\mu\frac{i(\not p_1-\not k)}{(p_1-k)^2}(+ig\gamma_\alpha T^a)q_1|P_1P_2\rangle$$

$$= +\frac{2\pi g^2 C_F}{1-\varepsilon}\mu_{\overline{MS}}^{2\varepsilon}\int\frac{d^Dk}{(2\pi)^D}\frac{\delta(k^2)\delta(q^2-Q^2)}{(2p_1\cdot k)^2}$$

$$\otimes\frac{1}{4}\mathrm{Tr}\,\not p_1\gamma^\alpha(\not k-\not p_1)\gamma^\mu\not p_2\gamma_\mu(\not k-\not p_1)\gamma_\alpha$$

$$=\frac{(D-2)^2}{4}\mathrm{Tr}\,\not p_1\not k\not p_2\not k = 2(D-2)^2 p_1\cdot k\, p_2\cdot k = 2(1-\varepsilon)(2p_1\cdot k)(2p_2\cdot k)$$

$$= 4\pi g^2 C_F\frac{(\mu^2 e^\gamma)^\varepsilon}{32\pi^3\Gamma(1-\varepsilon)}\int d\bar n\cdot k\, dn\cdot k\, d\vec k_\perp^2(\vec k_\perp^2)^{-\varepsilon}\frac{2p_2\cdot k}{2p_1\cdot k}\delta(k^2)\delta(q^2-Q^2)$$

$$= \frac{\alpha_s C_F(\mu^2 e^\gamma)^\varepsilon}{2\pi\Gamma(1-\varepsilon)}\int_0^{\sqrt{s}(1-\tau)}\frac{dn\cdot k}{\sqrt{s}}\frac{\bar n\cdot k}{n\cdot k}(\bar n\cdot k\, n\cdot k)^{-\varepsilon},\quad n\cdot k\equiv\sqrt{s}(1-\tau)z,\quad \bar n\cdot k=\sqrt{s}(1-\tau)(1-z)$$

$$=\frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2 e^\gamma}{s}\right)^\varepsilon\frac{(1-\tau)^{1-2\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^1 dz\,(1-z)^{1-\varepsilon}z^{-1-\varepsilon} = \frac{\Gamma(2-\varepsilon)\Gamma(-\varepsilon)}{\Gamma(2-2\varepsilon)}$$

$$
\begin{aligned}
M_c &= \frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2 e^{\gamma}}{s}\right)^{\varepsilon}\frac{(1-\varepsilon)\Gamma(-\varepsilon)}{\Gamma(2-2\varepsilon)}(1-\tau)^{1-2\varepsilon} \\
&= \frac{\alpha_s C_F}{2\pi}(1-\tau)\left[-\frac{1}{\varepsilon_{\mathrm{IR}}}-\ln\frac{\mu^2}{s}-1+2\ln(1-\tau)\right]
\end{aligned}
$$

- Final result

$$
\begin{aligned}
F_{DY}^{(1)}(\tau,Q^2) &= 2\mathrm{Re}M_a + 2M_b + 2M_c + 2[Z_q^{(1)} + R_q^{(1)}]\delta(1-x) \\
&= \frac{\alpha_s C_F}{\pi}\Bigg\{\delta(1-\tau)\left[-\frac{3}{2}\left(\frac{1}{\varepsilon_{\mathrm{IR}}}+\ln\frac{\mu^2}{Q^2}\right)-4+\frac{\pi^2}{3}\right] \\
&\quad -\frac{1+\tau^2}{(1-\tau)_+}\left(\frac{1}{\varepsilon_{\mathrm{IR}}}+\ln\frac{\mu^2}{Q^2}+\ln\tau\right)+4\tau\left(\frac{\ln(1-\tau)}{1-\tau}\right)_+ -(1-\tau)(1-2\ln(1-\tau))\Bigg\}
\end{aligned}
$$

- The same IR poles as PDF computation

$$
2f_{q/q}^{(1)}(x,\mu) = \frac{\alpha_s C_F}{\pi}\left(\frac{1}{\varepsilon_{\mathrm{UV}}}-\frac{1}{\varepsilon_{\mathrm{IR}}}\right)\left[\frac{3}{2}\delta(1-x)+\frac{1+x^2}{(1-x)_+}\right]
$$

- PDF reproduces low energy physics of full QCD

Hard Function at NLO

$$H_{DY}^{(1)}(\tau,Q^2,\mu) = F_{DY}^{(1)}(\tau,Q^2) - (f_{q/q}^{(1)}(\tau) + f_{\bar{q}/\bar{q}}^{(1)}(\tau)) = F_{DY}^{(1)}(\tau,Q^2) - 2f_{q/q}^{(1)}(\tau)$$

$$= \frac{\alpha_s C_F}{\pi}\Big\{\delta(1-\tau)\Big[-\frac{3}{2}\ln\frac{\mu^2}{Q^2} - 4 + \frac{\pi^2}{3}\Big] - \frac{1+\tau^2}{(1-\tau)_+}\Big(\ln\frac{\mu^2}{Q^2} + \ln\tau\Big)$$

$$+4\tau\Big(\frac{\ln(1-\tau)}{1-\tau}\Big)_+ - (1-\tau)(1-2\ln(1-\tau))\Big\}$$

- Hard function is the Wilson coefficient of low energy EFT

- Anomalous dimension of the hard function

$$\frac{d}{d\ln\mu}H_{DY}(\tau,Q^2,\mu) = \int_\tau^1 \frac{dz}{z}\gamma_{H_{DY}}(\tau,\mu)H_{DY}(\frac{\tau}{z},Q^2,\mu)$$

$$\gamma_{H_{DY}}(\tau,\mu) = \frac{\partial}{\partial\ln\mu}H_{DY}(\tau,Q^2,\mu) = 2\frac{\alpha_s C_F}{\pi}\Big[-\frac{3}{2}\delta(1-\tau) - \frac{1+\tau^2}{(1-\tau)_+}\Big] = -2\gamma_f(\tau)$$

Structure function is scale invariant