Perturbative QCD

Part IV : NLO computation in DIS

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Matching

- Factorization Theorem for DIS

$$F_1(x,Q^2) = \int_x^1 \frac{dz}{z} H(z,Q^2,\mu) f_{q/N}(\frac{x}{z},\mu)$$

- At NLO at the parton level

$$F_1^{(1)}(x) = \int_x^1 \frac{dz}{z} \left[H^{(0)}(z) f_{q/q}^{(1)}(\frac{x}{z}) + H^{(1)}(z) f_{q/q}^{(0)}(\frac{x}{z}) \right] = f_{q/q}^{(1)}(x) + H^{(1)}(x)$$

$$\therefore H^{(1)}(x) = F_1^{(1)}(x) - f_{q/q}^{(1)}(x)$$

- Hard function should be infrared finite
- RG behaviors of STF and PDF are each different

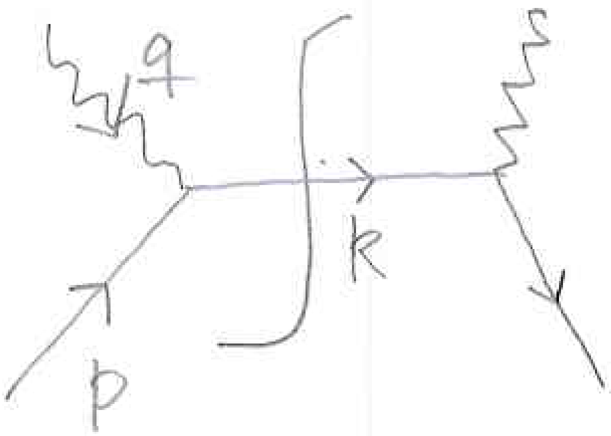
NLO Computation

- Structure function in D dimension

$$F_1(x,Q^2) = -\frac{(2\pi)^{D-1}}{D-2}\sum_X \delta^{(D)}(q+p-p_X)\langle q(p)|J^\dagger_{\perp\mu}|X\rangle\langle X|J^\mu_\perp|q(p)\rangle$$

At tree level,

$$F_1(x) = -\frac{(2\pi)^{D-1}}{D-2}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}\frac{1}{k^0}\delta^{(D)}(q+p-k)\langle p|\bar{q}\gamma^\perp_\mu q|X\rangle\langle X|\bar{q}\gamma^\perp_\mu q|p\rangle$$



$$= -\frac{1}{D-2}\delta(k^2)\Big|_{k=p+q}\frac{1}{2}\mathrm{Tr}\,\not{p}\gamma^\perp_\mu\not{k}\gamma_\perp^\mu,\qquad p=\bar{n}\cdot p\frac{n^\mu}{2},\quad k=p+q.$$

$$= -\frac{1}{2(D-2)}\delta(2p\cdot q-Q^2)\,\mathrm{Tr}\,\not{p}\gamma^\perp_\mu(\not{p}+\not{q})\gamma^\perp_\mu$$

$$x=\frac{Q^2}{2p\cdot q}=\frac{Q}{\bar{n}\cdot p}$$

$$= \bar{n}\cdot p\,Q\,\delta(Q\bar{n}\cdot p-Q^2)$$

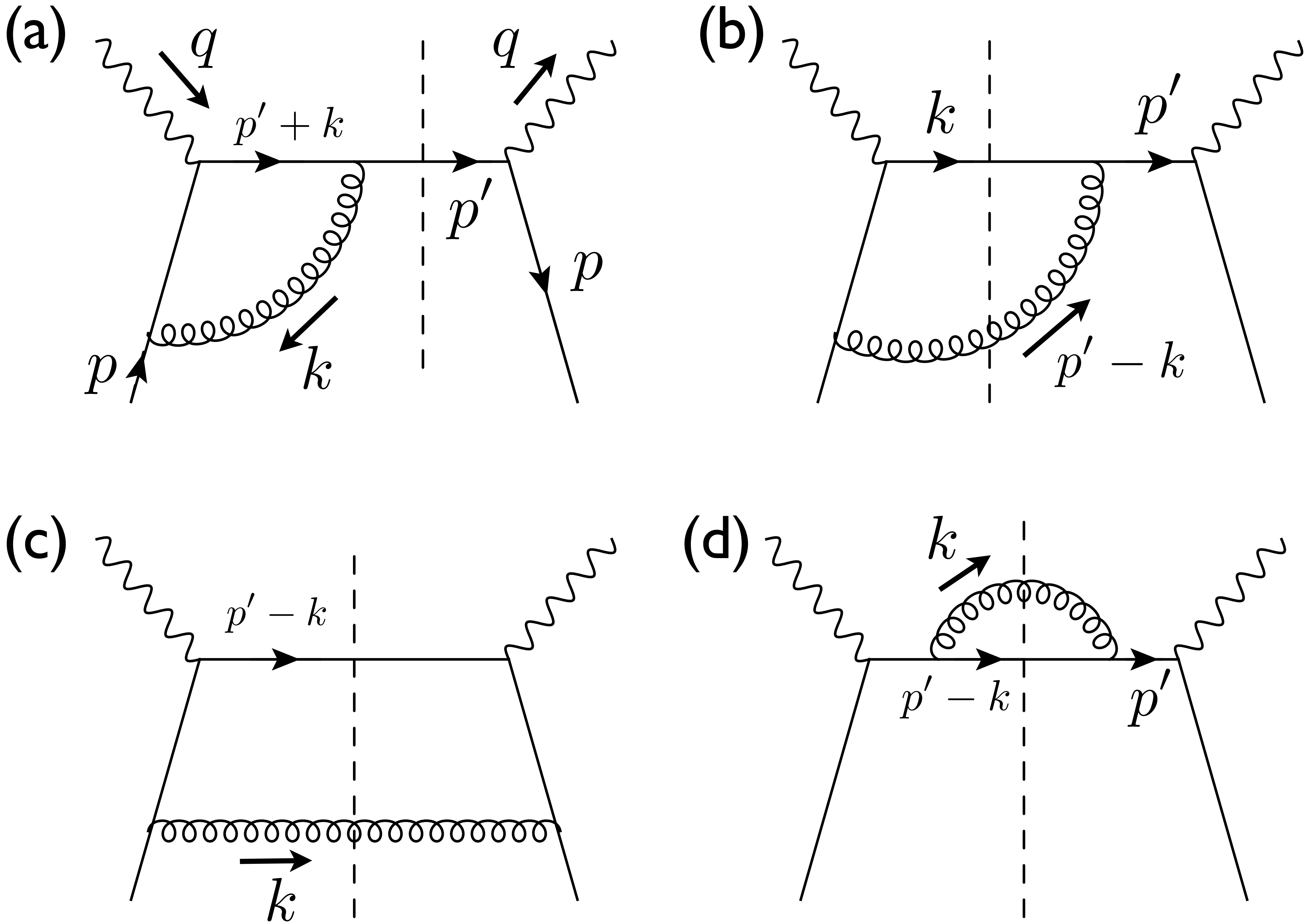
$$= \delta(1-x).$$

$$= +\bar{n}\cdot p\,Q\,\mathrm{Tr}\frac{\not{n}}{2}\gamma_\perp^\mu\frac{\not{\bar{n}}}{2}\gamma^\perp_\mu$$

$$= -(D-2)\bar{n}\cdot p\,Q\cdot\mathrm{Tr}\frac{\not{n}}{2}\frac{\not{\bar{n}}}{2}$$

$$= -2(D-2)\bar{n}\cdot p\,Q$$

- Feynman diagrams

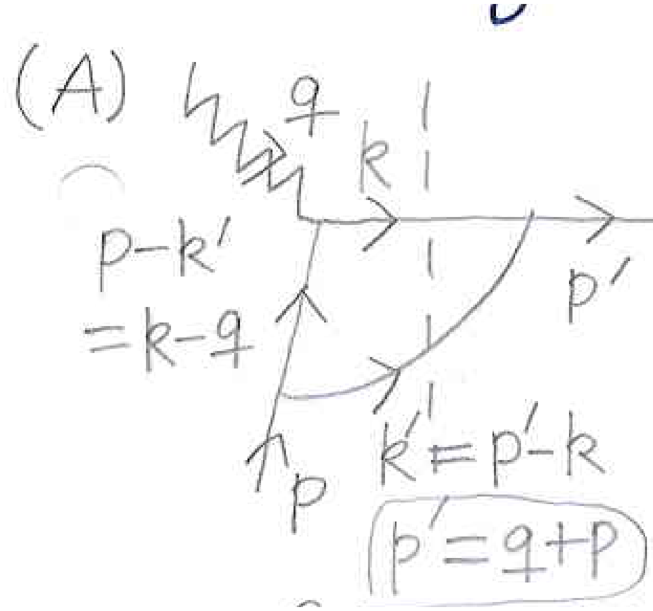


▪ Diagram (a)

$$M_a = \bar{M}_V \delta(1-x) = \frac{\alpha_s C_F}{2\pi}\Big[\frac{1}{2\,\varepsilon_{\rm UV}} - \frac{1}{\varepsilon_{\rm IR}^2} - \frac{1}{\varepsilon_{\rm IR}}(2+\ln\frac{\mu^2}{Q^2}) - 4 + \frac{\pi^2}{12} - \frac{3}{2}\ln\frac{\mu^2}{Q^2} - \frac{1}{2}\ln^2\frac{\mu^2}{Q^2}\Big]\delta(1-x)$$

$$M^2 \equiv 2p\cdot p' = 2p\cdot q = Q^2/x \to Q^2$$

▪ Diagram (b)



$$M_{RA} = -\frac{1}{D-2}\mu_{\overline{MS}}^{2\epsilon}\int\frac{d^Dk}{(2\pi)^D}\delta(k^2) \otimes \langle p|\bar{q}\gamma_\perp^\mu \frac{i\not p'}{p'^2}(+ig\gamma^\alpha T^a)\not k\gamma_\mu^\perp\frac{i(\not k-\not q)}{(k-q)^2} \otimes (+ig\gamma_\alpha T^a)q\rangle\cdot\left(-2\pi\delta(k'^2)\right)$$

$$M_b = \frac{g^2 C_F}{2(D-2)}\mu_{MS}^{2\varepsilon}\int\frac{d^Dk}{(2\pi)^{D-1}}\frac{\delta(k^2)\delta(p'^2-2p'\cdot k)}{p'^2(k-q)^2}\,{\rm Tr}\,\not p\gamma_\perp^\mu\not p'\gamma^\nu\not k\gamma_\mu^\perp(\not k-\not q)\gamma_\nu$$

- Computation of trace

$$\mathrm{Tr}\, \not{p}\gamma_\perp^\mu \not{p}'\gamma^\nu \not{k}\gamma_\mu^\perp(\not{k}-\not{q})\gamma_\nu = 8(D-2)p\cdot k k_\perp^2 + 4(D-2)(D-4)(2k\cdot q p\cdot q + Q^2 p\cdot k)$$
$$-4(D-2)Q^2\overline{n}\cdot p n\cdot k$$

- Integration of component by component

$$M_b = \frac{\alpha_s C_F}{2\pi}\frac{(\mu^2 e^\gamma)^\varepsilon}{\Gamma(1-\varepsilon)}\int d\overline{n}\cdot k\, dn\cdot k\, d\mathbf{k}_\perp^2(\mathbf{k}_\perp^2)^{-\varepsilon}\frac{\delta(\overline{n}\cdot k n\cdot k - \mathbf{k}_\perp^2)\delta(p'^2 - 2p'\cdot k)}{(-Q^2+2p\cdot q)(Q^2+2k\cdot q)}$$
$$\times\Big[2p\cdot k\mathbf{k}_\perp^2 + Q^2\overline{n}\cdot p n\cdot k + 2\varepsilon(Q^2 p\cdot k + 2k\cdot q p\cdot q)\Big]$$

$$\int\frac{d^D k}{(2\pi)^D} = \frac{(4\pi)^\varepsilon}{32\pi^3}\frac{1}{\Gamma(1-\varepsilon)}\int d\overline{n}\cdot k\, dn\cdot k\, d\mathbf{k}_\perp^2(\mathbf{k}_\perp^2)^{-\varepsilon}$$

- Momentum relations by kinematic conditions

$$q^2 = -Q^2,\ \overline{n}\cdot q = -Q,\ n\cdot q = Q$$

$$p'^2 = (p+q)^2 = -Q^2 + \overline{n}\cdot p Q = \frac{1-x}{x}Q^2,$$
$$p'^2 - 2p'\cdot k = \frac{1-x}{x}Q^2 - 2(p+q)\cdot k = \frac{1-x}{x}Q(Q-n\cdot k) - Q\overline{n}\cdot k$$

$\because \bar{n}\cdot k = \frac{1-x}{x}(Q - n\cdot k) \geq 0 \qquad \therefore 0 \leq n\cdot k \leq Q$

$$M_b = \frac{\alpha_s C_F}{2\pi}\frac{(\mu^2 e^{\gamma})^{\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^Q dn\cdot k \cdot x^{\varepsilon}(1-x)^{-\varepsilon}(n\cdot k)^{-\varepsilon}(Q-n\cdot k)^{-\varepsilon}\frac{1}{Q^4}\cdot\frac{x^2}{(1-x)(Q-n\cdot k)}$$

$$\otimes\left[\frac{Q^2}{x}\cdot\frac{1-x}{x}(Q-n\cdot k)n\cdot k + \frac{Q^3}{x}n\cdot k + \varepsilon\frac{1-x}{x^2}Q^3(Q-n\cdot k)\right].$$

$$= \frac{\alpha_s C_F}{2\pi}\frac{(\mu^2 e^{\gamma})^{\varepsilon}}{\Gamma(1-\varepsilon)}x^{\varepsilon}(1-x)^{-\varepsilon}\int_0^Q dn\cdot k(n\cdot k)^{-\varepsilon}(Q-n\cdot k)^{-\varepsilon}\frac{1}{Q}$$

$$\otimes\left[\frac{n\cdot k}{Q} + \varepsilon + \frac{x}{1-x}\frac{n\cdot k}{Q-n\cdot k}\right], \qquad n\cdot k \equiv Qz.$$

$$= \frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2 e^{\gamma}}{Q^2}\right)^{\varepsilon}\frac{x^{\varepsilon}(1-x)^{-\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^1 dz\, z^{-\varepsilon}(1-z)^{-\varepsilon}\left[z + \varepsilon + \frac{x}{1-x}\frac{z}{1-z}\right]$$

$$= \frac{\alpha_s C_F}{2\pi}\left\{\left(\frac{\mu^2 e^{\gamma}}{Q^2}\right)^{\varepsilon}\frac{x^{1+\varepsilon}(1-x)^{-1-\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^1 dz\, z^{1-\varepsilon}(1-z)^{-1-\varepsilon} + \frac{1}{2}\right\}$$

$$-\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} - \varepsilon\left(\frac{\ln(1-x)}{1-x}\right)_+ \qquad = \frac{\Gamma(2-\varepsilon)\Gamma(-\varepsilon)}{\Gamma(2-2\varepsilon)}$$

$$M_b = \frac{\alpha_s C_F}{2\pi}\left[\frac{1}{2} + \left(\frac{\mu^2 e^{\gamma}}{Q^2}\right)^{\varepsilon} x^{1+\varepsilon}(1-x)^{-1-\varepsilon}(1-\varepsilon)\frac{\Gamma(-\varepsilon)}{\Gamma(2-2\varepsilon)}\right]$$

$$\frac{1}{(1-x)^{1+\varepsilon}} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} - \varepsilon\left(\frac{\ln(1-x)}{1-x}\right)_+ + \mathcal{O}(\varepsilon^2)$$

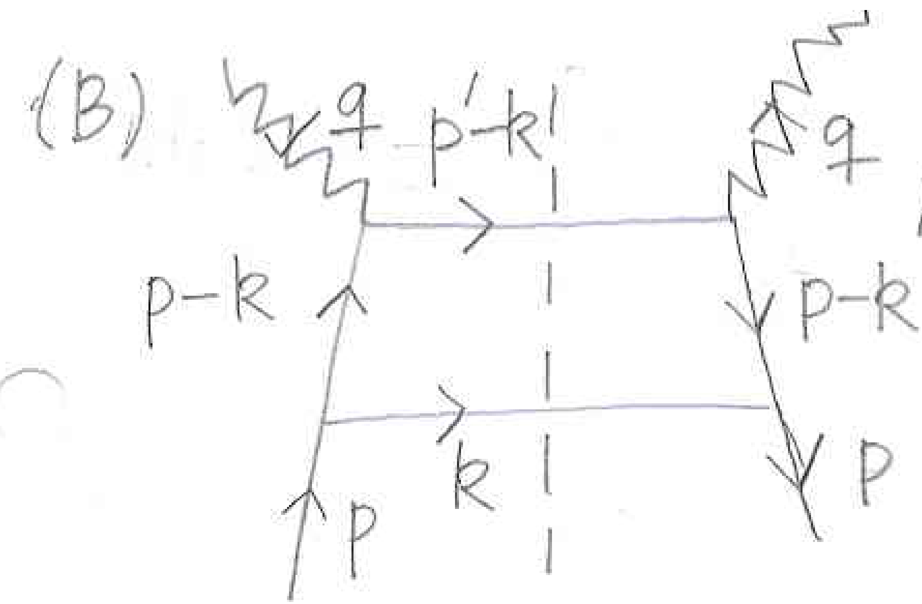
- Final result

Cancelled by the virtual contribution

$$M_b = \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x)\left[\frac{1}{\varepsilon_{\rm IR}^2} + \frac{1}{\varepsilon_{\rm IR}}\left(1+\ln\frac{\mu^2}{Q^2}\right) + 2 - \frac{\pi^2}{4} + \ln\frac{\mu^2}{Q^2} + \frac{1}{2}\ln\frac{\mu^2}{Q^2}\right]\right.$$
$$\left. - \frac{x}{(1-x)_+}\left[\frac{1}{\varepsilon_{\rm IR}} + \ln\frac{\mu^2}{Q^2} + 1 + \ln x\right] + x\left(\frac{\ln(1-x)}{1-x}\right)_+ + \frac{1}{2}\right\}$$

- In full theory, on-shell gluon emission contribution gives only infrared divergence

- Diagram (c)



$$M_{RB} = -\frac{1}{D-2}\mu_{\overline{MS}}^{2\varepsilon}\int\frac{d^Dk}{(2\pi)^D}\delta((p'-k)^2)$$

$$\otimes\langle p|\bar{q}(+ig\gamma^\alpha T^a)\frac{i(\not p-\not k)}{(p-k)^2}\gamma_\perp^\mu(\not p'-\not k)\gamma_\mu^\perp$$

$$\otimes\frac{i(\not p'-\not k)}{(p-k)^2}(+ig\gamma_\alpha T^a)q|p\rangle(-2\pi\delta(k^2))$$

$p' = p+q$

$$= +\frac{g^2C_F}{D-2}2\pi\mu_{\overline{MS}}^{2\varepsilon}\int\frac{d^Dk}{(2\pi)^D}\frac{\delta(k^2)\delta(p'^2-2p'\cdot k)}{(2p\cdot k)^2}$$

$$\otimes\frac{1}{2}\mathrm{Tr}\,\not p\gamma^\alpha(\not p-\not k)\gamma_\perp^\mu(\not p'-\not k)\gamma_\mu^\perp(\not p-\not k)\gamma_\alpha$$

$$= (2-D)\mathrm{Tr}\,\not p\not k\gamma_\perp^\mu(\not p'-\not k)\gamma_\mu^\perp\not k = 2(2-D)p\cdot k\,\mathrm{Tr}\,\gamma_\perp^\mu(\not p'-\not k)\gamma_\mu^\perp\not k$$

$$= (2-D)2p\cdot k\left[-(D-2)\mathrm{Tr}\,\not p'\not k - \mathrm{Tr}\,\gamma_\perp^\mu\not k\gamma_\mu^\perp\not k\right]$$

$$= 8\vec{k}_\perp^2$$

$$= (D-2)2p\cdot k\left[4(D-2)p'\cdot k - 8\vec{k}_\perp^2\right].$$

$$= +4\pi g^2 C_F \mu_{\overline{MS}}^{2\varepsilon} \int \frac{d^D k}{(2\pi)^D} \frac{\delta(k^2)\delta(p'^2 - 2p'\cdot k)}{2p\cdot k} \left[(D-2)p'\cdot k - 2\vec{k}_\perp^2\right]$$

$$= \frac{\alpha_s C_F}{2\pi} \frac{(\mu^2 e^{\gamma})^\varepsilon}{\Gamma(1-\varepsilon)} \int \frac{d\bar{n}\cdot k\, dn\cdot k\, d\vec{k}_\perp^2}{(\vec{k}_\perp^2)^\varepsilon} \frac{\delta(\bar{n}\cdot k\, n\cdot k - \vec{k}_\perp^2)\,\delta\left(\frac{1-x}{x}Q(Q-n\cdot k) - Q\bar{n}\cdot k\right)}{\bar{n}\cdot p\, n\cdot k}$$

$$\otimes \left[(1-\varepsilon)2p'\cdot k - 2\vec{k}_\perp^2\right], \quad \bar{n}\cdot k = \frac{1-x}{x}(Q - n\cdot k)$$

$$※\ 2p'\cdot k = 2p\cdot k + 2q\cdot k = (\bar{n}\cdot p - Q)n\cdot k + Q\bar{n}\cdot k$$

$$= \frac{1-x}{x}Qn\cdot k + \frac{1-x}{x}(Q - n\cdot k)Q = \frac{1-x}{x}Q^2 = p'^2.$$

$$= \frac{\alpha_s C_F}{2\pi} \frac{(\mu^2 e^{\gamma})^\varepsilon}{\Gamma(1-\varepsilon)} \frac{1}{Q\bar{n}\cdot p} \int_0^Q \frac{dn\cdot k}{n\cdot k} (\bar{n}\cdot k\, n\cdot k)^{-\varepsilon} \left[(1-\varepsilon)\frac{1-x}{x}Q^2 - 2\bar{n}\cdot k\, n\cdot k\right], \quad n\cdot k = Qz$$

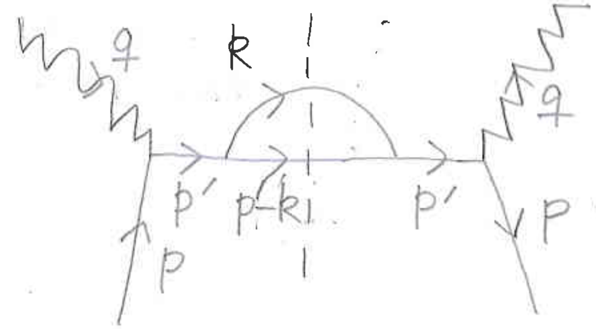
$$= \frac{\alpha_s C_F}{2\pi} \left(\frac{\mu^2 e^{\gamma}}{Q^2}\right)^\varepsilon \frac{x^\varepsilon (1-x)^{-\varepsilon}}{\Gamma(1-\varepsilon)} \int_0^1 \frac{dz}{z} z^{-\varepsilon}(1-z)^{-\varepsilon} \left[(1-\varepsilon)(1-x) - 2x\cdot\frac{1-x}{x}(1-z)z\right].$$

$$= \frac{\alpha_s C_F}{2\pi} \left(\frac{\mu^2 e^{\gamma}}{Q^2}\right)^\varepsilon \frac{x^\varepsilon (1-x)^{1-\varepsilon}}{\Gamma(1-\varepsilon)} \int_0^1 dz\, z^{-1-\varepsilon}(1-z)^{-\varepsilon} \left[(1-\varepsilon) - 2z(1-z)\right]$$

$$M_c = \frac{\alpha_s C_F}{2\pi}\left[(1-x)\left(-\frac{1}{\varepsilon_{\mathrm{IR}}} - \ln\frac{\mu^2}{Q^2} - 1 - \ln\frac{1-x}{x}\right) - 1\right]$$

- Diagram (d)

(c)



$$M_{Rc} = +\frac{1}{D-2}\mu_{\overline{MS}}^{2\epsilon}\int\frac{d^Dk}{(2\pi)^D}\delta((p'-k)^2)$$
$$\otimes \langle p|\bar q\gamma_\perp^{\mu}\frac{+i\not p'}{p'^2}(+ig\gamma^\alpha T^a)(\not p'-\not k)$$
$$\otimes(+ig\gamma_\alpha T^a)\frac{i\not p'}{p'^2}\gamma_\mu^\perp q|p\rangle(-2\pi\delta(k^2))$$

$$= +2\pi\frac{g^2C_F}{D-2}\mu_{\overline{MS}}^{2\epsilon}\int\frac{d^Dk}{(2\pi)^D}\frac{\delta(k^2)\delta(p'^2-2p'\cdot k)}{(p'^2)^2}$$
$$\otimes\frac{1}{2}\mathrm{Tr}\,\not p\gamma_\perp^\mu\not p'\gamma^\alpha(\not p'-\not k)\gamma_\alpha\not p'\gamma_\mu^\perp$$
$$= +(D-2)^2\mathrm{Tr}\,\not p\not p'(\not p'-\not k)\not p' = +(D-2)^2\mathrm{Tr}\,\not p\not q(\not p'-\not k)\not q$$
$$= +4(D-2)^2\left[2p\cdot q\, q\cdot(p'-k)+Q^2p\cdot(q-k)\right]$$

$$= 4\pi g^2C_F\mu_{\overline{MS}}^{2\epsilon}(1-\epsilon)\int\frac{d^Dk}{(2\pi)^D}\frac{\delta(k^2)\delta(p'^2-2p'\cdot k)}{(p'^2)^2} \rightarrow \bar n\cdot k=\frac{1-x}{x}(Q-n\cdot k)$$
$$p'^2=\frac{1-x}{x}Q^2$$
$$\otimes\left[2p\cdot q\,2q\cdot(p+q-k)+Q^2\,2p\cdot(q-k)\right]$$
$$= \frac{Q^2}{x}\left(\frac{Q^2}{x}-2Q^2+Qn\cdot k-Q\bar n\cdot k\right)+Q^2\left(\frac{Q^2}{x}-\frac{Q}{x}n\cdot k\right)$$
$$= \frac{Q^2}{x}\left[\frac{1-x}{x}Q^2-Q\bar n\cdot k\right]=\frac{1-x}{x^2}Q^3\left[Q-(Q-n\cdot k)\right]=\frac{1-x}{x^2}Q^3n\cdot k$$

$$= \frac{\alpha_sC_F}{2\pi}\frac{(\mu^2e^{\gamma_E})^\epsilon}{\Gamma(1-\epsilon)}(1-\epsilon)\int dn\cdot k\,d\bar n\cdot k\,(\vec k_\perp^2)^{-\epsilon}\delta(p'^2-2p'\cdot k)$$
$$\otimes\frac{1-x}{x^2}Q^3n\cdot k\,\frac{1}{\left(\frac{1-x}{x^2}\right)^2Q^4} \qquad n\cdot k\equiv Qz,\ (\vec k_\perp^2)^{-\epsilon}=(\bar n\cdot k\,n\cdot k)^{-\epsilon}$$
$$= x^\epsilon(1-x)^{-\epsilon}z^{-\epsilon}(1-z)^{-\epsilon}\otimes(Q^2)^{-\epsilon}$$
$$= \frac{\alpha_sC_F}{2\pi}\frac{1-\epsilon}{\Gamma(1-\epsilon)}\left(\frac{\mu^2e^{\gamma_E}}{Q^2}\right)^\epsilon x^\epsilon(1-x)^{-1-\epsilon}\int_0^1 dz\,z^{1-\epsilon}(1-z)^{-\epsilon} = \frac{\Gamma(2-\epsilon)\Gamma(1-\epsilon)}{\Gamma(3-2\epsilon)}$$

$$M_d=\frac{\alpha_sC_F}{4\pi}\left[\delta(1-x)\left(-\frac{1}{\varepsilon_{\rm IR}}-\ln\frac{\mu^2}{Q^2}-1\right)+\frac{1}{(1-x)_+}\right]$$

- Final result

$$
\begin{aligned}
F_1^{(1)}(x,Q^2) &= 2\mathrm{Re}(M_a+M_b)+M_c+M_d+2[Z_q^{(1)}+R_q^{(1)}]\delta(1-x) \\
&= \frac{\alpha_s C_F}{2\pi}\Bigg\{\delta(1-x)\Big[-\frac{3}{2}(\frac{1}{\varepsilon_{\rm IR}}+\ln\frac{\mu^2}{Q^2})+\frac{3}{2}-\frac{\pi^2}{3}\Big] \\
&\quad -\frac{1+x^2}{(1-x)_+}\Big(\frac{1}{\varepsilon_{\rm IR}}+\ln\frac{\mu^2}{Q^2}-\ln x\Big)-\frac{1+2x^2}{2(1-x)_+}+2x\Big(\frac{\ln(1-x)}{1-x}\Big)_+\Bigg\}
\end{aligned}
$$

- No UV pole

- The same IR poles as PDF computation

$$
\begin{aligned}
f_{q/q}^{(1)}(x,\mu) &= 2\mathrm{Re}(M_a+M_b)+M_c+(Z_q^{(1)}+R_q^{(1)})\delta(1-x) \\
&= \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\varepsilon_{\rm UV}}-\frac{1}{\varepsilon_{\rm IR}}\Big)\Big[\frac{3}{2}\delta(1-x)+\frac{1+x^2}{(1-x)_+}\Big]
\end{aligned}
$$

- PDF reproduces low energy physics of full QCD

Hard Function at NLO

$$
\begin{aligned}
H^{(1)}(x, Q^2, \mu) &= F_1^{(1)}(x, Q^2) - f_{q/q}^{(1)}(x) \\
&= \frac{\alpha_s C_F}{2\pi} \Big\{ \delta(1-x) \Big[-\frac{3}{2} \ln \frac{\mu^2}{Q^2} + \frac{3}{2} - \frac{\pi^2}{3} \Big] \\
&\quad - \frac{1+x^2}{(1-x)_+} \Big(\ln \frac{\mu^2}{Q^2} - \ln x \Big) - \frac{1+2x^2}{2(1-x)_+} + 2x \Big(\frac{\ln(1-x)}{1-x} \Big)_+ \Big\}
\end{aligned}
$$

- Hard function is the Wilson coefficient of low energy EFT

- Anomalous dimension of the hard function

$$
\frac{d}{d \ln \mu} H(x, Q^2, \mu) = \int_x^1 \frac{dz}{z} \gamma_H(z, \mu) H(\frac{x}{z}, Q^2, \mu)
$$

$$
\gamma_H(x, \mu) = \frac{\partial}{\partial \ln \mu} H(x, Q^2, \mu) = \frac{\alpha_s C_F}{\pi} \Big[-\frac{3}{2} \delta(1-x) - \frac{1+x^2}{(1-x)_+} \Big] = -\gamma_f(x, \mu)
$$

Structure function is scale invariant