Perturbative QCD

Part II : Structure functions for DY
One loop correction to current

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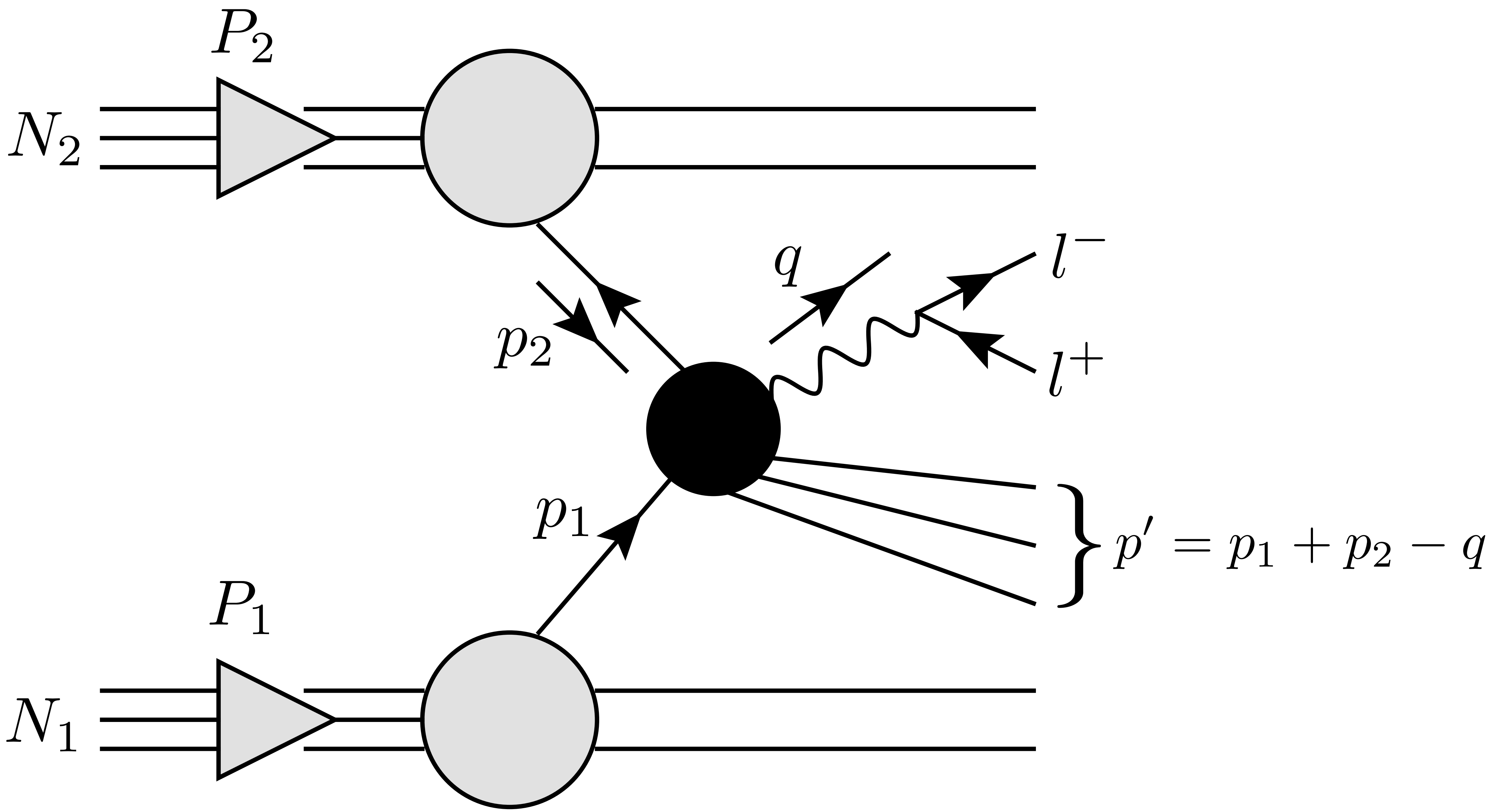
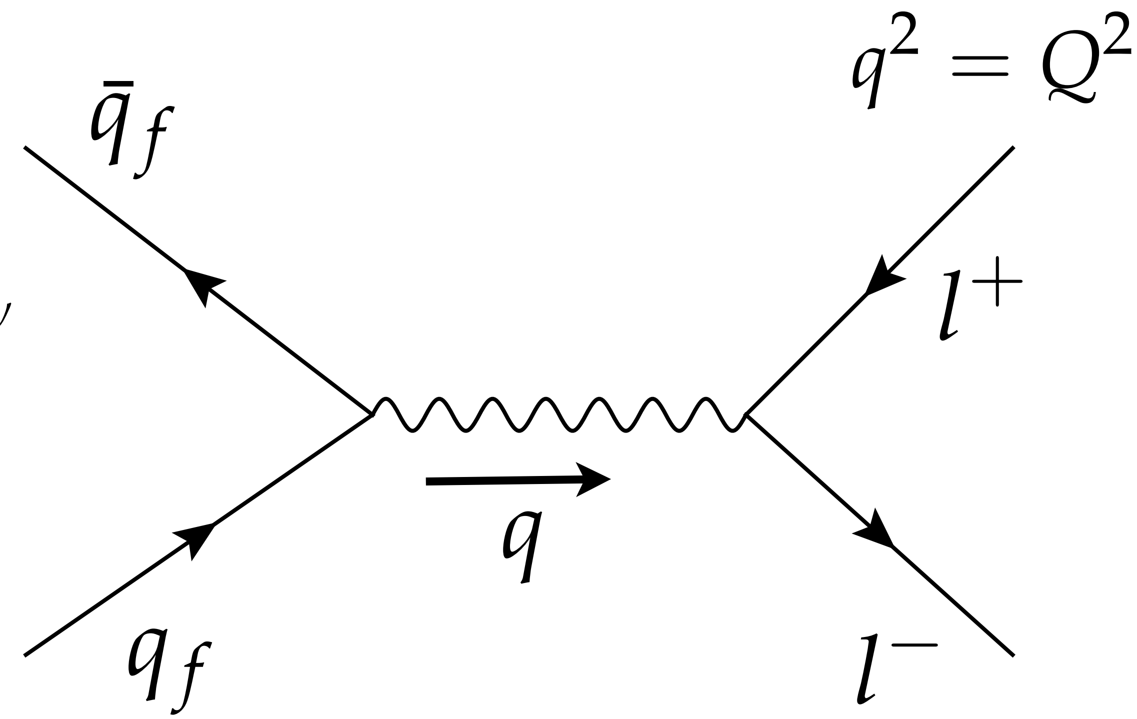
Drell-Yan Process



▪ Scattering Cross Section



$$i\mathcal{M} = \langle l^+ l^- X | (iQ_f e) \bar{q}_f \gamma^\mu q_f \cdot (ie) \bar{l} \gamma_\mu l | N_1 N_2 \rangle \frac{-i}{q^2}$$

$$|\mathcal{M}|^2 = \frac{e^4 Q_f^2}{4Q^4} \langle N_1 N_2 | J_\mu^\dagger | X \rangle \langle X | J_\nu | N_1 N_2 \rangle L^{\mu\nu}$$

$$\begin{aligned} L^{\mu\nu}(k_1, k_2) &= \frac{1}{4} \sum_{s,s'} \bar{u}_s(k_1) \gamma^\nu v_{s'}(k_2) \bar{v}_{s'}(k_2) \gamma^\mu u_s(k_1) \\ &= \frac{1}{4} \mathrm{Tr} \not{k}_1 \gamma^\nu \not{k}_2 \gamma^\mu = k_1^\mu k_2^\nu - k_1^\nu k_2^\mu - g^{\mu\nu} k_1 \cdot k_2 \end{aligned}$$

$$\begin{aligned} \sigma(N_1 N_2 \to l^+ l^- X) &= \frac{1}{2s} \sum_X \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{1}{2k_1^0 \cdot 2k_2^0} (2\pi)^4 \delta(P_1 + P_2 - q - p_X) \sum_f |\mathcal{M}|^2 \qquad s = (P_1 + P_2)^2 \\ &= \frac{32\pi^2 \alpha^2}{Q^4 s} \sum_f Q_f^2 W_{\mu\nu}(P_1, P_2, q) \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{L^{\mu\nu}(k_1, k_2)}{2k_1^0 \cdot 2k_2^0} \\ &= \frac{32\pi^2 \alpha^2}{Q^4 s} \sum_f Q_f^2 \int d^4q \, W_{\mu\nu}(P_1, P_2, q) \boxed{\int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{L^{\mu\nu}(k_1, k_2)}{2k_1^0 \cdot 2k_2^0} \delta(q - k_1 - k_2)}. \end{aligned}$$

Homework:

$$\begin{aligned} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{L^{\mu\nu}(k_1, k_2)}{2k_1^0 \cdot 2k_2^0} \delta(q - k_1 - k_2) &= -\frac{\pi}{6} (-q^2 g^{\mu\nu} + q^\mu q^\nu) \\ &\longrightarrow \int dQ^2 \delta(q^2 - Q^2) \left(-\frac{\pi}{6} Q^2 g^{\mu\nu} \right). \end{aligned}$$

$q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

- Hadronic tensor

$$W_{\mu\nu}(P_1,P_2,q) = \sum_X (2\pi)^4 \delta(P_1+P_2-q-p_X)\langle N_1 N_2 | J_\mu^\dagger | X\rangle \langle X | J_\nu | N_1 N_2\rangle$$

- Structure function

$$\frac{d\sigma}{dQ^2} = \sum_f Q_f^2 \frac{4\pi\alpha^2}{3Q^2 s}\int \frac{d^4q}{(2\pi)^4}\delta(q^2-Q^2)(-g^{\mu\nu})W_{\mu\nu}(P_1,P_2,q) = \frac{\sigma_0}{s}F_{DY}(\tau,Q^2)$$

LO x-section at parton level: $\sigma_0 = \sum_f Q_f^2 \frac{4\pi\alpha^2}{3N_c Q^2}$ **Homework**

$$\begin{aligned} F_{DY}(\tau,Q^2) &= -N_c \sum_X \int d^4q\,\delta(q^2-Q^2)\delta(P_1+P_2-q-p_X)\langle N_1N_2|J_\mu^\dagger|X\rangle\langle X|J^\mu|N_1N_2\rangle \\ &= -N_c\int\frac{d^4q}{(2\pi)^4}\delta(q^2-Q^2)\int d^4z e^{-iq\cdot z}\underline{\langle N_1N_2|J_\mu^\dagger(z)J^\mu(0)|N_1N_2\rangle}\end{aligned}$$

will perform short distance expansion

- Matching of EM current

$$J^\mu = \bar{q}\gamma^\mu q \to \underline{\bar{q}_{\bar{n}}W_{\bar{n}}\gamma_\perp^\mu W_n^\dagger q_n}$$

$$\not{n}\, q_n = \mathcal{O}(\lambda),\ \ \not{\bar{n}}\, q_{\bar{n}} = \mathcal{O}(\lambda)$$

Derivation of FT Theorem

$$
\begin{aligned}
F_{DY}(\tau) &= -N_c \delta(q^2 - Q^2) \langle N_1 N_2 | \bar{q}_n W_n \gamma_\perp^\mu W_{\bar{n}}^\dagger q_{\bar{n}} \cdot \bar{q}_{\bar{n}} W_{\bar{n}} \gamma_\mu^\perp W_n^\dagger q_n | N_1 N_2 \rangle \\
&= -N_c \int dy_1 dy_2 \delta(q^2 - Q^2) \langle N_1 N_2 | \bar{q}_n W_n \gamma_\perp^\mu W_{\bar{n}}^\dagger q_{\bar{n}} \\
&\quad \times \left[\bar{q}_{\bar{n}} W_{\bar{n}} \delta(y_2 + \frac{n \cdot \mathcal{P}^\dagger}{n \cdot P_2}) \right] \gamma_\mu^\perp \left[\delta(y_1 - \frac{\bar{n} \cdot \mathcal{P}}{\bar{n} \cdot P_1}) W_n^\dagger q_n \right] | N_1 N_2 \rangle
\end{aligned}
$$

- PDF projection

$$
\langle N | \left[\delta(y - \frac{\bar{n} \cdot \mathcal{P}}{\bar{n} \cdot P}) \Psi_n \right]_a^\alpha \left(\bar{\Psi}_n \right)_b^\beta | N \rangle = \frac{\bar{n} \cdot P}{2N_c} \delta^{\alpha\beta} \left(\frac{\not{n}}{2} \right)_{ab} f_{q/N}(x), \; \Psi_n = W_n^\dagger q_n
$$

$$
\langle N | \left(\Psi_{\bar{n}} \right)_a^\alpha \left[\bar{\Psi}_{\bar{n}} \delta(y + \frac{n \cdot \mathcal{P}^\dagger}{n \cdot P}) \right]_b^\beta | N \rangle = \frac{n \cdot P}{2N_c} \delta^{\alpha\beta} \left(\frac{\not{\bar{n}}}{2} \right)_{ab} f_{\bar{q}/N}(y)
$$

$$
\langle N_1 N_2 | \bar{q}_n W_n \gamma_\perp^\mu W_{\bar{n}}^\dagger q_{\bar{n}} \cdot \left[\bar{q}_{\bar{n}} W_{\bar{n}} \delta(y_2 + \frac{n \cdot \mathcal{P}^\dagger}{n \cdot P_2}) \right] \gamma_\mu^\perp \left[\delta(y_1 - \frac{\bar{n} \cdot \mathcal{P}}{\bar{n} \cdot P_1}) W_n^\dagger q_n \right] | N_1 N_2 \rangle
$$

$$
= \frac{s}{4N_c^2} N_c f_{q/N_1}(y_1) f_{q/N_2}(y_2) \mathrm{Tr} \frac{\not{n}}{2} \gamma_\perp^\mu \frac{\not{\bar{n}}}{2} \gamma_\mu^\perp = -\frac{s}{N_c} f_{q/N_1}(y_1) f_{q/N_2}(y_2)
$$

▪ LO factorization theorem

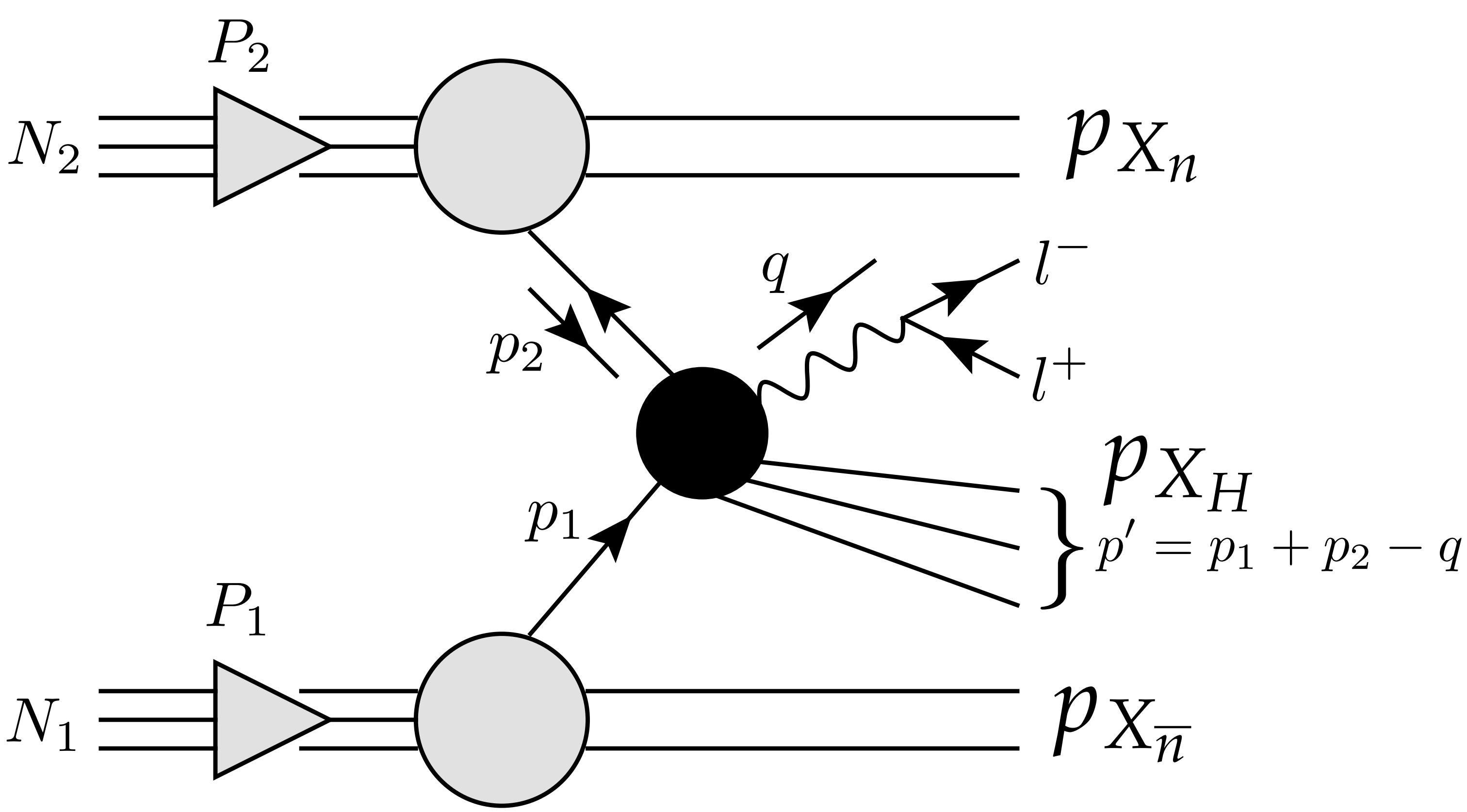
$$
\begin{aligned}
F_{DY}(\tau) &= s\int dy_1 dy_2 f_{q/N_1}(y_1) f_{\bar{q}/N_2}(y_2)\delta(q^2-Q^2) \\
&= \int \frac{dy_1}{y_1}\frac{dy_2}{y_2} f_{q/N_1}(y_1) f_{\bar{q}/N_2}(y_2)\delta(1-z) = \int_\tau^1 \frac{dz}{z}\delta(1-z) L_{q\bar{q}}(\frac{\tau}{z})
\end{aligned}
$$

$$
L_{q\bar{q}}(\tau) = \int_\tau^1 \frac{dy_1}{y_1} f_{q/N_1}(y) f_{\bar{q}/N_2}(\frac{\tau}{z y_1})
$$

- Kinematic variables

$$
q = P_1 + P_2 - p_X = (P_1 - p_{X_n}) + (P_2 - p_{X_{\overline{n}}}) + p_{X_H} = p_1 + p_2 - p_{X_H}
$$

$$
q^2 - Q^2 = \hat{s} - 2p_{X_H}\cdot(p_1+p_2) + p_{X_H}^2 - Q^2
$$



$$
\hat{s} = (p_1+p_2)^2 = \overline{n}\cdot p_1 n\cdot p_2 = y_1 y_2 s
$$

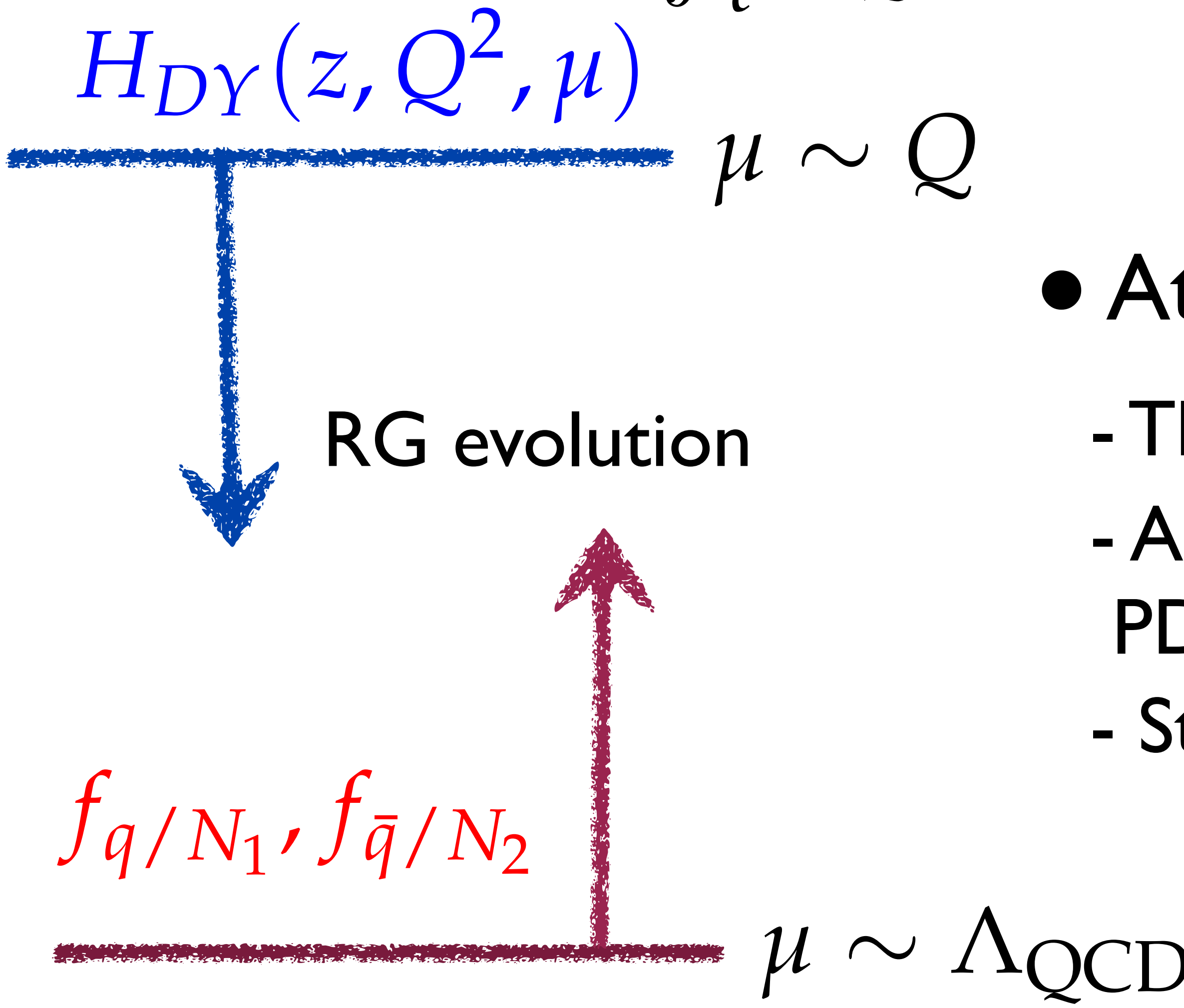
$$
z \equiv \frac{Q^2}{\hat{s}}, \quad \tau \equiv \frac{Q^2}{s}
$$

At LO,

$$
p_{X_H} = 0 \to q^2 - Q^2 = \hat{s}(1-z)
$$

▪ Factorization theorem

$$F_{DY}(\tau) = \int_\tau^1 \frac{dz}{z} \boxed{H_{DY}(z, Q^2, \mu)} \boxed{L_{q\bar{q}}(\frac{\tau}{z}, \mu)}$$



- At the higher order
  - The hard function should be IR-finite
  - All the IR divergence should reside in PDFs
  - Structure fn should be scale invariant

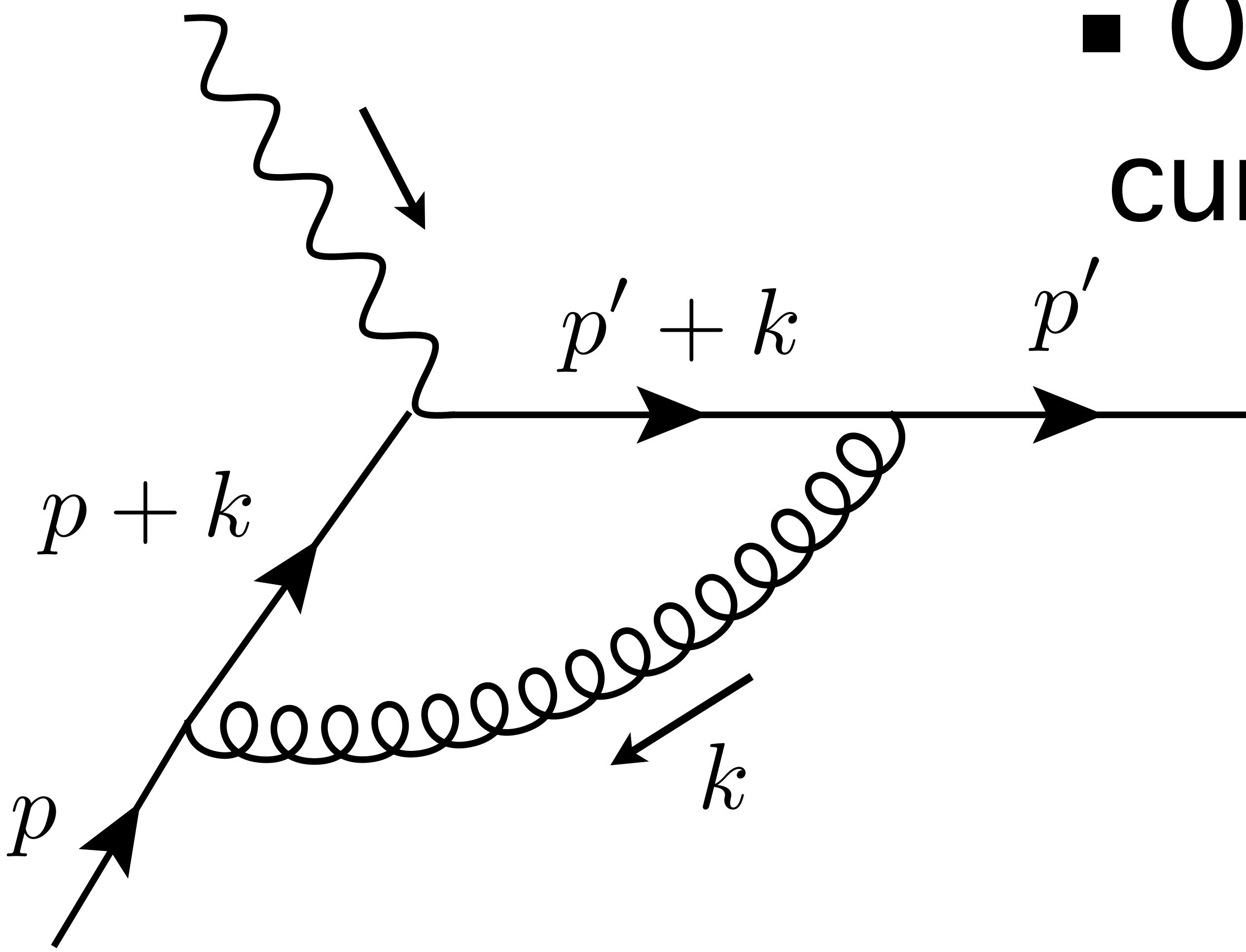
※ LO structure function at parton level

$$H_{DY}^{(0)} = \delta(1-z)$$
$$f_{q/q}^{(0)}(y) = f_{\bar{q}/\bar{q}}^{(0)}(y) = \delta(1-y)$$

$$\Longrightarrow \quad F_{DY}^{(0)}(\tau) = \delta(1-\tau)$$

One loop virtual computation in full QCD

- One loop correction to current



$$\bar{M}_V \bar{q}\gamma^\mu q = -ig^2 C_F \mu^{2\varepsilon}_{\overline{MS}} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2(k+2p\cdot k)(k+2p'\cdot k)} \bar{q}\gamma^\nu(\not{k}+\not{p}')\gamma^\mu(\not{k}+\not{p})\gamma_\nu q$$

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2(k+2p\cdot k)(k+2p'\cdot k)} = 2\int_0^1 d\alpha \int_{1-\alpha}^1 d\beta \int \frac{d^D l}{(2\pi)^D} \frac{1}{(l^2-\Delta)^3}$$

$$l = k + \alpha p + \beta p'$$

$$\Delta = \alpha\beta M^2, \;\; M^2 \equiv 2p\cdot p' = 2p\cdot q$$

- Numerator

$$\bar{q}\gamma^\nu(\not{k}+\not{p}')\gamma^\mu(\not{k}+\not{p})\gamma_\nu q = \Big[\frac{(D-2)^2}{D}l^2 + 2M^2(1-\alpha-\beta+(1-\varepsilon)\alpha\beta)\Big]\bar{q}\gamma^\mu q$$

$$\begin{aligned}
\gamma_\mu\gamma^\mu &= D, \gamma_\mu\gamma^\alpha\gamma^\mu = (2-D)\gamma^\alpha,\ \gamma_\mu\gamma_5\gamma^\mu = -D\gamma_5, \\
\gamma^\mu\gamma^\alpha\gamma^\beta\gamma_\mu &= 4g^{\alpha\beta} + (D-4)\gamma^\alpha\gamma^\beta, \\
\gamma^\mu\gamma^\alpha\gamma^\beta\gamma^\delta\gamma_\mu &= -2\gamma^\delta\gamma^\beta\gamma^\alpha + (4-D)\gamma^\alpha\gamma^\beta\gamma^\delta.
\end{aligned}$$

- Loop integrals

In D-dimension, after Wick rotation such as $l^0 = il_E^0$, $l^2 = -l_E^2$, we have

$$\int \frac{d^D l_E}{(2\pi)^D} = \int \frac{d\Omega_D}{(2\pi)^D}\int_0^\infty dl_E l_E^{D-1}, \quad \int d\Omega_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}. \tag{1}$$

Therefore Feynman integrals becomes

$$\int \frac{d^D l_E}{(2\pi)^D}\frac{l_E^{2m}}{(l_E^2+\Delta)^n} = \frac{1}{(4\pi)^{D/2}}\frac{\Gamma(m+D/2)}{\Gamma(D/2)}\frac{\Gamma(n-m-D/2)}{\Gamma(n)}\Big(\frac{1}{\Delta}\Big)^{n-D/2-m}. \tag{2}$$

Usually $D = 4-2\epsilon$, and Gamma function can be expanded in powers of ϵ such as

$$\Gamma(1+\epsilon) = \epsilon\Gamma(\epsilon) = 1 - \gamma_E\epsilon + \frac{1}{2}(\gamma_E^2 + \frac{\pi^2}{6})\epsilon^2 + \cdots. \tag{3}$$

- Result at one loop

$$\Delta' = \Delta/M^2 = \alpha\beta$$

$$\begin{aligned}\bar{M}_V &= \frac{\alpha_s C_F}{4\pi}\left(\frac{\mu^2 e^\gamma}{M^2}\right)^\varepsilon \int_0^1 d\alpha \int_{1-\alpha}^1 d\beta \Big[2(1-2\varepsilon)\Gamma(\varepsilon)\Delta'^{-\varepsilon} \\ &\quad - 2\Gamma(1+\varepsilon)\Delta'^{-1-\varepsilon}(1-\alpha-\beta+(1-\varepsilon)\alpha\beta)\Big] \\ &= \frac{\alpha_s C_F}{2\pi}\left[\frac{1}{2\,\varepsilon_{\rm UV}} - \frac{1}{\varepsilon_{\rm IR}^2} - \frac{1}{\varepsilon_{\rm IR}}\left(2+\ln\frac{\mu^2}{M^2}\right) - 4 + \frac{\pi^2}{12} - \frac{3}{2}\ln\frac{\mu^2}{M^2} - \frac{1}{2}\ln^2\frac{\mu^2}{M^2}\right].\end{aligned}$$

Cancelled by field strength renormalization of quark fields

- Double pole (logarithm) results from Sudakov effect
- For totally inclusive process, all the IR divergences cancel, but....

- Can be matched onto EFT (SCET), Remaining finite terms gives the hard corrections